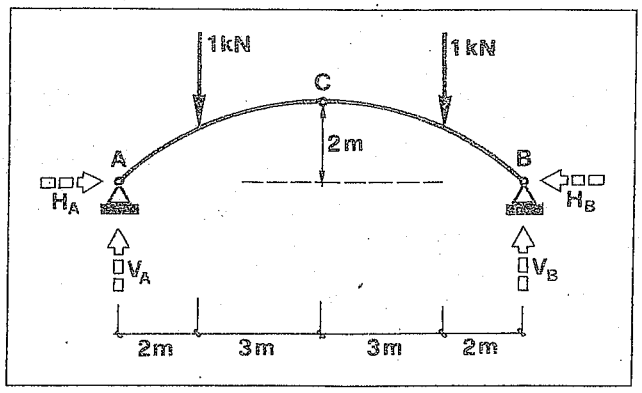
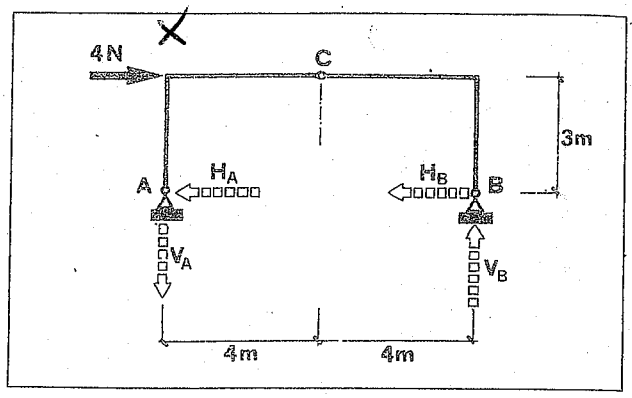
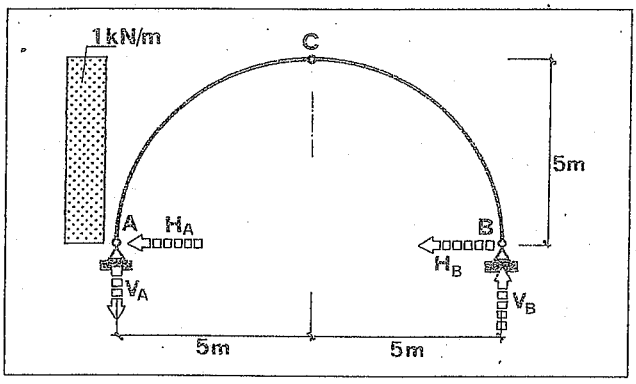


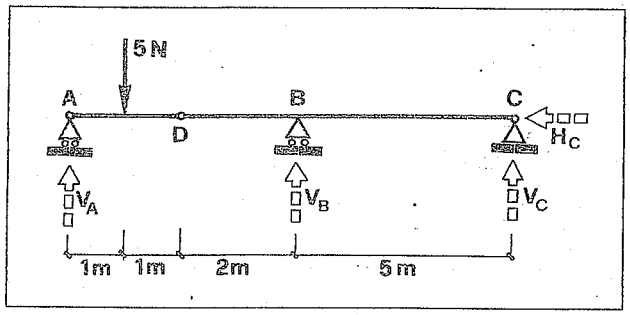
[Risp.: $V_A = V_B = 1$ kN; $H_A = H_B = 1$ kN]



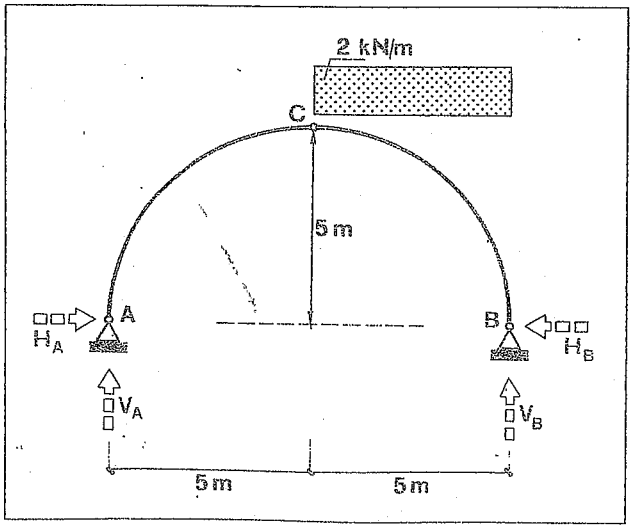
[Risp.: $H_A = H_B = 2$N; $V_A = V_B = 1{,}5$ N]



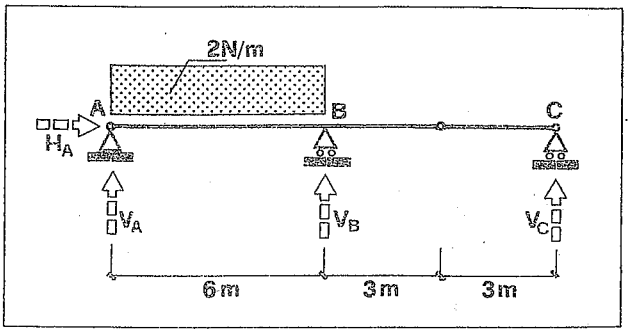
[Risp.: $H_A = 3{,}75$ kN; $H_B = 1{,}25$ kN; $V_A = 2{,}5$ kN; $V_B = 2{,}5$ kN]



[Risp.: $V_A = 2{,}5$ N; $V_B = 3{,}5$ N; $V_C = -1$ N; $H_C = 0$]



[Risp.: $H_A = H_B = 2{,}5$ kN; $V_A = 2{,}5$ kN; $V_B = 7{,}5$ kN]



[Risp.: $H_A = 0$; $V_A = 6$ N; $V_B = 6$ N; $V_C = 0$]

METODO DELLE TENSIONI AMMISSIBILI

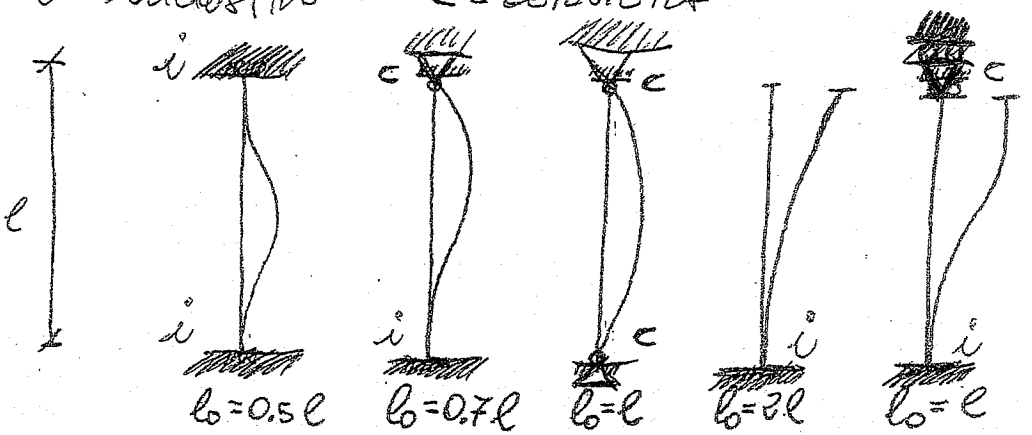
$$\lambda = \frac{l_0}{\rho_{(MINIMO)}}$$ SNELLEZZA (λ = lambda)

l_0 = lunghezza libera di inflessione

i = incastro - c = cerniera

CARICO DI PUNTA

Sollecitazione di compressione



METODO ω — METODO OMEGA

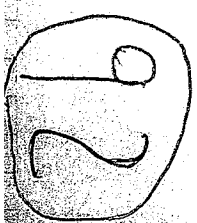
ω = FATTORE DI AMPLIFICAZIONE DI N

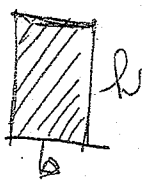
VERIFICA

$$N_{RES} \geq (\omega \cdot N)$$

ρ RAGGIO DI INERZIA MINIMO DELLA SEZIONE

$$\rho = \sqrt{\frac{I}{A}}$$





$$\rho = \sqrt{\frac{b \cdot h^3}{12} \cdot \frac{1}{b \cdot h}} = \frac{h}{\sqrt{12}}$$

SEZ. RETT.

$$\rho_x = \frac{a}{\sqrt{12}} \qquad \rho_y = \frac{b}{\sqrt{12}}$$

λ	ω
50	1.00
70	1.08
85	1.32
100	1.62

$\lambda < 50 \Rightarrow \omega = 1.00$

$\lambda > 100 \Rightarrow$ TROPPO PERICOLOSO

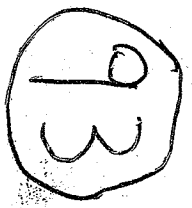
Tabella dei coefficienti ω in funzione della snellezza λ

PER IL CALCESTRUZZO

TABELLA 9.2

λ	0	1	2	3	4	5	6	7	8	9
50	1,00	1,00	1,01	1,01	1,02	1,02	1,02	1,03	1,03	1,04
60	1,04	1,04	1,05	1,05	1,06	1,06	1,06	1,07	1,07	1,08
70	1,08	1,10	1,11	1,13	1,14	1,16	1,18	1,19	1,21	1,22
80	1,24	1,26	1,27	1,29	1,30	1,32	1,34	1,36	1,38	1,40
90	1,42	1,44	1,46	1,48	1,50	1,52	1,54	1,56	1,58	1,60
100	1,62	1,64	1,66	1,68	1,70	1,72	1,76	1,79	1,83	1,87
110	1,91	1,94	1,98	2,02	2,06	2,09	2,13	2,17	2,21	2,24
120	2,28	2,32	2,35	2,39	2,42	2,46	2,50	2,53	2,57	2,60
130	2,64	2,68	2,71	2,75	2,78	2,82	2,86	2,89	2,93	2,92
140	3,00									

W = COMPRESSIONE

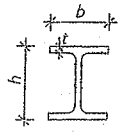


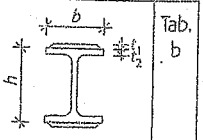
La normativa prescrive che i fenomeni di instabilità (ossia la verifica a carico di punta) devono essere presi in considerazione solo per snellezze λ > di 50.

La normativa prescrive in oltre che snellezze maggiori di 100 sono da considerare con particolari cautele di progettazione e di calcolo.

Acciai — Coefficienti ω per acciaio S235 (ex Fe 360) (CNR-UNI 10011)

ACCIAI S235 / S275

Aste semplici	Profili cavi quadri, rettangoli o tondi saldati o laminati $t \le 40$ mm	Tab. a
Aste semplici	Laminati $\frac{h}{b} \ge 1{,}2$ $t \le 40$ 	Tab. b

Aste semplici	Laminati rinforzati con piatti saldati $t_1 \le 40$ mm $t_2 \le 40$ mm 	Tab. b
Aste semplici	Sezione chiusa, a cassone, saldata $t \le 40$ 	Tab. b

Aste semplici o composte	Sezione di forma generica	$t \le 40$ mm	Tab. c
	Tutte le forme di sezione	$t > 40$ mm	Tab. d

λ	0	1	2	3	4	5	6	7	8	9
0	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
10	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
20	1,00	1,01	1,01	1,01	1,01	1,01	1,02	1,02	1,02	1,03
30	1,03	1,03	1,03	1,04	1,04	1,04	1,05	1,05	1,05	1,06
40	1,06	1,06	1,07	1,07	1,07	1,08	1,08	1,09	1,09	1,10
50	1,10	1,11	1,11	1,12	1,12	1,13	1,13	1,14	1,14	1,15
60	1,16	1,16	1,17	1,17	1,18	1,18	1,19	1,20	1,20	1,21
70	1,22	1,23	1,24	1,24	1,25	1,26	1,27	1,28	1,29	1,30
80	1,31	1,32	1,33	1,34	1,36	1,37	1,38	1,40	1,41	1,42
90	1,44	1,45	1,47	1,48	1,50	1,52	1,53	1,55	1,57	1,59
100	1,61	1,63	1,65	1,67	1,69	1,71	1,73	1,75	1,77	1,79
110	1,82	1,84	1,86	1,89	1,91	1,94	1,96	1,99	2,01	2,04
120	2,06	2,09	2,12	2,14	2,17	2,20	2,22	2,25	2,28	2,31
130	2,34	2,37	2,40	2,43	2,46	2,49	2,52	2,55	2,58	2,61
140	2,65	2,68	2,71	2,74	2,78	2,81	2,84	2,88	2,91	2,95
150	2,98	3,02	3,05	3,08	3,12	3,16	3,19	3,23	3,27	3,30
160	3,34	3,38	3,41	3,45	3,49	3,53	3,56	3,60	3,64	3,68
170	3,72	3,76	3,80	3,84	3,88	3,92	3,96	4,01	4,05	4,09
180	4,14	4,18	4,22	4,27	4,31	4,35	4,40	4,44	4,49	4,53
190	4,58	4,62	4,67	4,72	4,77	4,81	4,85	4,90	4,94	4,99
200	5,03	5,08	5,13	5,18	5,22	5,27	5,32	5,37	5,42	5,47

λ	0	1	2	3	4	5	6	7	8	9
0	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
10	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
20	1,01	1,01	1,01	1,02	1,02	1,02	1,03	1,03	1,04	1,04
30	1,05	1,05	1,06	1,06	1,07	1,07	1,08	1,08	1,09	1,09
40	1,10	1,10	1,11	1,11	1,12	1,12	1,13	1,13	1,14	1,15
50	1,15	1,16	1,17	1,17	1,18	1,19	1,20	1,20	1,21	1,22
60	1,23	1,24	1,25	1,25	1,26	1,27	1,28	1,29	1,30	1,31
70	1,33	1,34	1,35	1,36	1,37	1,38	1,40	1,41	1,42	1,44
80	1,45	1,47	1,48	1,50	1,51	1,53	1,55	1,56	1,58	1,60
90	1,62	1,63	1,65	1,67	1,69	1,71	1,73	1,75	1,77	1,79
100	1,81	1,83	1,86	1,88	1,90	1,92	1,95	1,97	1,99	2,02
110	2,04	2,07	2,09	2,12	2,14	2,17	2,20	2,23	2,25	2,28
120	2,31	2,34	2,37	2,40	2,42	2,45	2,48	2,51	2,54	2,58
130	2,61	2,64	2,67	2,70	2,74	2,77	2,80	2,84	2,87	2,91
140	2,94	2,97	3,01	3,04	3,08	3,11	3,15	3,19	3,22	3,26
150	3,30	3,34	3,37	3,41	3,45	3,49	3,53	3,57	3,62	3,66
160	3,70	3,74	3,79	3,83	3,86	3,90	3,94	3,99	4,03	4,07
170	4,11	4,15	4,20	4,24	4,28	4,33	4,37	4,42	4,47	4,51
180	4,56	4,60	4,65	4,70	4,74	4,79	4,84	4,88	4,93	4,98
190	5,02	5,07	5,12	5,16	5,21	5,26	5,31	5,36	5,41	5,46
200	5,51	5,56	5,60	5,65	5,70	5,76	5,81	5,87	5,92	5,97

CAMPO IN PUNTA MATERIALE LEGNO ω

Legno — Coefficienti ω per legno lamellare e per legno massiccio (DIN 1052)

λ	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
LAMELLARE	1,00	1,00	1,00	1,00	1,03	1,13	1,28	1,51	1,92	2,43	3,00	3,63	4,32	5,07	5,88	6,75	7,68	8,67	9,72	10,83
MASSICCIO	1,00	1,04	1,08	1,15	1,26	1,42	1,62	1,88	2,20	2,58	3,00	3,63	4,32	5,07	5,88	6,75	7,68	8,67	9,72	10,83

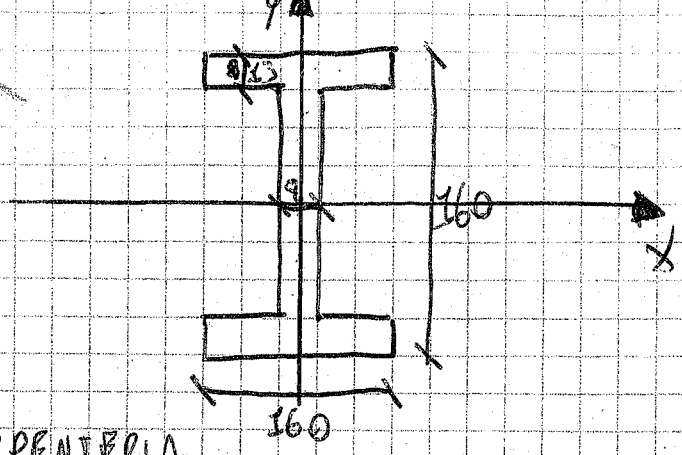
INTERPOLAZIONE LINEARE

ESEMPIO = $\lambda = 36 \Rightarrow \omega = ?$

$$X = \frac{(\omega_{MAG} - \omega_{MIN}) \cdot (\lambda_{calc.} - \lambda_{MIN})}{(\lambda_{MAG} - \lambda_{MIN})}$$

ω_{FINALE} PER $\lambda = 36$ $\Rightarrow \omega_{MINIMO} + X$

4 ESERCIZIO



$HB = 160\ mm$

$H = 6,00\ ml$

$P = 10\ KN = N$

$\sigma = \frac{P}{A} \leq \sigma_{AMMISSIBILE}$

ACCIAIO DA CARPENTERIA

S235 → $\sigma_{AMMISSIBILE} = \frac{\sigma_{SNERVAMENTO}}{\gamma_{materiale}} = \frac{235}{1,1} = 213,64\ N/mm^2$

$\sigma = \frac{10000}{5430} = 1,8\ N/mm^2$ $\quad A = 54,3\ cm$

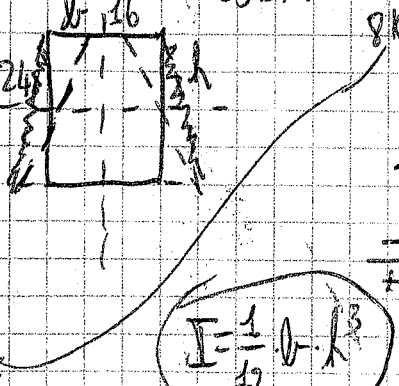
$\lambda = \frac{l_0}{\rho} = \frac{0,7 \cdot l_{cm}}{\sqrt{\frac{I_{MIN}}{A_{cm}}}} = \frac{4,2}{4,04} = 103,80$

$I_x = 2492\ cm^4$ $\quad W = 1,90$

$I_y = 889\ cm^4$

$\sigma = \frac{10000 \cdot 1,90}{5430} = \frac{19000}{5430} = 3,5\ N/mm^2$

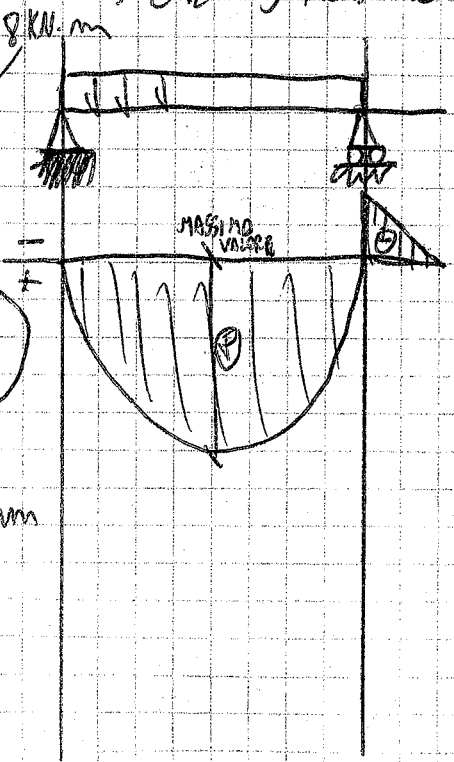
ESEMPIO FLESSIONE LEGNO



$I = \frac{1}{12} \cdot b \cdot l^3$

$8 \cdot 10^3$.

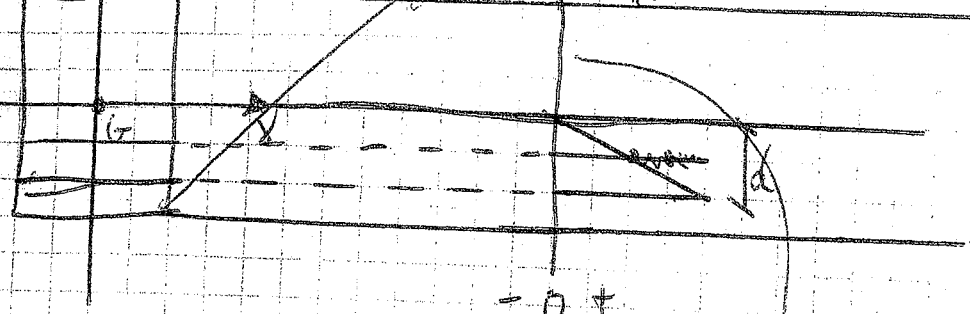
$8\ KN \cdot m = 8 \cdot 10^6\ N \cdot mm$



$\sigma_{AMMISSIBILE} = 10\ N/mm^2$

$\frac{b}{l} = 0,7$

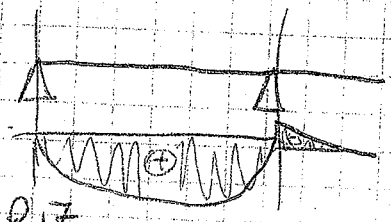
$\sigma_f = \frac{M}{I} \cdot Y\ N/mm^2$



$\frac{24}{0,7}$

$\sigma_{f\oplus} < \sigma_{AMMISSIBILE}$

$\sigma_f = \frac{M}{I} \cdot y$



$\frac{b}{h} = 0,7$

$M = 122 KN \cdot m$

$A = 16 \times 24\ cm$

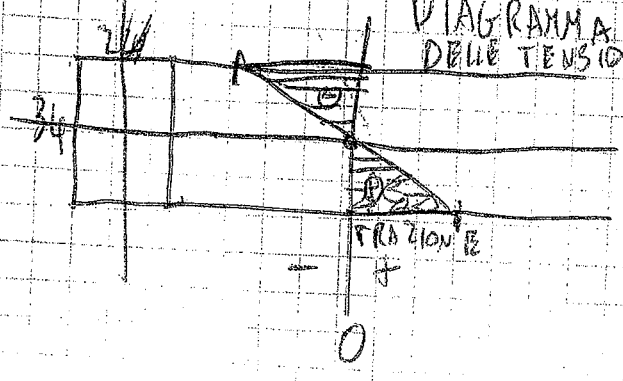
$\sigma_{AMMISSIBILE} = 100\ N/mm^2$

$I = \frac{1}{12} b \cdot h^3 = \frac{1}{12} \cdot 160 \cdot 240^3 = 184320000\ cm^4$

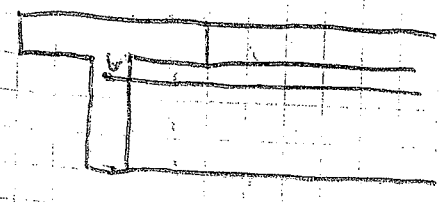
$\sigma = \frac{122 \cdot 10^6}{184320000} \cdot 120 = 79,3\ N/m$

NON È VERIFICATO
SI RIPROGETTA

$\sigma = \frac{122 \cdot 10^6}{\frac{1}{12} \cdot 240 \cdot 340^3} \cdot 170 = 26,31$



$\sigma = \frac{122 \cdot 10^6}{\frac{1}{12} \cdot 180 \cdot 400^3} = 16\ N/mm^2$



W_x = modulo di resistenza $= \frac{I_x}{y}$

$W_y = \frac{I_y}{y}$

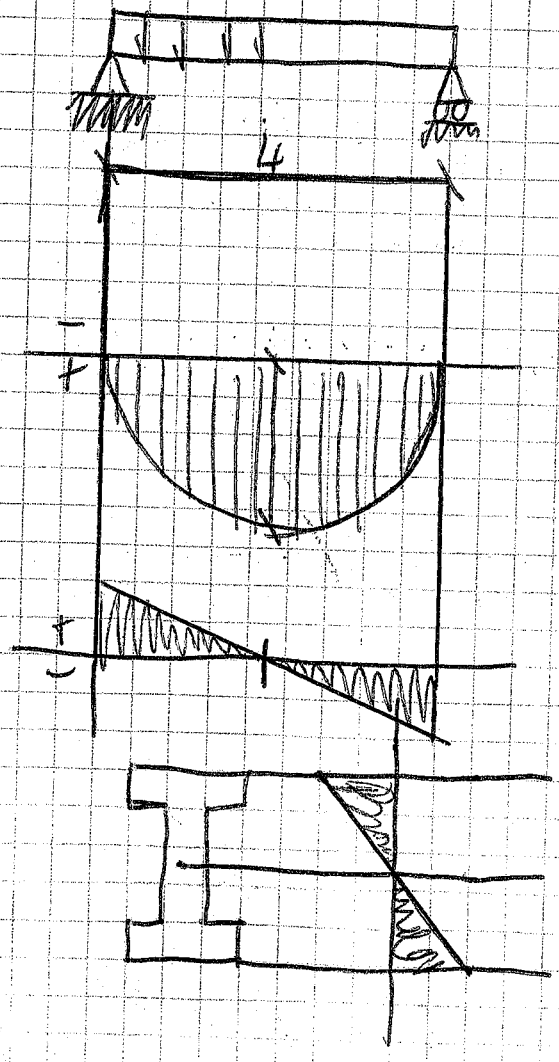
$\sigma_f = \frac{M}{W}$

$W = \frac{1}{6} \cdot \frac{B \cdot H^3}{H} \cdot 2 = \frac{1}{6} B \cdot h^2 = cm^3$

$I = \frac{BH^3 - bh^3}{12}$

$W = \frac{BH^3 - bh^3}{6H} \cdot 2$

$b/2 + b/2 = b$



$q \cdot l = 5\ KN/ml$

FERRO S235

$I = 160$

$\sigma_{AMMISSIBILE} = \frac{235}{1,05} = 223,$

$M = \frac{1}{8} \cdot q \cdot l^2 = \frac{1}{8} \cdot 5 \cdot 4^2 = 10\ KN \cdot m = 10 \cdot 10^6\ N_{/m}$

$\sigma_f = \frac{M}{I} \cdot y = \frac{10 \cdot 10^6}{869 \cdot 10^4} \cdot 80 = 92,05 < \sigma_{AMM}$

VERIFICATO

$T = \frac{1}{2} q \cdot l = 10\ KN$

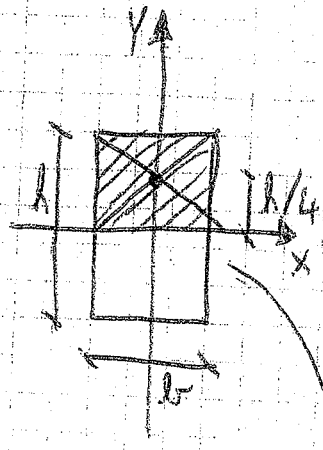
$\tau = t_{xy} = \frac{T \cdot S}{I \cdot b}$ = momento statico della sezione / larghezza trave

$\boxed{\tau = \frac{T \cdot S}{I \cdot b}}$

sez rettangolare

$\tau = \frac{3}{2} \cdot \frac{T}{A}$

$$\tau = \frac{T \cdot S}{I \cdot b}$$



$$S = A \cdot d$$

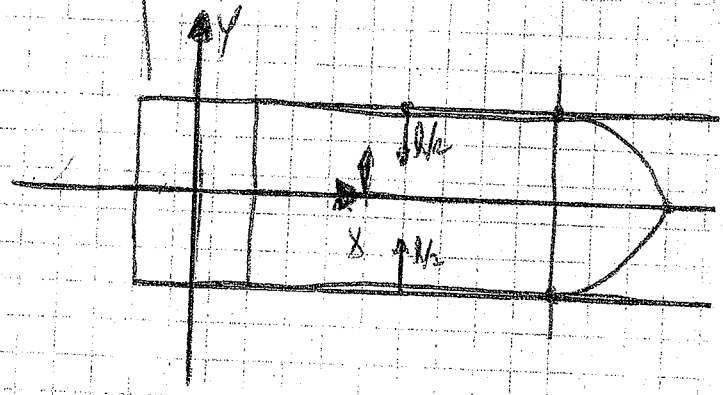
$$S_x = \left(b \cdot \frac{h}{2}\right) \cdot \frac{h}{4} =$$

$$I = \frac{1}{12} \cdot b \cdot h^3$$

$$\tau = \frac{T \cdot b \cdot h^2 \cdot 12^3}{28 \cdot h \cdot h^2 \cdot b} =$$

$$= \frac{3}{2} \cdot \frac{T}{A}$$ TRAVE RETTANGOLARE

DIAGRAMMA T = TAGLIO



TRAVE DI FERRO

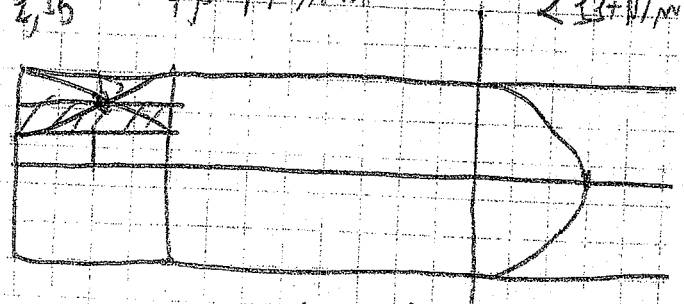
$$S = \frac{1}{8} \cdot b \cdot h^2$$

$T = 10\,kN$ $\quad \tau = 0{,}577 \cdot \sigma_{AMMISSIBILE} = 117\,N/mm^2$

$b = 20\,cm$ $\quad S235$ $\quad \sigma_{AMMISSIBILE} = \frac{235}{1{,}15} = 204{,}34\,N/mm^2$

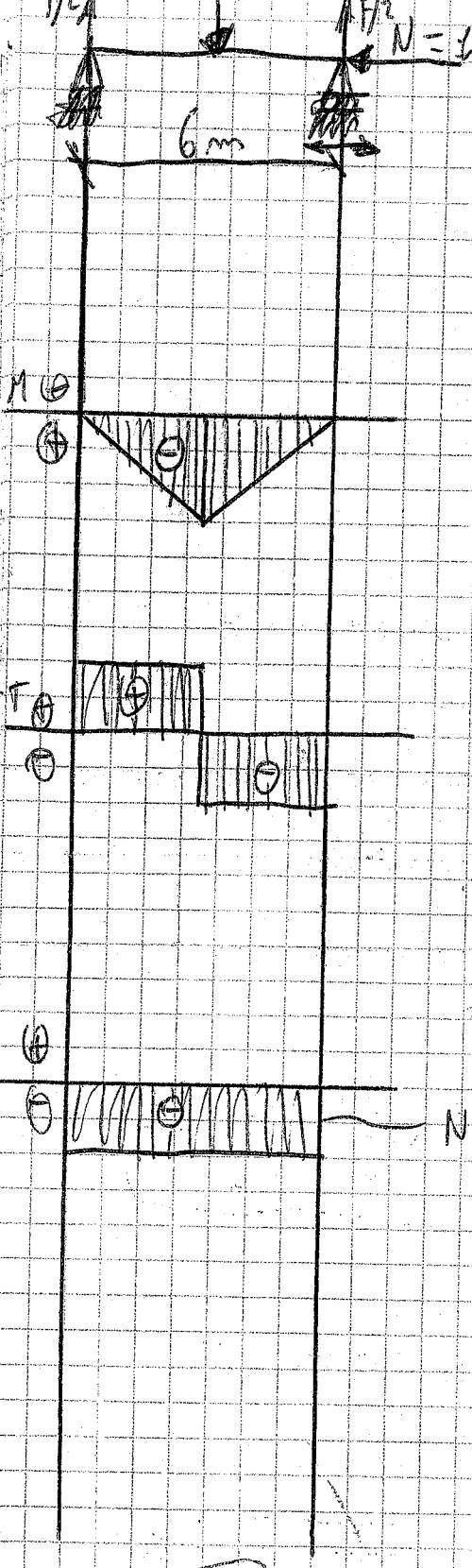
$h = 30\,cm$

$$\tau = \frac{3}{2} \cdot \frac{10 \cdot 10^3}{200 \cdot 300} = 69\,N/mm^2$$



$$S = b \cdot h \cdot 12{,}5$$

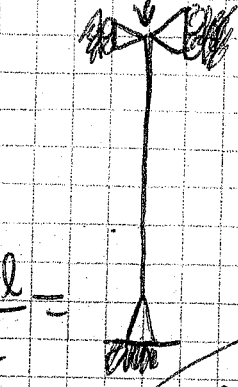
DIAGRAMMA TENSIONI DI TAGLIO



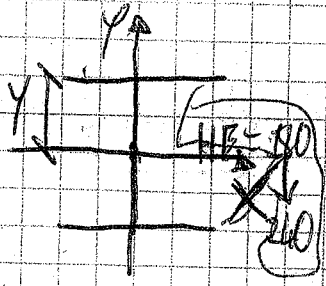
$$M = \frac{F}{2} \cdot \frac{l}{2} = \frac{F \cdot l}{4} =$$

$$= 120 \text{ KN/m}$$

$$T = F = 80 \text{ KN}$$



1429 1700



$M_{MAX} = 120 \text{ KN·m}$

$T_{MAX} = 8 \text{ KN}$

$N_{MAX} = 100 \text{ KN}$

$S = 275$

$\sigma_{AMM} = \frac{275}{1,15} = 239 \text{ N/mm}^2$

$\sigma_c = \frac{1000 \cdot 10^3 \cdot 1,47}{4530} = 32,45 \leq \sigma_{AMM.}$

$\tau_{AMM} = 239 \cdot 0,577 = 138$

VON MISES

$\sigma_{ideale} = \sqrt{\sigma_f^2 + 3\tau^2} \leq \sigma_{AMM.}$

$\sigma_c = \frac{N}{A} \cdot W \leq \sigma_{AMM.}$

$\lambda = \frac{l_0}{\rho} = \frac{600}{7,4} = 81$

$\rho = \sqrt{\frac{I}{A}} = \sqrt{\frac{2510}{45,3}} = 7,4$ cm

$\sigma_f = \frac{M}{I} \cdot Y$

$W = \frac{I}{Y}$

$\sigma_f = \frac{M}{W} \leq \sigma_{AMM.}$ 408

$\sigma_f = \frac{120 \cdot 10^6}{294 \cdot 10^3} =$

$\tau = \frac{I \cdot S}{I \cdot b}$

$\tau = \frac{80 \cdot 10^3 \cdot 372 \cdot 10^3}{7763 \cdot 10^4 \cdot 240} =$

$\tau = \frac{80000 \cdot 372}{77630 \cdot 240} = 1,6 \text{ N/mm}^2 < 138$

$\sigma_{id} = \sqrt{177^2 + 3 \cdot 1,6^3} = 177,02 \text{ N/mm}^2$

99

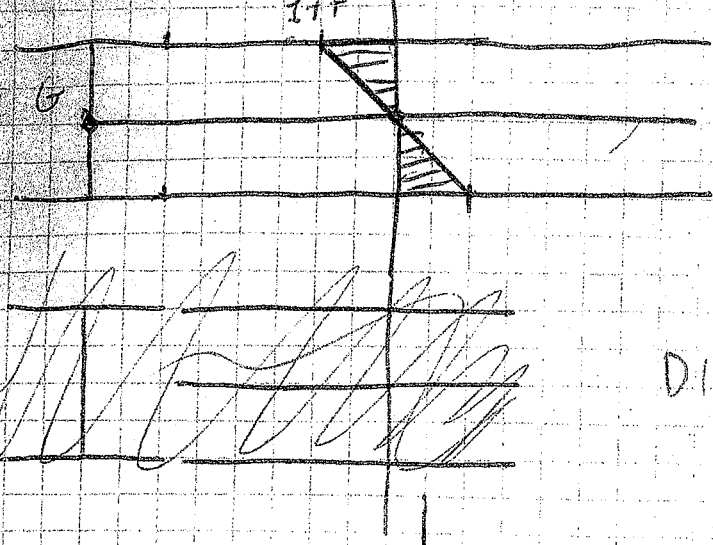


DIAGRAMMA DI FLESSIONE

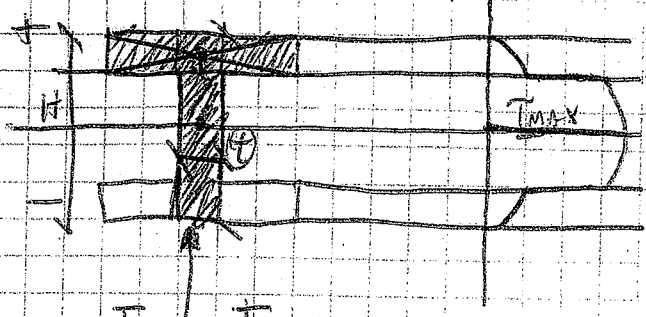


DIAGRAMMA TAGLIO

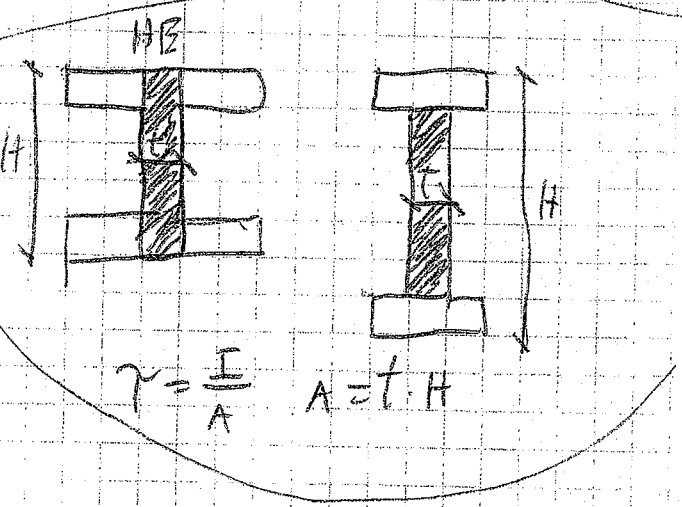
0,3468909S3

$\tau = \frac{I}{A} \frac{T}{t \cdot H}$

$\tau = \frac{T \cdot S_x}{I_x \cdot b}$ JOURAWSKI

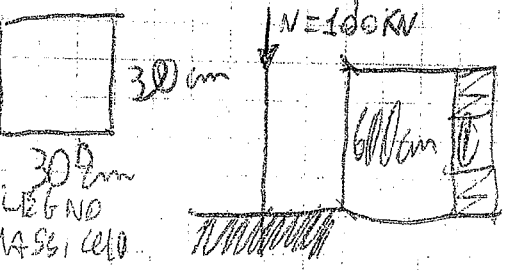
$\tau = \frac{3}{2} \frac{I}{A}$

$\tau = \frac{4}{3} \frac{I}{A}$



$\tau = \frac{I}{A}$ $A = t \cdot H$

Sia dato un pilastro in legno a sezione quadrata



$\sigma_{amm.\,C} = 8\ N/mm^2$

$\sigma_{AMMISSIBILE}$

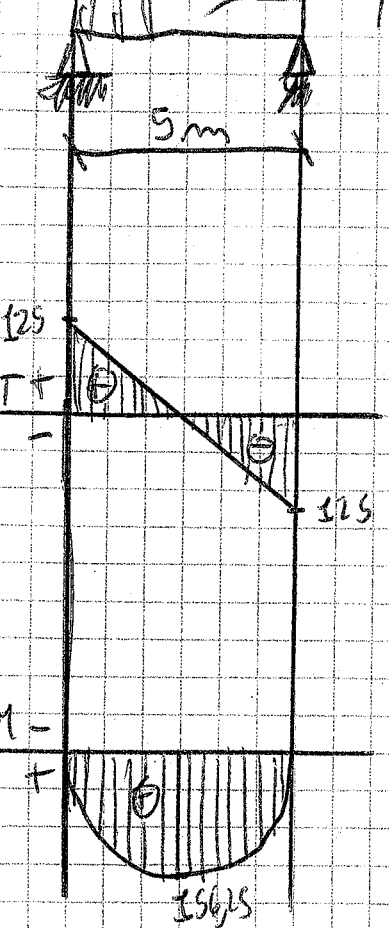
$\sigma_C = \frac{N}{A} \cdot w \leq \sigma_{amm.} \rightarrow 8\ N/mm$

$\lambda = \frac{l_0}{\rho} = \frac{2l}{\rho \sqrt{\frac{I}{A}}} = 138,57$

$x = \frac{(5,88 - 5,07) \cdot (138,57 - 130)}{(140 - 130)} = 0,69$

$w = 5,07 + 0,69 = 5,76$

100



$T_{MAX} = 125\,KN$

$M = \frac{1}{8} q \cdot l^2 = 156,25\,KN$

$\sigma_{AMM.} = 10\,KN/mm^2$

$\sigma_f = \frac{M}{I} \cdot Y = \frac{156,25 \cdot 10^6}{I_x} \cdot 150 =$

$= \frac{1}{12} \cdot 304 \cdot 10^4 =$

$I_x = \frac{1}{12} \cdot b \cdot h^4$

$\frac{156,25 \cdot 10^6}{\frac{1}{12} \cdot b^4} \cdot 150 = \frac{156,25 \cdot 10^6}{\frac{1}{12} \cdot 81 \cdot 10^8} \cdot 150$

$= \frac{1562,5}{675} \cdot 150 = 34,72\,N/mm^2$

TRAVE NON - VERIFICATA

$\frac{156,25 \cdot 10^6}{\frac{1}{12} \cdot b^4} \cdot 150 = \frac{156,25 \cdot 10^6}{\frac{1}{12} \cdot 81 \cdot 10^8} \cdot 150 =$

$= \frac{1562,5}{675} \cdot 150 = 34,72\,N/mm^2$

S 275

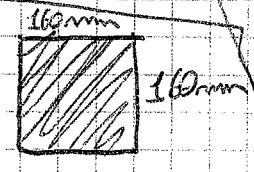
$\sigma_{AMMISSIBILE} = \frac{235}{1,10} = 213,64\,N/mm^2$

$M = 156,25 \cdot 10^6\,N \cdot mm$

$\sigma_f = \frac{M}{W} = \frac{156,25 \cdot 10^6}{212 \cdot 10^3} =$

$= 736,68\,N/mm^2$

$\sigma_f = \frac{M}{W} = \frac{156,25 \cdot 10^6}{\frac{1}{12} \cdot 160^4} =$



$= 228,88\,N/mm^2$

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29/05/2017

VERIFICA PILASTRO FERRO (COMPRESSIONE + CARICO DI PUNTA)

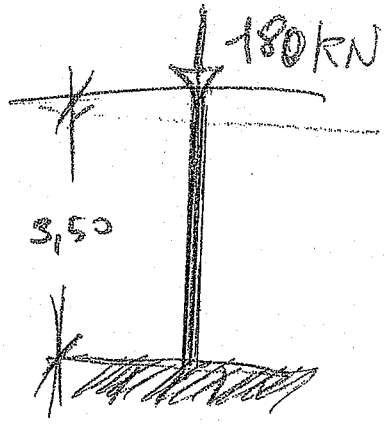
CARICO SUL PILASTRO CIRCA: $4 \cdot 3 = 12$ mq totali

Terra	0,8
Solaio	0,25
S.m.	0,2
	≃ 1,3 t/mq

Travi: $0,3 \cdot 0,8 \cdot 1 \cdot 2,5 = 0,6$ t/ml

tot. $12 \cdot 1,3 + 0,6 \cdot 4 = 18$ t $= 180$ KN

S235
HEA 100



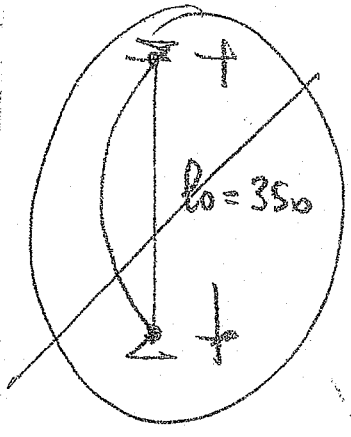
con $\sigma_{adm} = 235$ kg/cm² $\quad A = \frac{N}{\sigma}$

$A = \frac{18000}{2350} = 9$ cm²

HEA 100 con $A = 21$ cm² è il profilato + piccolo. Ho σ effettiva visibilmente + piccola

σ = [illegible]

Verifica al carico di punta: nella ipotesi + sfavorevole di $l_0 = h$



λ snellezza $= \frac{l_0}{\rho_{min}} \qquad i_{min} = \rho_{min}$

i_y (di HEA 100) $= 2,5$

$\lambda = \frac{350 \times 2}{2,5} = 280$

nella Tabella $\lambda \rightarrow \omega$

al valore 140 corrisponde $\omega =$ [illegible]

$\sigma = \frac{N}{A} \cdot \omega \qquad \sigma = \frac{18000}{21 \cdot 10^2} \cdot$ [illegible] $\qquad \sigma = 857 \cdot$ [illegible] $=$ (5700) ?

N.B. devo prendere una HEA maggiore e rifare la verifica

tensione $\sigma_c < \sigma_{ammissibile}$

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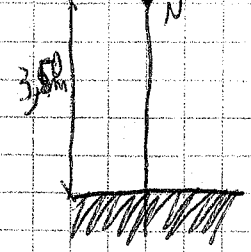


$\sigma_c = \frac{N+w}{A} \leq \sigma_{AMMISSIBILE}$

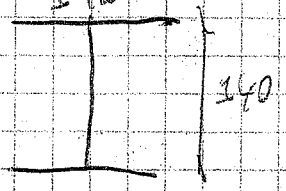
$\sigma_{AMM.} = \frac{235}{1,05} = 323,8 N/mm^2$

S235

$HE_A \Rightarrow 100$

$w \to \lambda = \frac{l_0}{\rho} = \frac{2l}{\sqrt{\frac{I}{A}}} =$

$= \frac{2 \cdot 350}{\sqrt{\frac{349}{23,2}}} = \frac{700}{4,06} = 172,41$



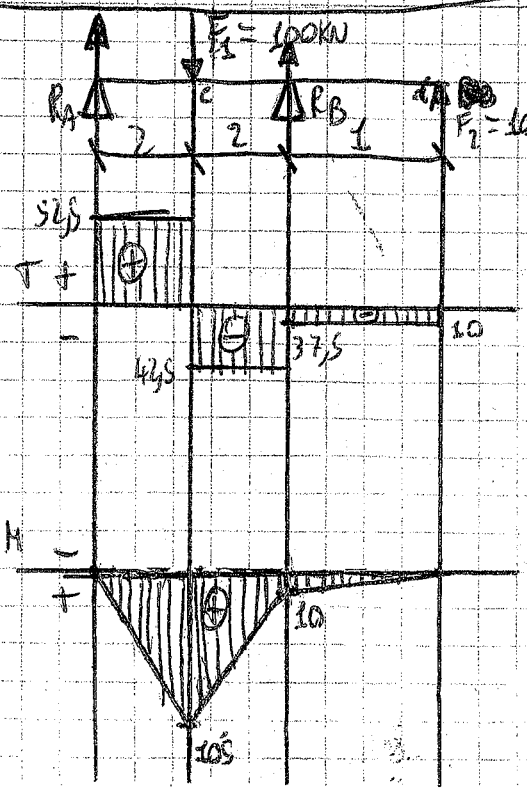
$w = 4,24$

$\frac{180000 \cdot 4,24}{2120} = \frac{763200}{2120} = 360 N/mm^2$

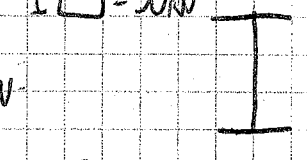
NON È VERIFICATO

$180000 \cdot$

$\frac{1}{12} \cdot b \cdot 4 - 2 \frac{1}{2} b \cdot h^3$



1 □ = 50KN



$I_x = 1943 cm^4$

$A = 28,5 cm^2$

S275

$\sigma_{AMM.} = \frac{275}{1,05} = 261$

$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \Rightarrow -R_A + F_1 - R_B - F_2 = 0 \\ \Sigma M_A = 0 \Rightarrow \end{cases}$

$\Rightarrow -R_A \cdot 0 + F_1 \cdot 2 - R_B \cdot 4 - F_2 \cdot 5 =$

$= R_B = \frac{200 - 50}{4} = 37,5 KN/ml$

$R_A = 100 - 37,5 - 10 = -52,5 KN/ml$

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$T_{RA} = R_A = 52,5\ KN/mm$

$T_C = R_A - F_1 = -42,5\ KN/mm$

$T_{R_B} = R_A - F_1 + R_B = -10\ KN/mm$

$T_d = 0$

$T_{MAX} = 100\ KN$

$M_A = 0$

$M_C = 52,5 \cdot 2 = 105\ KN \cdot m$

$M_B = F_2 \cdot 1 = 10\ KN \cdot m$

$M_d = 0$

$M_{MAX} = 105\ KN$

$\sigma_f = \frac{M}{I} \cdot Y = \frac{105 \cdot 10^2}{1943 \cdot 10^4} \cdot 100 = 540,40$ NON VA BENE PRENDO LA 240

$\sigma_f = \frac{M}{I} \cdot Y = \frac{105 \cdot 10^2}{3892 \cdot 10^4} \cdot 120$

$I_x = 3892$

IPE 300

$I_x = 5790$

$A = 45,9\ cm^4$

$W_x = 242\ cm^3$

$\sigma_f = \frac{M}{I} \cdot Y = \frac{105 \cdot 10^6}{5790 \cdot 10^4} \cdot 135 = 244,81\ N/mm^2$

$\tau = 0,577 \cdot \sigma_{AMM.}$

$\sigma_I = \sqrt{244,81^2 + 3 \cdot 3,09^2} = \sqrt{59935,9361 + 28,6443} = 244,86\ N/mm^2$

$\tau = \frac{T \cdot S}{I \cdot b} = \frac{100 \cdot 10^2 \cdot 242 \cdot 10^2}{5790 \cdot 10^4 \cdot 135} = 3,09\ N/mm^2$

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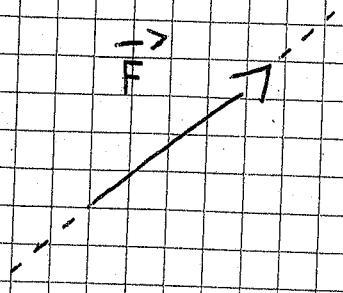
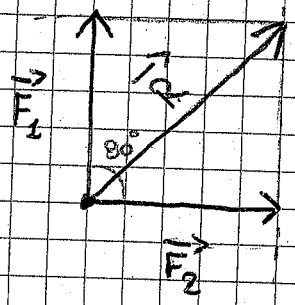
GRANDEZZE VETTORIALI



1 Punto di applicazione
2 Modulo o intensità
3 Direzione o verso
4 Linea di azione o traiettoria



$\vec{R}$ = Risultante

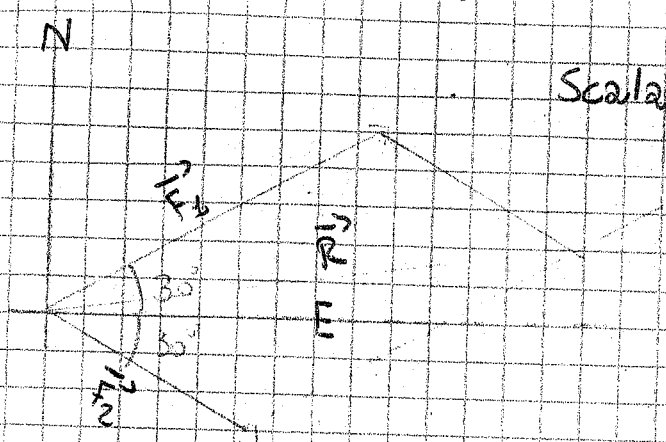
$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2\vec{F_1}\cdot\vec{F_2}\cos 90^\circ}$$

$$\cos 90^\circ = 0$$

Misoni Antonio

REGOLA DEL PARALLELOGRAMMO

$\vec{F_1} = 10N$ (NE) $\quad \vec{F_2} = 6N$



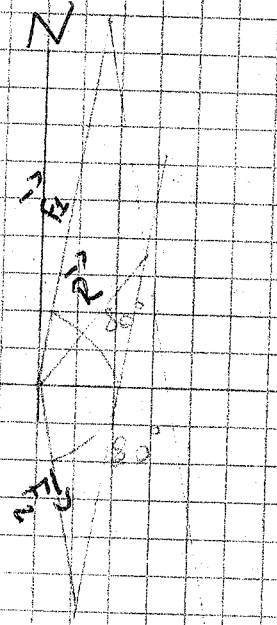
Scala: 1 cm = 2N

$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2F_1F_2 \cdot \cos\alpha}$$

$$\sqrt{10^2 + 6^2 + 2 \cdot 10 \cdot 6 \cdot \underbrace{\cos 60°}_{0,5}} = 14N$$

FARE ALMENO 3 ESERCIZI CHE ABBIA ALMENO UN ANGOLO DI 160°, 81°, 90°

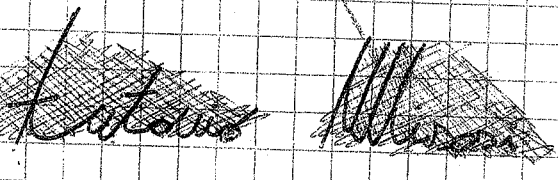
$\vec{F_1} = 10 \quad \vec{F_2} = 6N$



Scala: 1 cm = 2N

$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2F_1F_2 \cdot \cos\alpha}$$

$$\sqrt{10^2 + 6^2 + 2 \cdot 10 \cdot 6 \cdot (-0,336)} = 4,82N$$



UNITÀ DI MISURA

NEWTON (N)

Rappresenta la decima parte del kilo

1 N = 0,081 K_g

1 K_g = 9,81 N = 10 N

1 daN

1 kN

1 MN

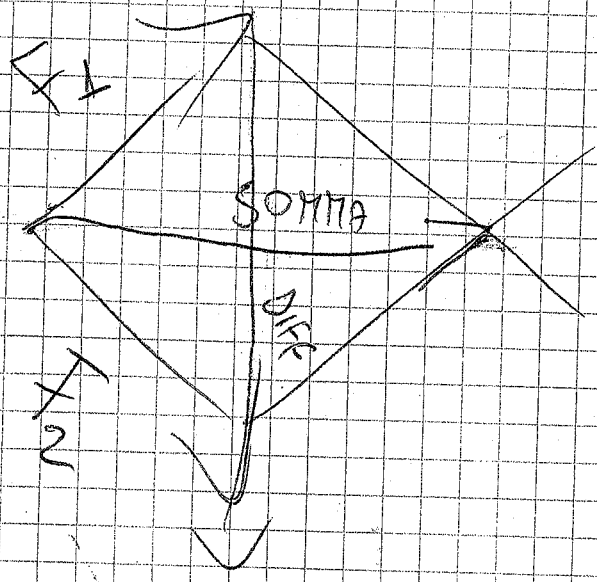
1 GN

da = Deca (DIECI)

K = Kilo (MILLE)

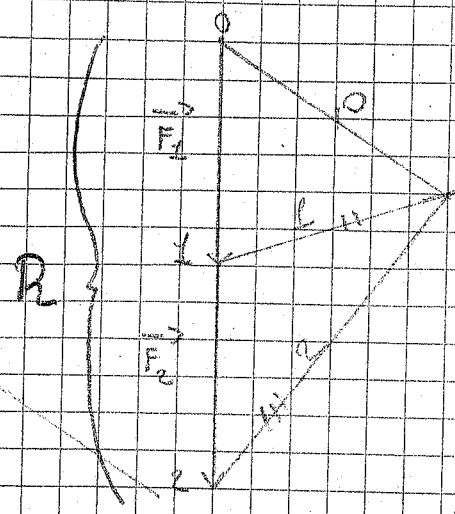
M = Mega (MILIONE)

G = Miliarda

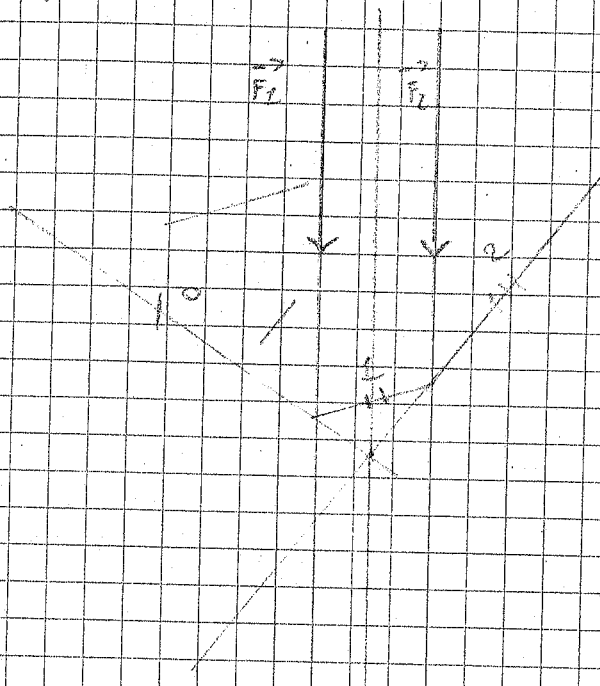


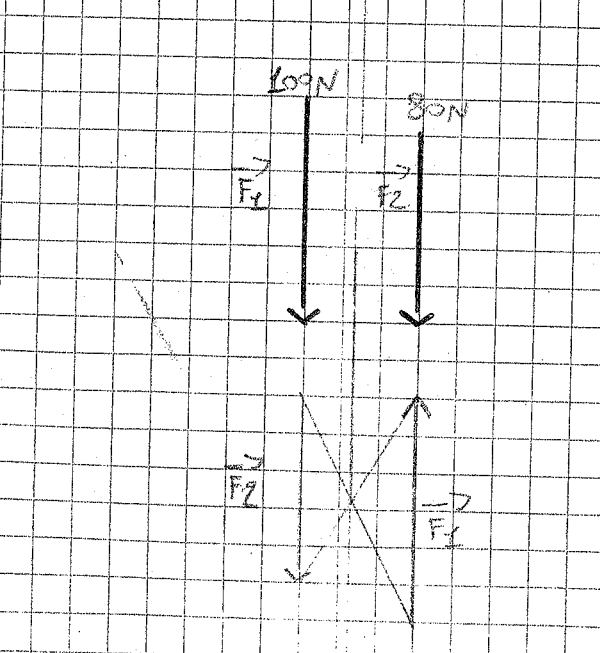
RISULTANTE?

PUNTO DI PASS?



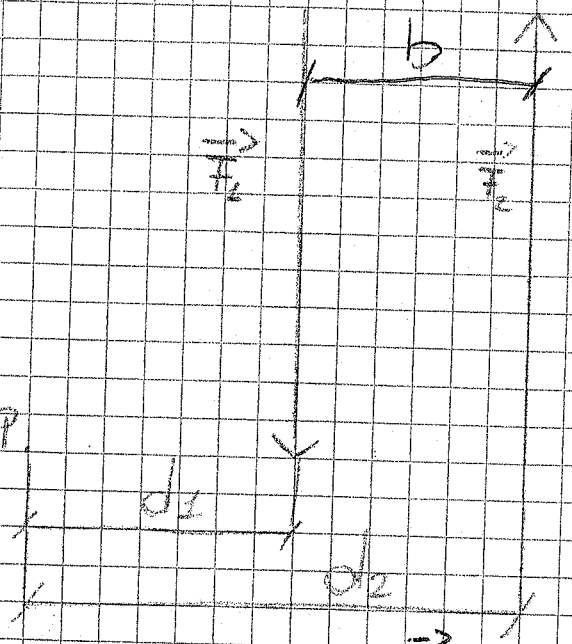
$\vec{R} = \vec{F_1} + \vec{F_2} = 3 + 3 = 6N$





$R = \vec{F_1} + \vec{F_2} = 100 + 80 = 180\,N$

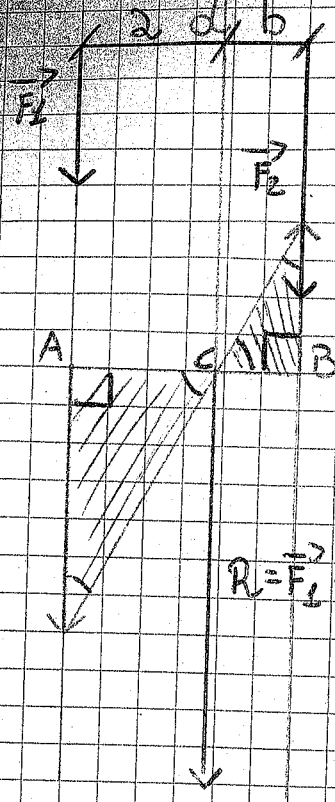
20 OTTOBRE COMPITO SU VETTORI, SOMME DI VETTORI, DIFFERENZE, SCOMPOSIZIONE DI VETTORI, TEOREMA DI PITAGORA E DI CARNOT, POLIGONO FUNICOLARE, FORZE POLIGENE CONCORDI E DISCORDI (NOTE VERIGNON PORSE)



$F = V_{GV2}$

$$M_P = \vec{F}_1 \cdot d_1 - F_2 \cdot d_2$$
$$M_P = F(d_2 - d_1)$$

NO COPPIO DI FORZE RISPETTO a QUALSIASI PUNTO DEL PIANO HA UNO MOMENTO UGUALE A $F \cdot b$



DATI

$\vec{F_1} = 30\ N$

$\vec{F_2} = 50\ N$

$d = 120\ cm$

DUE SCALE:

$\begin{cases} 1\ cm = 20\ cm \\ 1\ cm = 10\ N \end{cases}$

$R = \vec{F_1} + \vec{F_2} = 30 + 50 = 80\ N$

$F_2 : 2 = F_1 : b$

$b = \frac{F_2 \cdot d}{R}; \quad 2 = \frac{F_2 \cdot d}{R}$

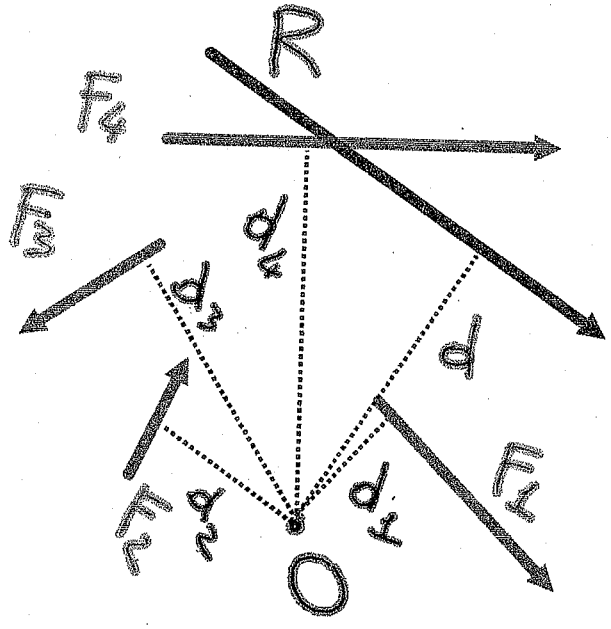
$b = \frac{30 \cdot 120}{80} = 45\ cm; \quad 2 = \frac{50 \cdot 120}{80} = 75\ cm$

Teorema di Varignon 3°B

Dato un sistema di forze complanari e scelto un punto nel piano, si può calcolare il momento di ciascuna forza e determinare il momento risultante ; ma i singoli momenti e il momento risultante devono soddisfare il teorema di Varignon, per il quale:

"In un sistema di forze complanari il momento della risultante , rispetto a un punto "O" qualsiasi nel piano, è uguale alla somma algebrica dei momenti delle singole forze rispetto al piano stesso."

es. $F_1 \times d_1$ ....



$$F_1 \times d_1 + F_2 \times d_2 + F_3 \times d_3 + F_4 \times d_4 = R \times d$$

$$d = \frac{F_1 \times d_1 + F_2 \times d_2 + F_3 \times d_3 + F_4 \times d_4}{R}$$

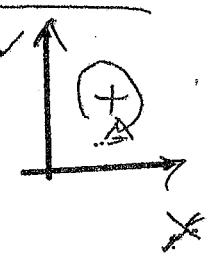
EQUAZIONI CARDINALI DELLA STATICA 3°B

SISTEMA LINEARE

$$\begin{cases} \sum \bar{F} = 0 \Rightarrow \text{NO TRASLAZIONI} \\ \sum \bar{M}_p = 0 \Rightarrow \text{NO ROTAZIONI} \end{cases}$$

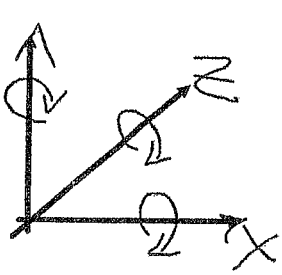
Condizione necessaria e sufficiente affinché un corpo sia in quiete (STATICA) è che la $\sum \bar{F} = 0$ e che la $\sum \bar{M}_p = 0$

NEL PIANO — 2D



$$\begin{cases} \sum \bar{F} = 0 \\ \sum \bar{M}_p = 0 \end{cases} \Rightarrow \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_p = 0 \end{cases}$$

NELLO SPAZIO — 3D



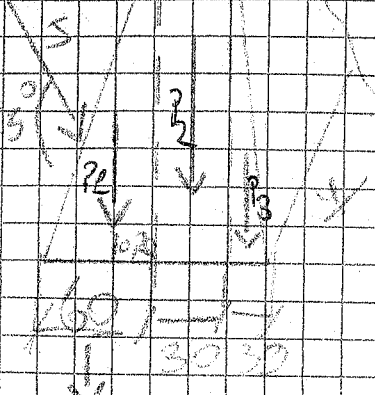
$$\begin{cases} \sum \bar{F} = 0 \\ \sum \bar{M}_p = 0 \end{cases} \Rightarrow \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \\ \sum M_{(xy)p} = 0 \\ \sum M_{(yz)p} = 0 \\ \sum M_{(zx)p} = 0 \end{cases}$$

N° 5

PESO CALCESTRUZZO 24 KN

S = 25 KN

PESO?



$P_{PL} = \underbrace{\frac{0,60 \cdot 3,60}{2}}_{\text{SUPERFICIE (AREA)}} \cdot \underbrace{1,00}_{\text{PROFONDITÀ}} \cdot 24\ KN/m^3 = 25,92\ KN$

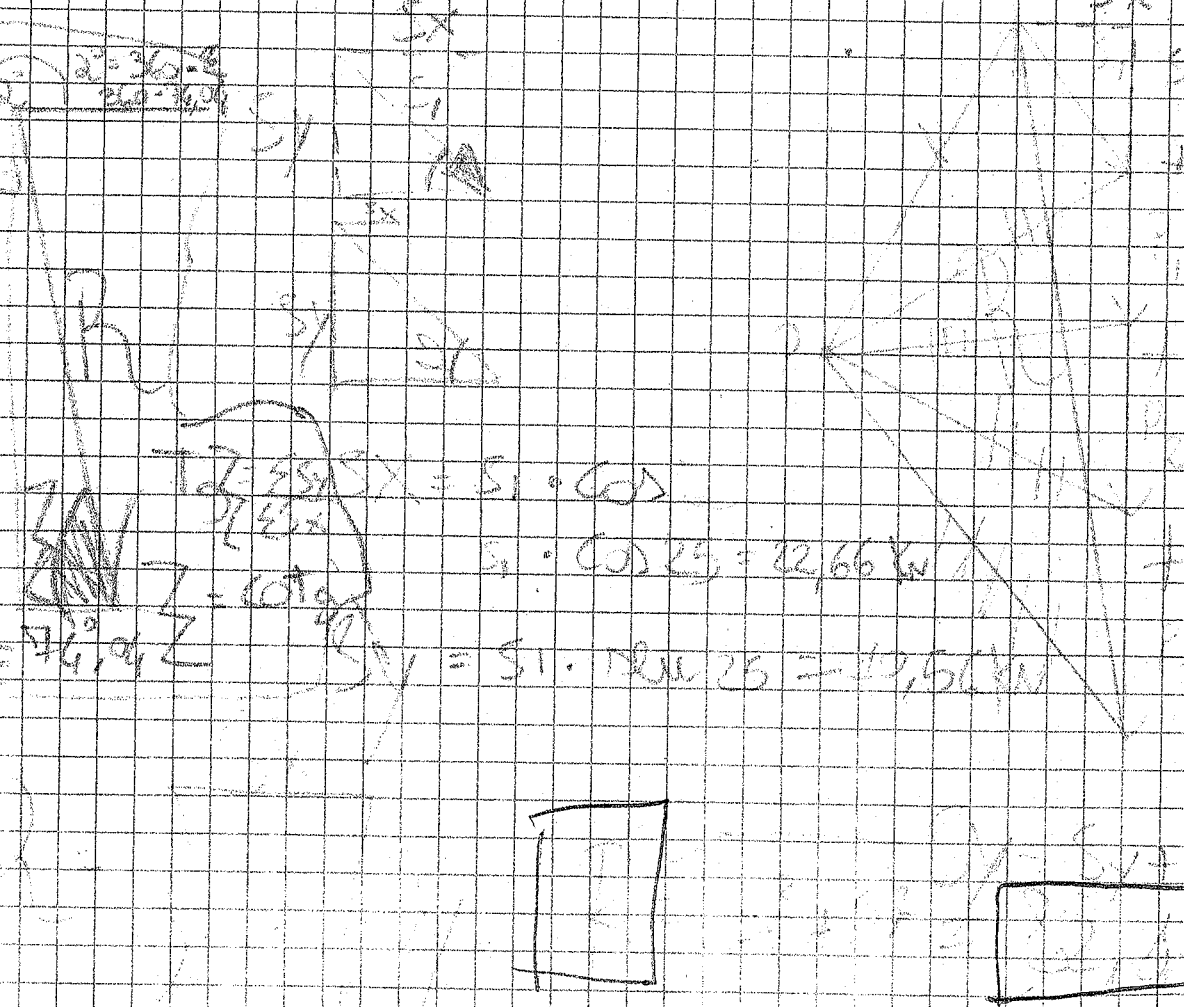
VOLUME

PESO SPECIFICO

PESO PEZZO

$m^3 \cdot \frac{KN}{m^3}$

$\frac{0,60}{3}$



$S_1 = 25\ KN$

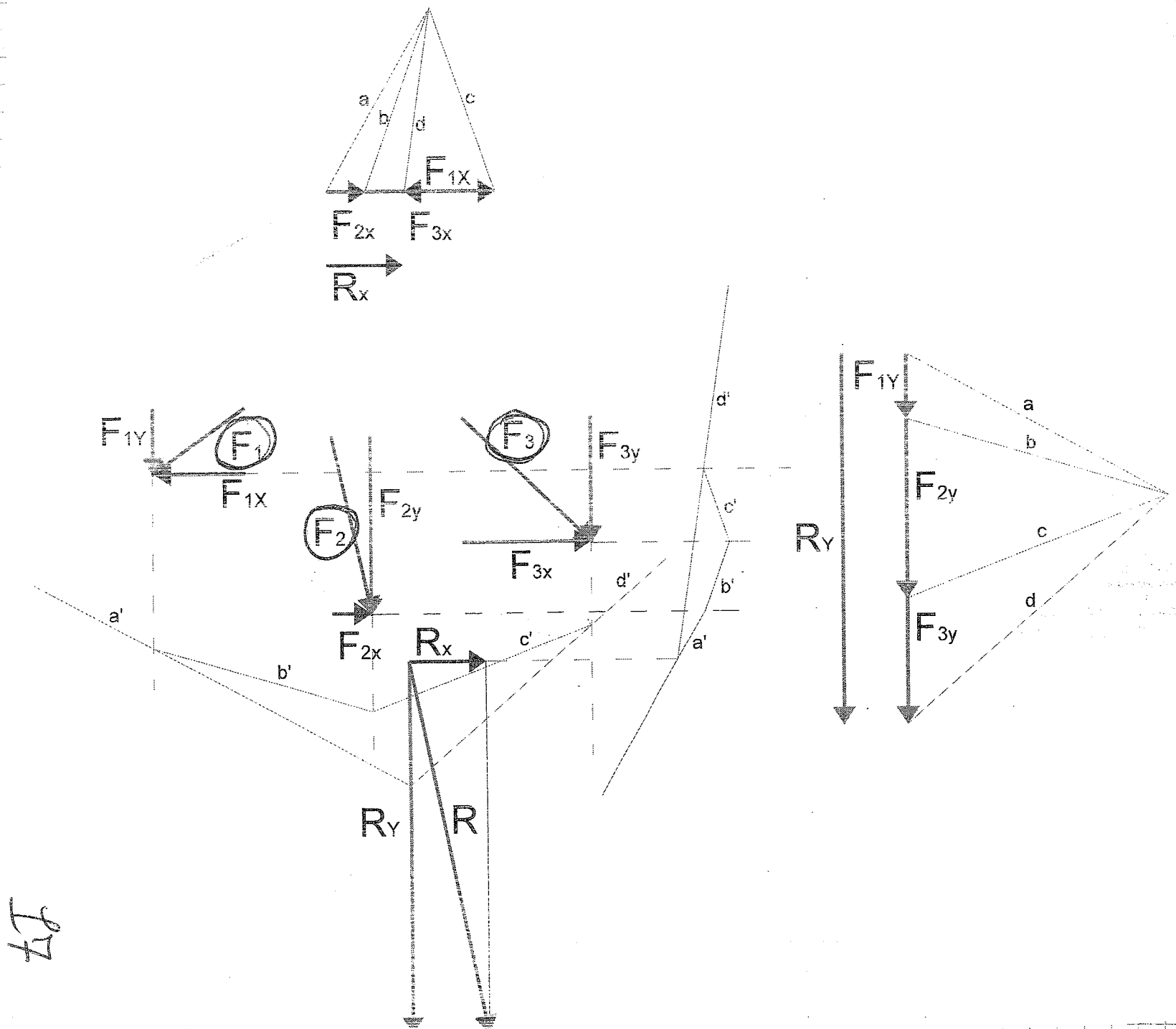
$S_1 \cdot \cos 45 = 12,66\ KN$

$S_1 \cdot \text{sen } 45 = 12,66\ KN$

$P_1 + P_2 + P_3 = 70,2$

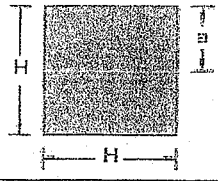
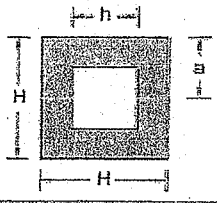
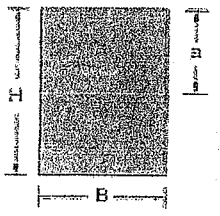
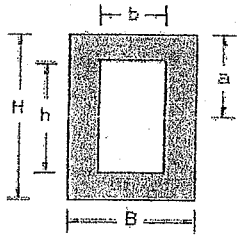
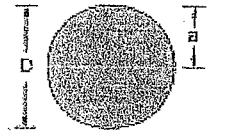
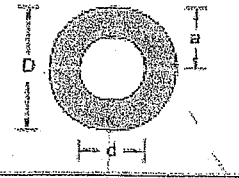
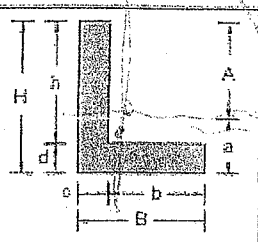
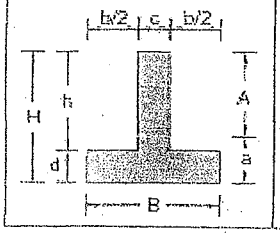
KN

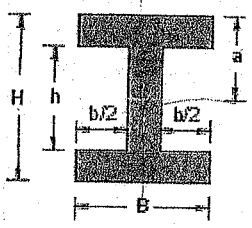
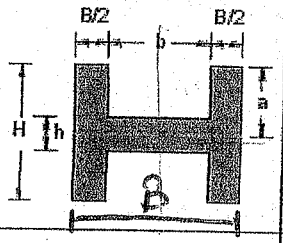
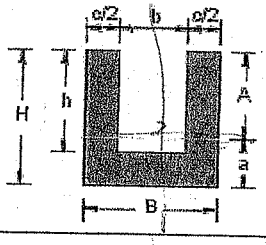
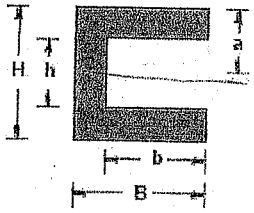
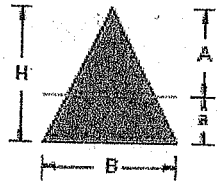
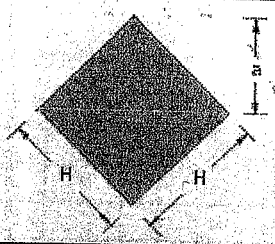
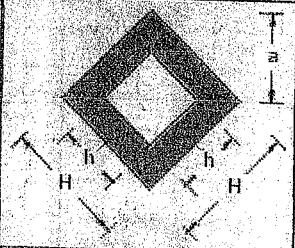
15


a
b
d
c
F1X
F2x
F3x
Rx
F1Y
F1
F1X
F2y
F2
F3
F3y
F3x
d'
c'
b'
a'
F2x
Rx
RY
R
F1Y
a
b
F2y
c
d
F3y
RY

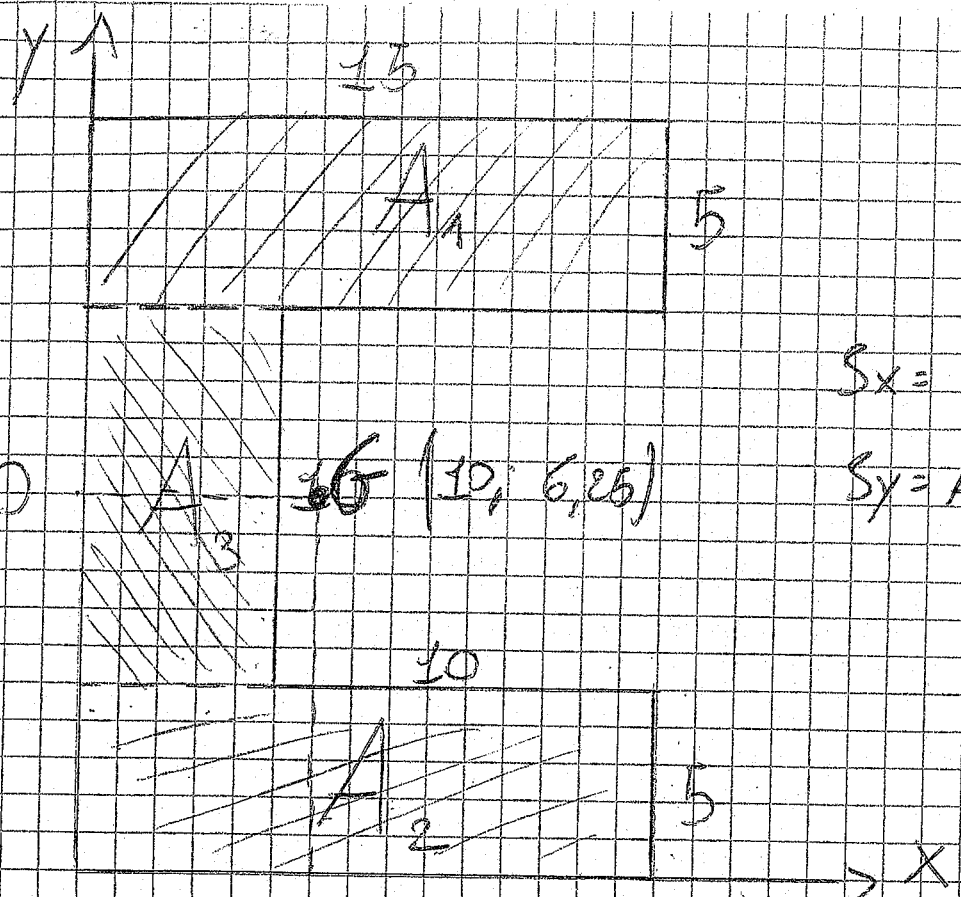
47

Tabella figure geometriche piane

Sezione	Area della sezione A cm^2	Distanza dal baricentro a cm	Momento di inerzia J cm^4	Modulo di resistenza W cm^3
	H^2	$\frac{H}{2}$	$\frac{H^4}{12}$	$\frac{H^3}{6}$
	$H^2 - h^2$	$\frac{H}{2}$	$\frac{H^4 - h^4}{12}$	$\frac{H^4 - h^4}{6\,H}$
	$B \cdot H$	$\frac{H}{2}$	$\frac{B \cdot H^3}{12}$	$\frac{B \cdot H^2}{6}$
	$BH - bh$	$\frac{H}{2}$	$\frac{1}{12} \cdot (BH^3 - bh^3)$	$\frac{1}{6H} \cdot (BH^3 - bh^3)$
	$\frac{\pi \cdot D^2}{4}$	$\frac{D}{2}$	$\frac{\pi \cdot D^4}{64}$	$\frac{\pi \cdot D^3}{32}$
	$\frac{\pi \cdot (D^2 - d^2)}{4}$	$\frac{D}{2}$	$\frac{\pi \cdot (D^4 - d^4)}{64}$	$\frac{\pi \cdot (D^4 - d^4)}{32 \cdot D}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{I}{A}$ $W_a = \frac{I}{a}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{J}{A}$ $W_a = \frac{J}{a}$

Sezione	Area della sezione A cm^2	Distanza dal baricentro a cm	Momento di inerzia J cm^4	Modulo di resistenza W cm^3
	$BH - bh$	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$
	$BH - bh$	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{I}{A}$ $W_a = \frac{I}{a}$
	$BH - bh$	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$
	$\frac{B \cdot H}{2}$	$A = \frac{2H}{3}$ $a = \frac{H}{3}$	$\frac{B \cdot H^3}{36}$	$W_A = \frac{B \cdot H^2}{24}$ $W_a = \frac{B \cdot H^2}{12}$
	H^2	$\frac{H}{2} \cdot \sqrt{2}$	$\frac{H^4}{12}$	$\frac{H^3}{6\sqrt{2}}$
	$H^2 - h^2$	$\frac{H}{2} \cdot \sqrt{2}$	$\frac{H^4 - h^4}{12}$	$\frac{H^4 - h^4}{6H\sqrt{2}}$

I = raggio di inerzia = $\sqrt{\frac{\text{Momento di inerzia}}{\text{Area della sezione}}} = \sqrt{\frac{J}{A}}$



$S_x = A_t \cdot y_G$

$S_y = A_t \cdot x_G$

$S_x = A_1 \cdot b + A_2 \cdot b + A_3 \cdot b$

$S_x = (5 \cdot 15) \cdot 17{,}5 + (15 \cdot 5) \cdot 2{,}5 + (5 \cdot 10) \cdot 1{,}5 =$

$75 \cdot 17{,}5 + 750 + 187{,}5 = 2000 \text{ cm}^3$

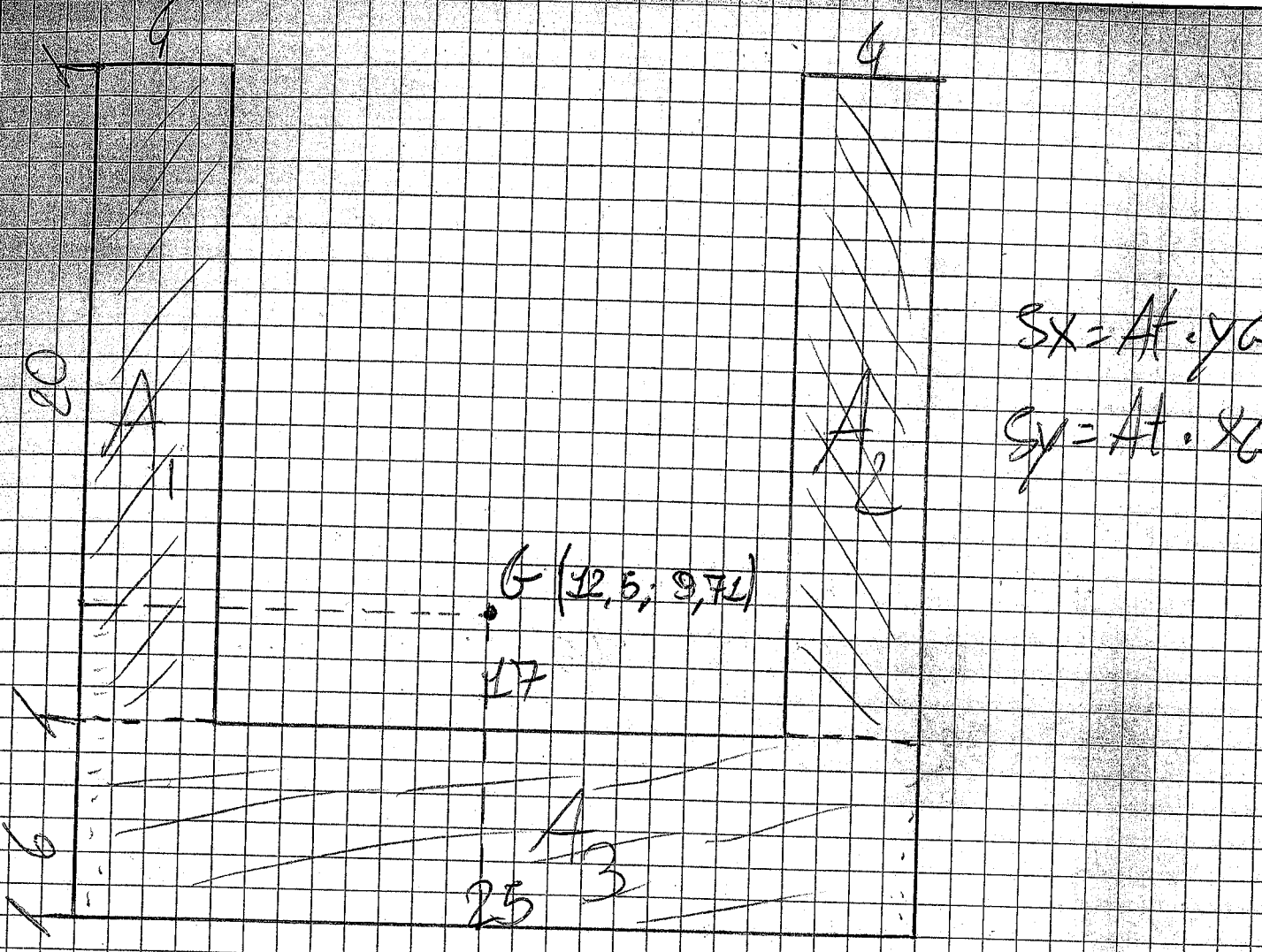
$y_G = \frac{S_x}{A_t} = \frac{2000}{200} = 10{,}00 \text{ cm}$

$S_y = A_1 \cdot b + A_2 \cdot b + A_3 \cdot b$

$S_y = (5 \cdot 15) \cdot 7{,}5 + (15 \cdot 5) \cdot 7{,}5 + (5 \cdot 10) \cdot 2{,}5 =$

$562{,}5 + 562{,}5 + 125 = 1250 \text{ cm}^2$

$x_G = \frac{1250}{200} = 6{,}25 \text{ cm}$



$S_x = A_t \cdot y_G$

$S_y = A_t \cdot x_G$

$S_x = A_1 \cdot b + A_2 \cdot b + A_3 \cdot b$

$S_x = (20 \cdot 4) \cdot 16 + (20 \cdot 4)(16) + (25 \cdot 6) \cdot 3 =$

$S_x = 1280 + 1280 + 450 = 3010\ cm^3$

$y_G = \frac{S_x}{A_t} = \frac{3010}{310} = 9,71\ cm$

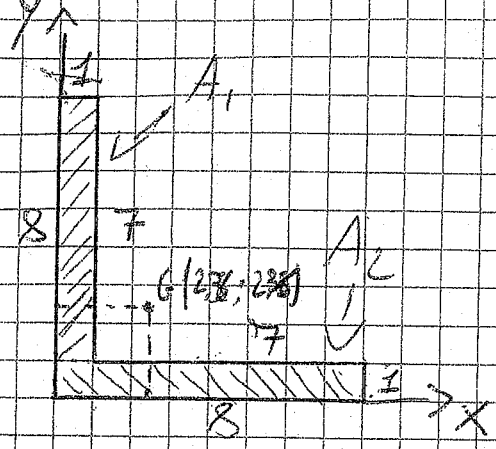
$S_y = A_1 \cdot b + A_2 \cdot b + A_3 \cdot b$

$S_y = (20 \cdot 4) \cdot 2 + (20 \cdot 4) \cdot 23 + (25 \cdot 6) \cdot 12,5 =$

$160 + 1840 + 1875 = 3875\ cm^3$

$x_G = \frac{S_y}{A_t} = \frac{3875}{310} = 12,5\ cm$

(1)



$S_x = A_t \cdot y_C$

$S_y = A_t \cdot x_C$

$S_x = A_1 \cdot d + A_2 \cdot d$

$S_x = (7 \cdot 1) \cdot 4,5 + (8 \cdot 1) \cdot 0,5$

$31,5 + 4 = 35,5\ cm^3$

$y_C = \frac{S_x}{A_t} = \frac{35}{16} = 2,36\ cm$

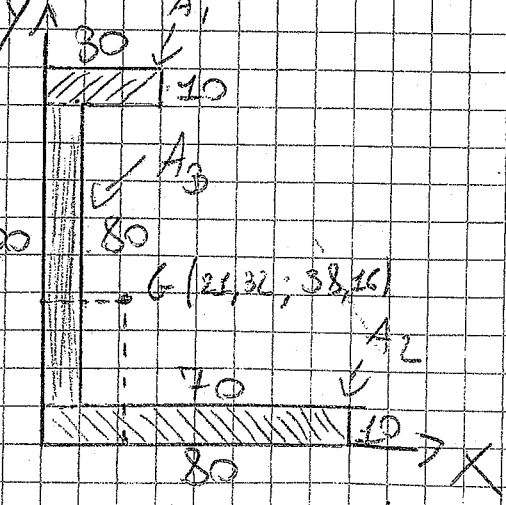
$S_y = A_1 \cdot d + A_2 \cdot d$

$S_y = (8 \cdot 1) \cdot 0,5 + (8 \cdot 1) \cdot 4,5$

$4 + 36 = 40\ cm^3$

$x_C = \frac{S_y}{A_t} = \frac{40}{16} = 2,5\ cm$

(2)



$S_x = A_t \cdot y_C$

$S_y = A_t \cdot x_C$

$S_x = A_1 \cdot d + A_2 \cdot d + A_3 \cdot d$

$S_x = (30 \cdot 10) \cdot 95 + (80 \cdot 10) \cdot 5 + (80 \cdot 10) \cdot 50$

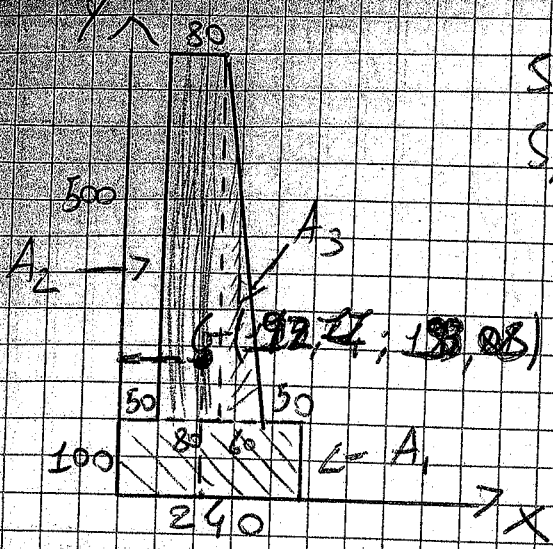
$28500 + 4000 + 40000 = 72500\ cm^3$

$y_C = \frac{S_x}{A_t} = \frac{72500}{1900} = 38,$

$S_y = A_1 \cdot d + A_2 \cdot d + A_3 \cdot d$

$S_y = 300 \cdot 1 + 800 \cdot 60 + 800 \cdot 5 = 60500\ cm^3$

$x_C = \frac{S_x}{A_t} = \frac{60500}{1900} = 31,$



(3)

$S_x = A_t \cdot y_G$

$S_y = A_t \cdot x_G$

$S_x = A_1 \cdot d + A_2 \cdot d + A_3 \cdot d$

$(240 \cdot 100) \cdot 30 + (80 \cdot 500) \cdot 350 + \left(\frac{60 \cdot 500}{2}\right) \cdot 36$

$1200000 + 14000000 + 54000 = 15254000\ cm^3$

$y_G = \frac{S_x}{A_t} = \frac{15254000}{79000} = 193,08\ cm$

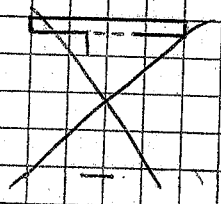
$S_y = A_1 \cdot d + A_2 \cdot d + A_3 \cdot d$

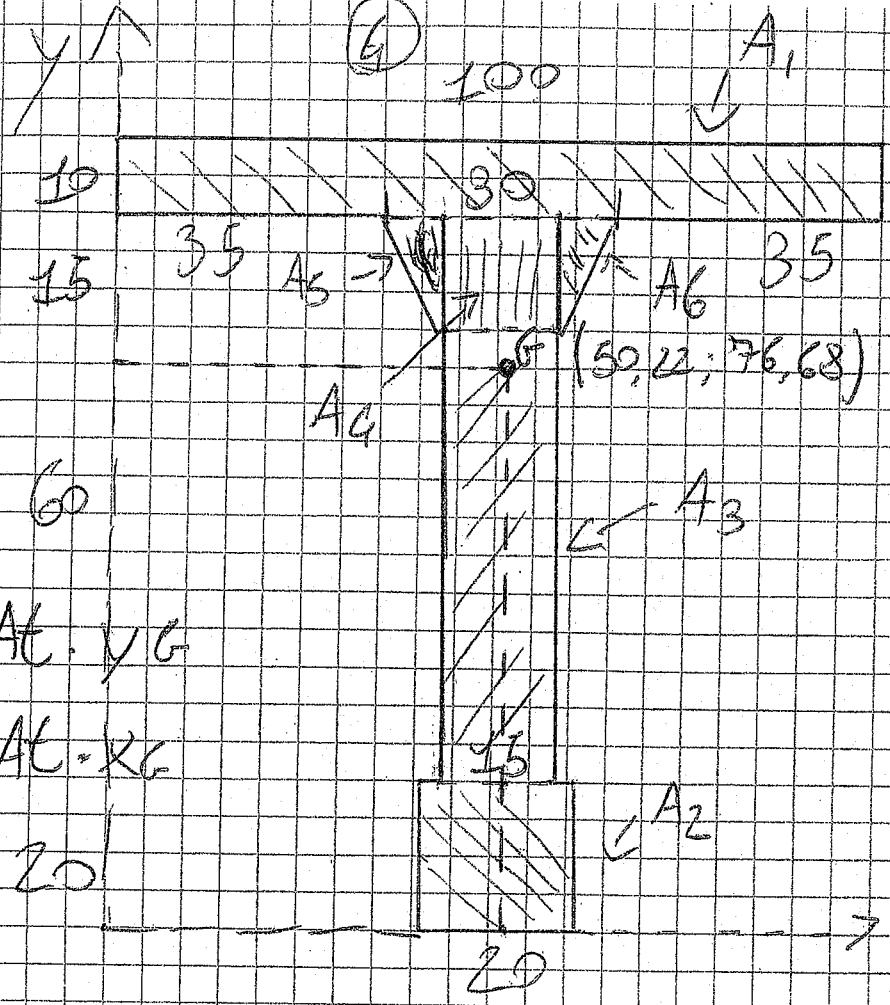
$2880000 \quad 3600000 \quad 2400000$

$24000 \cdot 120 + 40000 \cdot 90 + 15000 \cdot 160 = 8880000\ cm^3$

$x_G = \frac{S_y}{A_t} = \frac{8880000}{79000} = 112,41\ cm$

(4)





$Sx = A_1 \cdot d + A_2 \cdot d + A_3 \cdot d + A_4 \cdot d + A_5 \cdot d + A_6 \cdot d$

$1000 \cdot 100 + 40 \cdot 10 + 900 \cdot 30 + 225 \cdot 87,5 + (56,25 \cdot 85) \cdot 2$

$100000 + 400 + 45000 + 19687,5 + 4781,25 + 4781,25 = 174650$

$yC = \frac{Sx}{At} = \frac{174650}{2277,5} = 76,68\ cm$

$Sy = A_1 \cdot d + A_2 \cdot d + A_3 \cdot d + A_4 \cdot d + A_5 \cdot d + A_6 \cdot d$

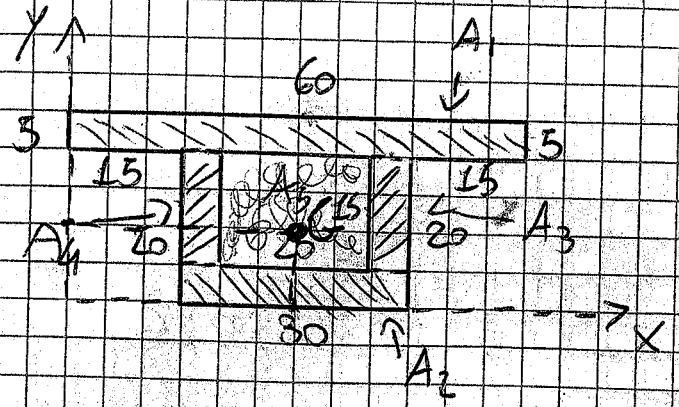
$1000 \cdot 50 + 40 \cdot 50 + 900 \cdot 50 + 225 \cdot 50 + 56,25 \cdot 38,75 +$

$56,25 \cdot 70 =$

$50000 + 2000 + 45000 + 11250 + 2179,69 + 3937,5 =$

$114367,19\ cm^2$

$xC = \frac{Sx}{At} = \frac{114367,19}{2277,5} = 50,22\ cm$



$S_x = A_t \cdot y_G$

$S_y = A_t \cdot x_G$

$S_x = A_1 \cdot d + A_2 \cdot d + A_3 \cdot d + A_4 \cdot d - A_5 \cdot d$

$(60 \cdot 5) \cdot 22,5 + (30 \cdot 5) \cdot 2,5 + (15 \cdot 15) \cdot 12,5 + (15 \cdot 15) \cdot 12,5 - (20 \cdot 15) \cdot 17,5$

$6750 + 375 + 937,5 + 937,5 = 9000 - 3750 = 5250$ cm

$y_G = \frac{S_x}{A_t} = \frac{9000}{600} = 15$ cm

$S_y = A_1 \cdot d + A_2 \cdot d + A_3 \cdot d + A_4 \cdot d - A_5 \cdot d$

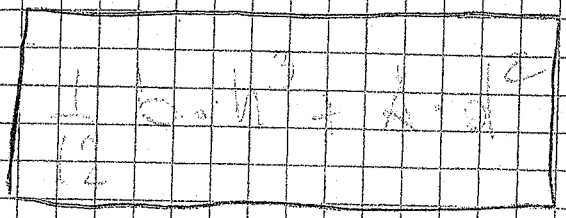
$(60 \cdot 5) \cdot 30 + (30 \cdot 5) \cdot 30 + (15 \cdot 5) \cdot 42,5 + (15 \cdot 5) \cdot 17,5 =$

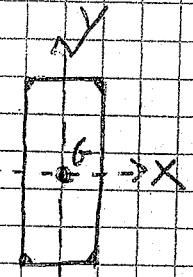
$9000 + 4500 + 3187,5 + 1312,5 = 18000$ cm^3

$x_G = \frac{S_y}{A_t} = \frac{18000}{600} = 30$ cm

Momento di Inerzia

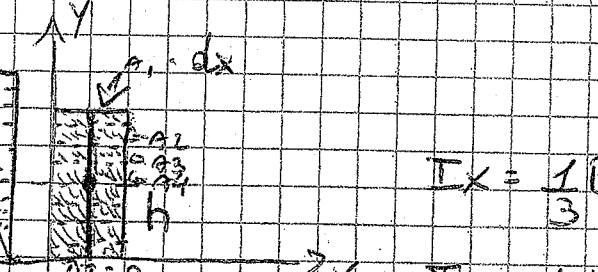
PRODOTTO DOVUTO ALL'AREA PER LA DISTANZA AL QUADRATO





$$I_{xG} = A_t \cdot d_y^2 = \omega^2 \cdot \omega^2 = \omega^4$$

$$I_{yG} = A_t \cdot d_x^2 = \omega^2 \cdot \omega^2 = \omega^4$$



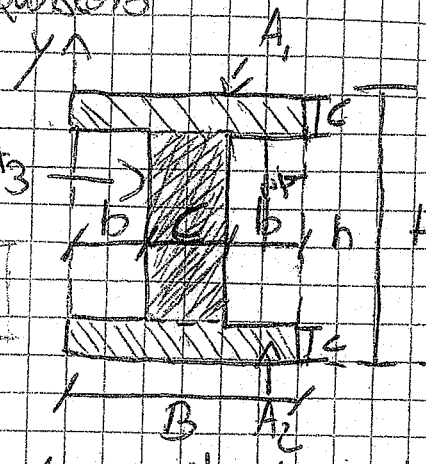
$$I_x = \frac{1}{3} B \cdot h^3$$

$$I_y = \frac{1}{3} h \cdot B^3$$

$$I_x = \sum A_i \cdot d_i^2$$

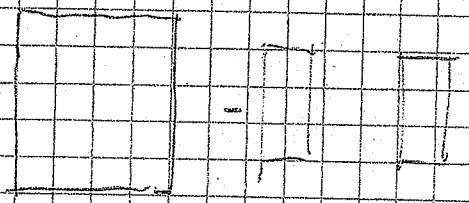
$$I_{xG} = A_t \cdot d_y^2$$

$$I_{yG} = A_t \cdot d_x^2$$

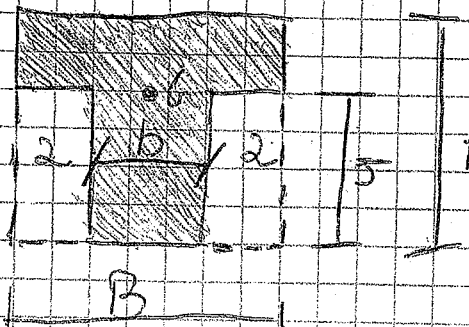


$$I_{xG} = \left(\frac{1}{12} B \cdot H^3\right) - 2\left(\frac{1}{12} b \cdot h^3\right) =$$

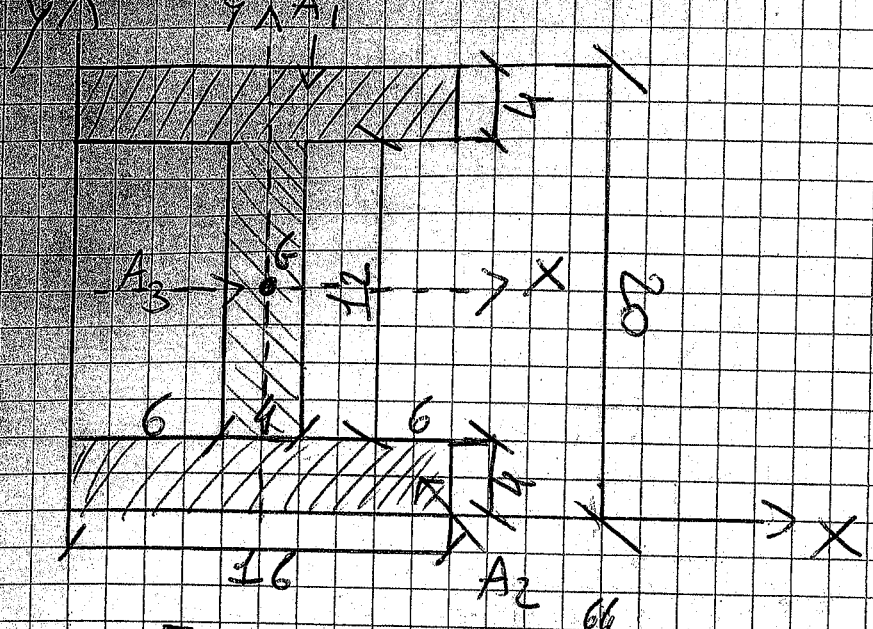
$$I_{totx_G} = \left[\frac{1}{12} \cdot B e^3 + A_1 \left(\frac{b}{2} + \frac{c}{2}\right)^2 \cdot 2 + \frac{1}{12} c\right.$$



RETTANGOLO

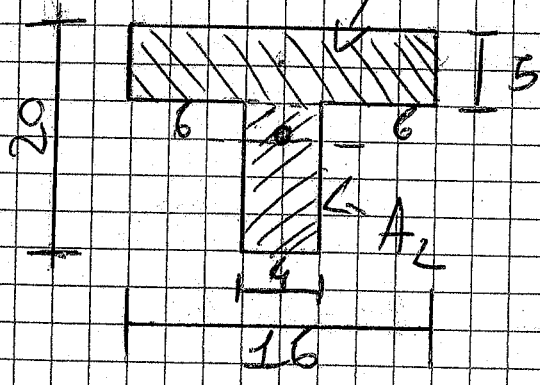


$$I_{xG} = \left(\frac{1}{12} B \cdot H^3\right) - 2\left(\frac{1}{12} 2 \cdot 5^3\right)$$



$$I_{xG} = \left[\left(\frac{1}{12} \cdot 16 \cdot 4^3\right) + \underset{64}{A_1}\left(\frac{4}{2} + \frac{6}{2}\right)^2\right] \cdot 2 + \frac{1}{12} \cdot 4 \cdot 12^3 = 8938,1$$

$$I_{xG} = \frac{1}{12} \cdot 16 \cdot 25^3 - 2\left(\frac{1}{12} \cdot 6 \cdot 12^3\right) = 8938,6$$



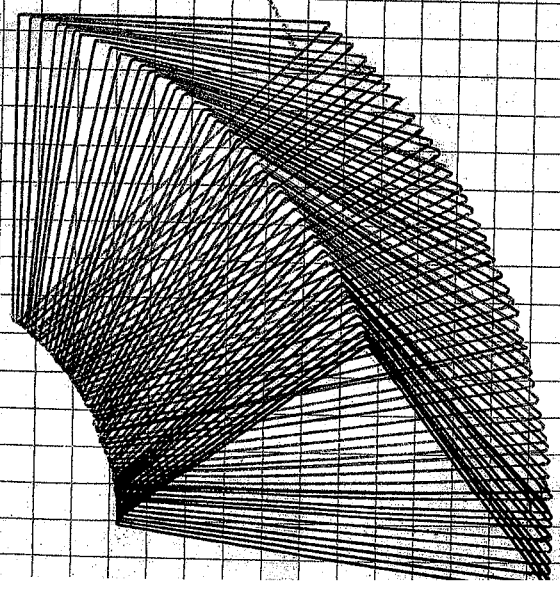
$$S_x = 80 \cdot 17,5 + 60 \cdot 7,5 = 1400 + 450 = 1850$$

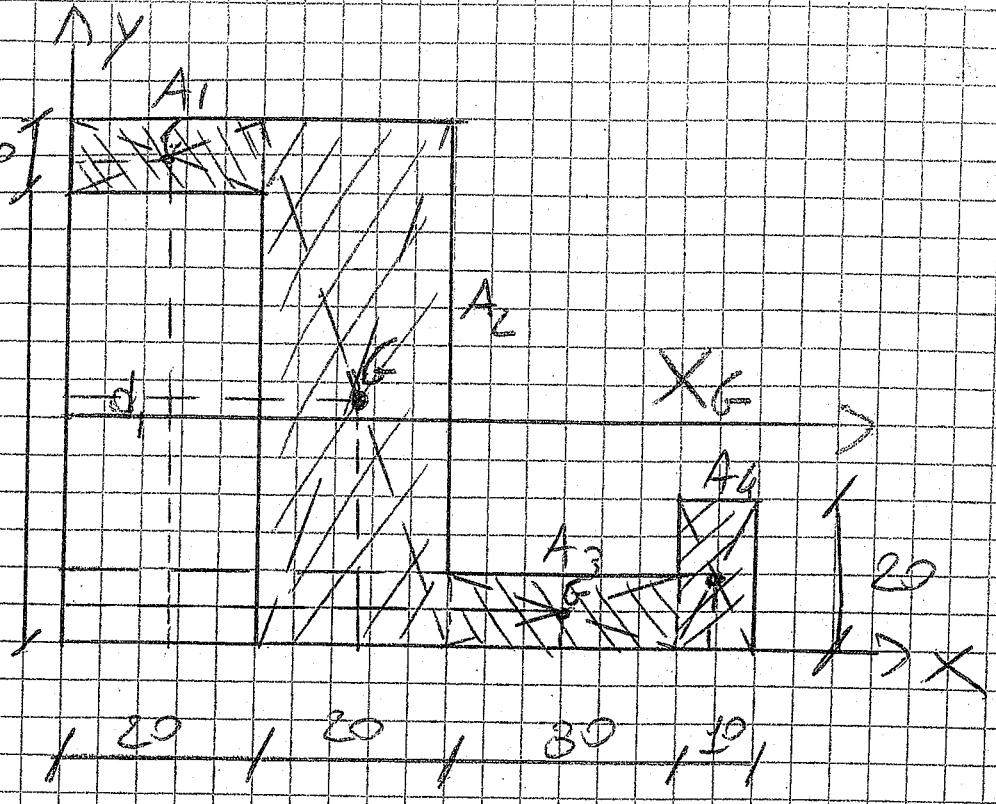
$$y_G = \frac{S_x}{A_G} = \frac{1850}{140} = 13,21 \text{ cm}$$

$$I_{xG} = \left[\left(\frac{1}{12} \cdot 16 \cdot 5^3\right) + 80\left(4,29\right)^2\right] + \frac{1}{12} \cdot 4 \cdot 1$$

$$I_{xG} = \left[\left(166,67\right) + 1472,33\right] + 1125 + 60$$

$$1639 + 3084,25 = 4723,25$$





$A_{tot} = A_1 + A_2 + A_3 + A_4$

$= 200 + 1400 + 300 + 200 = 2100 \text{ cm}^2$

$Sx_{tot} = Sx^1 + Sx^2 + Sx^3 + Sx^4$

$= A_1 \cdot d_1 + A_2 \cdot d_2 + A_3 \cdot d_3 + A_4 \cdot d_4$

$200 \cdot 65 + 1400 \cdot 35 + 300 \cdot 5 + 200 \cdot 10 = 65500 \text{ cm}^3$

$y_c = \frac{Sx}{At} = \frac{65500}{2100} = 31,19 \text{ cm}$

$Sy_{tot} = 200 \cdot 10 + 1400 \cdot 30 + 300 \cdot 55 + 200 \cdot 75 = 75500 \text{ cm}^3$

$x_c = \frac{Sy}{At} = \frac{75500}{2100} = 35,95 \text{ cm}$

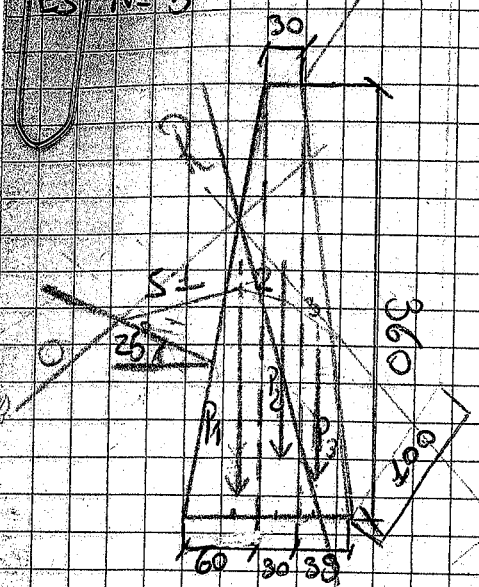
ES. N°6

Scala: 1 cm = 60 cm Peso calcestruzzo = 24

1 cm = 10 KN Spinta (S) = 25 KN

Peso?

Tutte le quote sono in cm



$P = V \cdot ps$

60 cm = 0,60 m

360 cm = 3,6 m

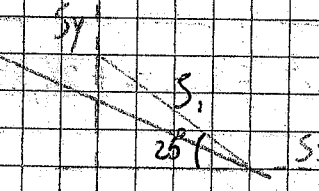
30 cm = 0,30 m

39 cm = 0,39 m

$P_{p1} = \frac{0,60\,m \cdot 3,60\,m}{2} \cdot 1,00\,m \cdot 24\,KN/m^3 = 25,92\,KN$

$P_{p2} = 0,30\,m \cdot 3,60\,m \cdot 1,00\,m \cdot 24\,KN/m^3 = 25,92\,KN$

$P_{p3} = \frac{0,39\,m \cdot 3,60\,m}{2} \cdot 1,00\,m \cdot 24\,KN/m^3 = 16,85\,KN$

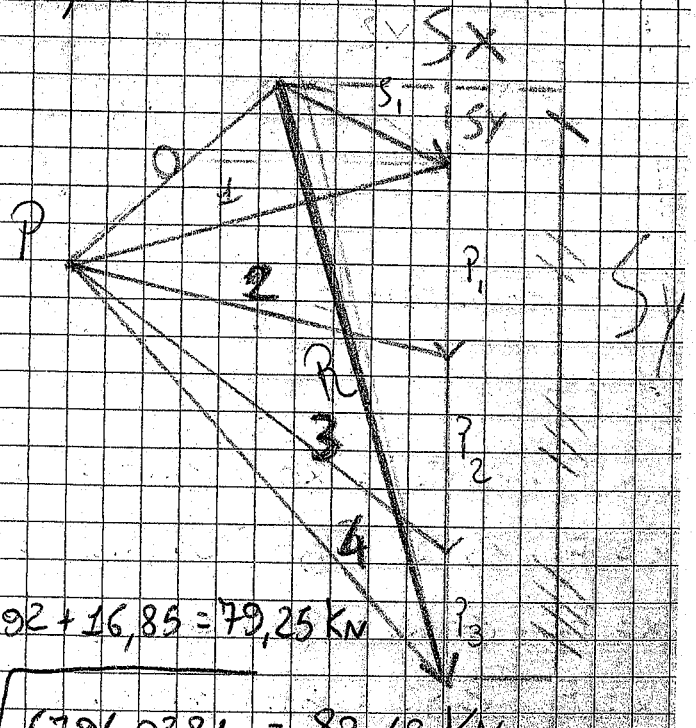


$Sx = S_1 \cdot \cos 2^\circ$

$25 \cdot \cos 25^\circ = 22,66\,KN$

$Sy = S_1 \cdot \text{sen}\,2^\circ$

$25 \cdot \text{sen}\,25^\circ = 10,56\,KN$



$\Sigma Sy = Sy + P_1 + P_2 + P_3 = 10,56 + 25,92 + 25,92 + 16,85 = 79,25\,KN$

$R = \sqrt{Sx^2 + \Sigma Sy^2} = \sqrt{22,66^2 + 79,25^2} = \sqrt{6796,0381} = 82,43\,KN$

$2^\circ = 2\text{rctg} * \frac{Ry}{Rx} = \text{arctg}\frac{79,25}{22,66} = 74^\circ,0431$

№ 4

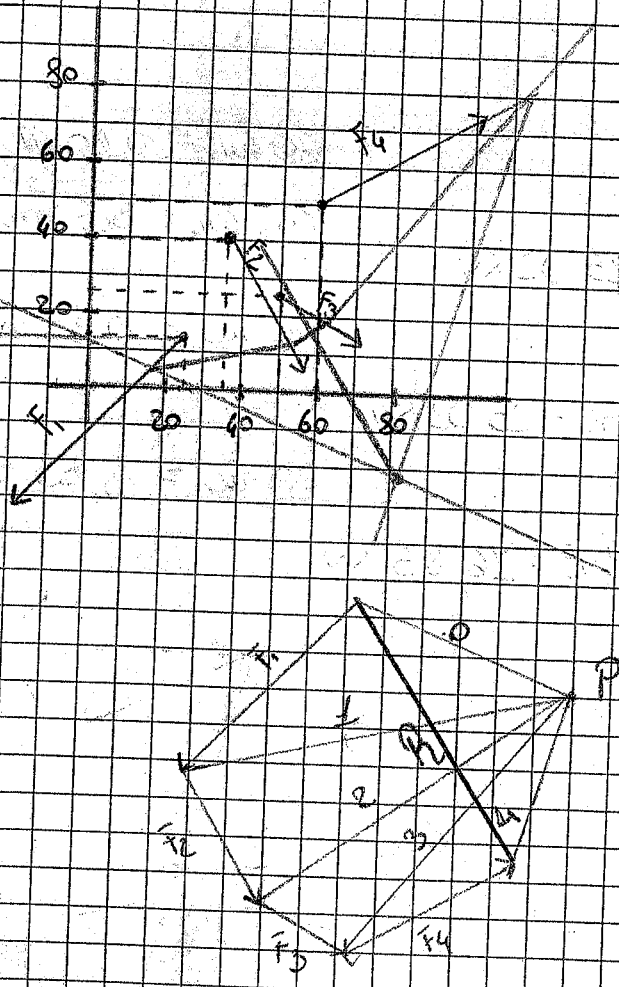
$F_1 = 80N$ $F_2 = 50N$ $F_3 = 30N$ $F_4 = 60N$

$F_1 = (25,25)$ $F_2 = (35,80)$ $F_3 = (80,35)$ $F_4 = (60,50)$

$F_1 = 225°$ $F_2 = 300°$ $F_3 = 330°$ $F_4 = 30°$

Skala: $\frac{1cm}{20cm}$

$\frac{1cm}{25N}$



$R = \sqrt{R_x^2 + R_y^2} = \sqrt{46,37^2 + 84,87^2}$

$= 96,76N$

$\alpha = arctg \frac{R_y}{R_x} = \frac{84,87}{46,37} =$

$F_{1x} = F_1 \cdot \cos\alpha = 80 \cdot \cos 225° = -56,57N$

$F_{2x} = F_2 \cdot \cos\alpha = 50 \cdot \cos 300° = +25,00N$

$F_{3x} = F_3 \cdot \cos\alpha = 30 \cdot \cos 330° = +25,98N$

$F_{4x} = F_4 \cdot \cos\alpha = 60 \cdot \cos 30° = +51,96N$

$F_{1y} = F_1 \cdot \sin\alpha = 80 \cdot \sin 225° = -56,57N$

$F_{2y} = F_2 \cdot \sin\alpha = 50 \cdot \sin 300° = -43,30N$

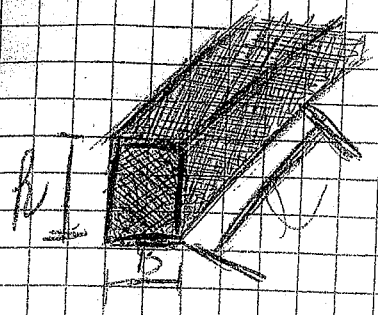
$F_{3y} = F_3 \cdot \sin\alpha = 30 \cdot \sin 330° = -15,00N$

$F_{4y} = F_4 \cdot \sin\alpha = 60 \cdot \sin 30° = +30,00N$

$\Sigma x = 46,37N$

$\Sigma y = 84,87N$

TRAVE = ELEMENTO LINEARE

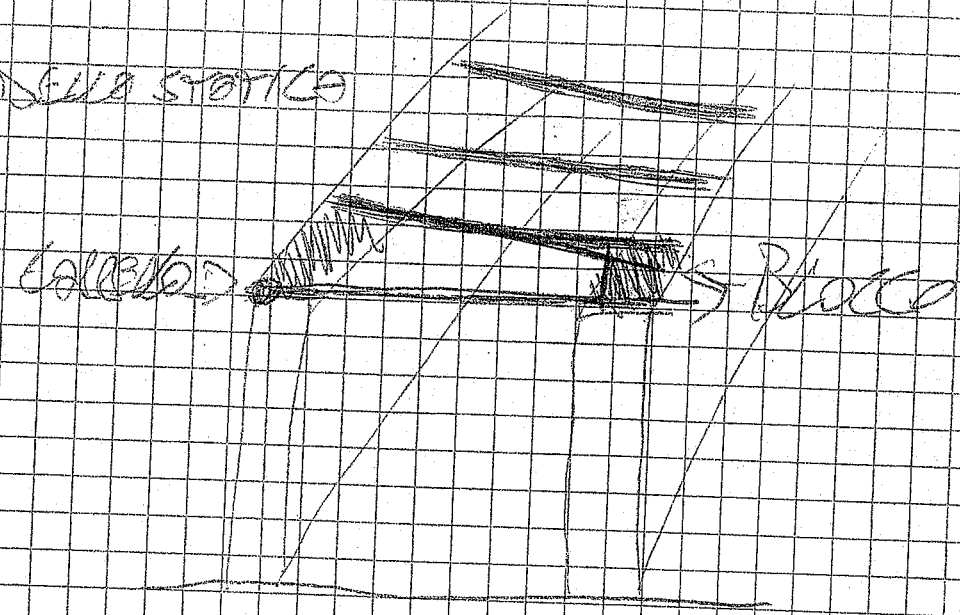


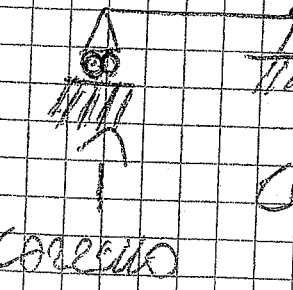
PER RAPPRESENTARLA USIAMO STATICA

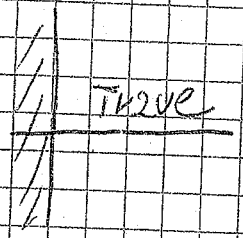
$\sum F_x = 0$

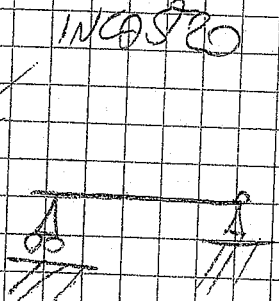
$\sum F_y = 0$

$\sum M = 0$









$3 \cdot 1 = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 0$

$3 = 3$

STRUTTURE LABILE, ISOSTATICA, IPERSTATICA

Per capire se una struttura è labile isostatica

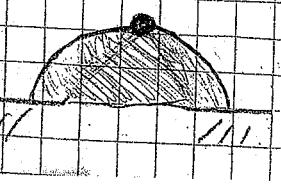
$3m \lessgtr \sum$ Vincoli $(a + c + i)$

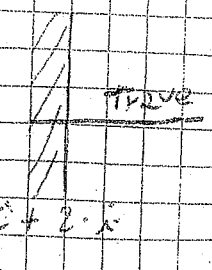
N.B. m = NUMERO TRAVI

a = APPOGGIO

c = CERNIERA

i = INCASTRO





$3 \cdot 1 = a \cdot 0 + c \cdot 0 + i \cdot 3$

$3 = 3 \Rightarrow$ ISOSTATICA

$3 \cdot 2 = a \cdot 0 + 1 \cdot c + 2 \cdot i$

$6 = 2 + 4$

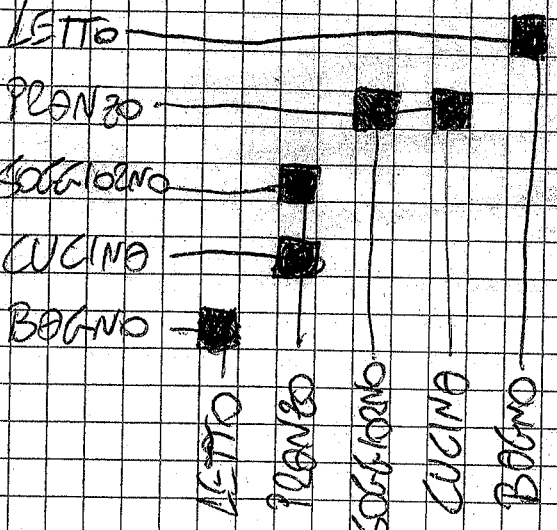
$6 < 8 \Rightarrow$ IPERSTATICA

Spazio Globale

=

Spazio usato dall'oggetto + Spazio per usare l'oggetto

COLLEGAMENTI CORRETTI



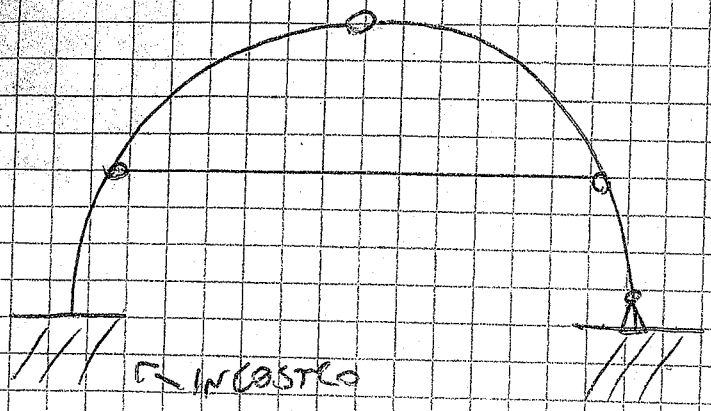
RAPPORTO AEREOILLUMINANTE

$\frac{1}{8}$

ES. 40 m² di [illegible] NANO.

AREA ILLUMINANTE = $40 \cdot \frac{1}{8} = 5$ m² di FINESTRA

$\frac{1}{1}$



$3n \geq \Sigma$ VINCOLI $(2+c+i)$ $\quad 3n \leq \Sigma\ 2\cdot 1 + c\cdot 2 + i\cdot 3$

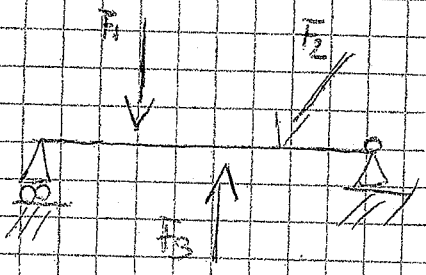
$9 \geq \Sigma\ 4\cdot 2 + 1\cdot 3$

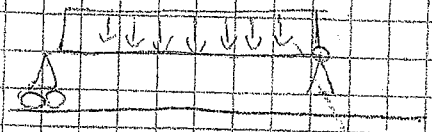
$9 > 8+3$

$9 > 11$

$F_1 = 20$ KN

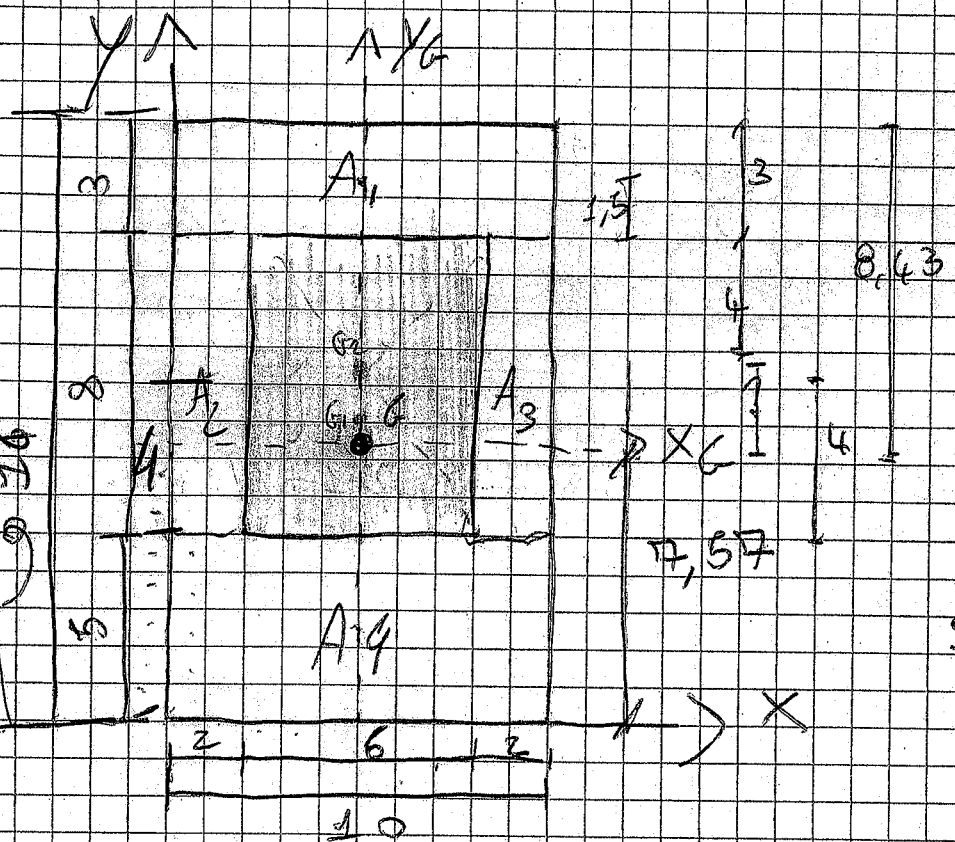
$F_2 = 30$ KN





42

Es.



$A_1 = 10 \cdot 3 = 30\ cm^2$

$A_2 = 2 \cdot 8 = 16\ cm^2$

$A_3 = 2 \cdot 8 = 16\ cm^2$

$A_4 = 10 \cdot 5 = 50\ cm^2$

$S_X = 30 \cdot 14,5 + 16 \cdot 9 + 16 \cdot 9 + 50 \cdot 2,5 = 435 + 144 + 144 + 125 = 848\ cm^3$

$Y_G = \frac{848}{112} = 7,57\ cm$

$S_Y = 30 \cdot 5 + 16 \cdot 1 + 16 \cdot 9 + 50 \cdot = 560\ cm^3$

$X_G = \frac{560}{112} = 5\ cm$

$I_{XG\,TOT} = \left[\frac{1}{12} \cdot 10 \cdot 3^3 + 30 \cdot (6,93)^2\right] + \left[\frac{1}{12} \cdot 2 \cdot 8^3 + 16 \cdot (1,43)^2\right] + \left[\frac{1}{12} \cdot 2 \cdot 16 \cdot (1,43)^2\right] + \left[\frac{1}{12} \cdot 10 \cdot 5^3 + 50 \cdot (5,07)^2\right] = [22,5 + 1440,74] + [85,33 + 32,7136] + [85,33 + 32,7136] + [104,17 + 1285,24] = 1463,247 + 118,0486 + 118,0486 + 1389,415 = \boxed{3088,76\ cm^4}$

$I_{YG\,TOT} = \left[\frac{1}{12} \cdot 3 \cdot 10^3 + 30 \cdot 0\right] + \left[\frac{1}{12} \cdot 8 \cdot 2^3 + 16 \cdot (4)^2\right] + \left[\frac{1}{12} \cdot 8 \cdot 2^3 + 16 \cdot (4)^2\right] + \left[\frac{1}{12} \cdot 5 \cdot 10^3 + 500\right] = [250 + 0] + [5,3 + 256] + [5,3 + 256] + [416,67 + 0] = 250 + 261,3 + 261,3 + 416,67 = \boxed{1189,33\ cm^4}$

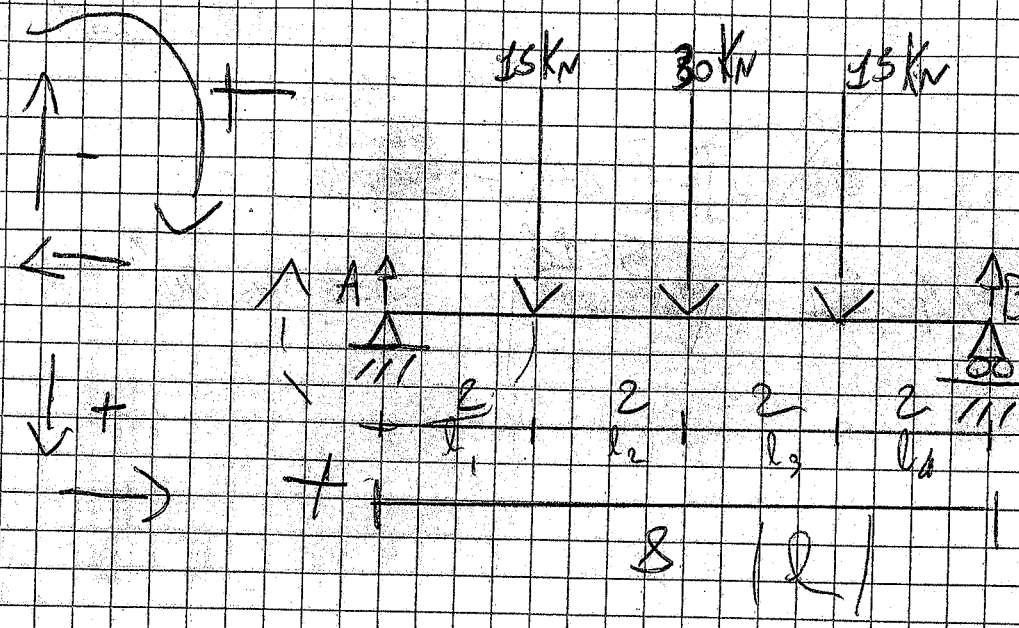
43

$$I_{xG\,TOT} = \left[\frac{1}{12} \cdot 10 \cdot 16^3 + (10 \cdot 16)(0{,}43)^2\right] - \left[\frac{1}{12} \cdot 6 \cdot 8^3 + (8 \cdot 6) \cdot (1{,}43)^2\right] =$$

$$\left[3413{,}33 + 29{,}58\right] - \left[256 + 38{,}16\right] =$$

$$3442{,}91 - 354{,}16 = 3088{,}76 \text{ cm}^4$$

$$I_{yG\,TOT} = \left[$$



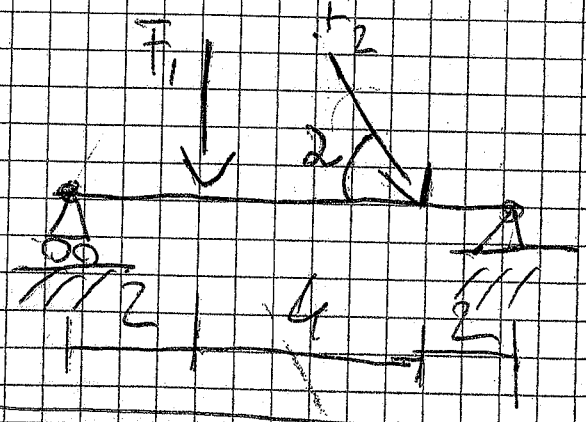
Дано:
$F_1 = 15$ kN
$F_2 = 30$ kN
$F_3 = 15$ kN
$l_1 = 2$ м
$l_2 = 4$ м
$l_3 = 6$ м
$l = 8$ м
R_A = ?
R_B = ?

$R_A = 60 - 30 = 30$ kN

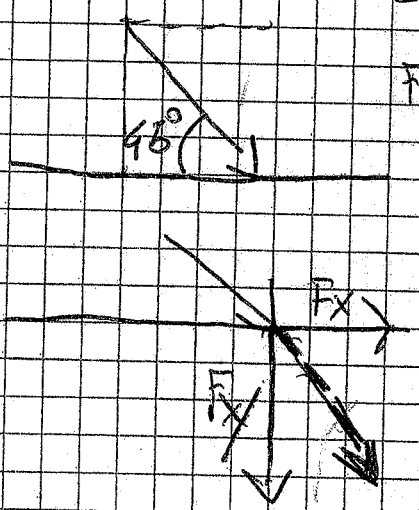
$$\begin{cases} \sum F_y = F_1 + F_2 + F_3 - R_A - R_B = 0 \Rightarrow 60 - R_A - 30 = 0 \\ \sum M_A = R_A \cdot 0 + F_1 \cdot l_1 + F_2 \cdot l_2 + F_3 \cdot l_3 + R_B \cdot l = 0 \end{cases}$$

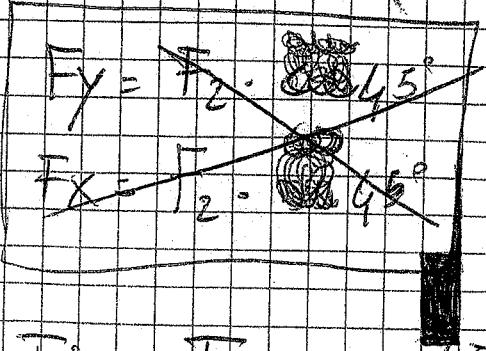
$0 + 30 + 120 + 90 - R_B \cdot 8 = 0$

$240 - R_B \cdot 8 = 0 \Rightarrow R_B = \frac{240}{8} = 30$ kN



$F_1 = 30$ kN
$F_2 = 40$ kN
↓
F_{y2} ; F_x





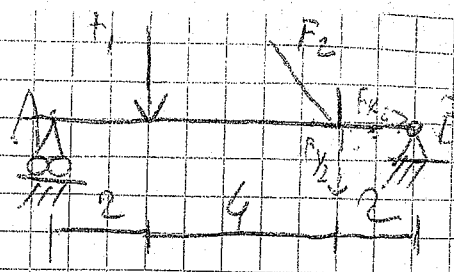
45

$F_y = F_2 \cdot \sin 45$

$F_x = F_2 \cdot \cos 45$

$F_{y_2} = F_2 \cdot \sin 45^\circ = 28,28$ kN

$F_{x_2} = F_2 \cdot \cos 45^\circ = 28,28$ kN



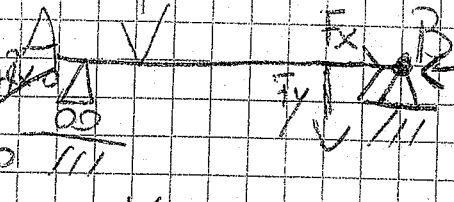
$\Sigma F_x = 0 \Rightarrow F_x + R_x = 0 \quad 28,28 + R_x = 0 \quad R_x = -28,28$ kN

$\Sigma F_y = 0 \Rightarrow R_A + F_1 + F_y + R_B = 0$

$\Sigma M_A = 0 \Rightarrow R_A \cdot 0 + F_1 \cdot 2 + F_y + 6 + F_x \cdot 0 - R_B \cdot 8 + 0 = 0$

$60 + 169,68 - R_B \cdot 8 = 0$

$-R_B = \frac{229,68}{8} = -28,71$ kN



$\Sigma F_y \Rightarrow R_A = F_1 + F_y - R_B = 30 + 28,28 - 28,71 = 29,57$ kN

Verifica $\Rightarrow$ Somma Forze verticali = somma Reazioni

$28,28 + 30 = 28,71 + 29,57$

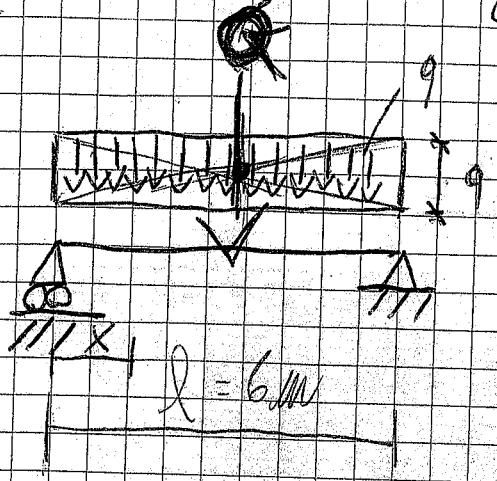
$58,28 = 58,28$

CARICO UNIFORMEMENTE DISTRIBUITO

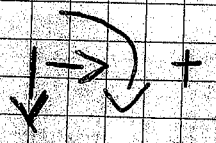
$q = 300$ N/ml (metro lineare)

$l = 6$ m

$Q = q \cdot l = 300 \cdot 6 = 1800$ N



$x \begin{cases} 0 \\ l \end{cases}$



$\Sigma F_x = 0 \rightarrow = 0$

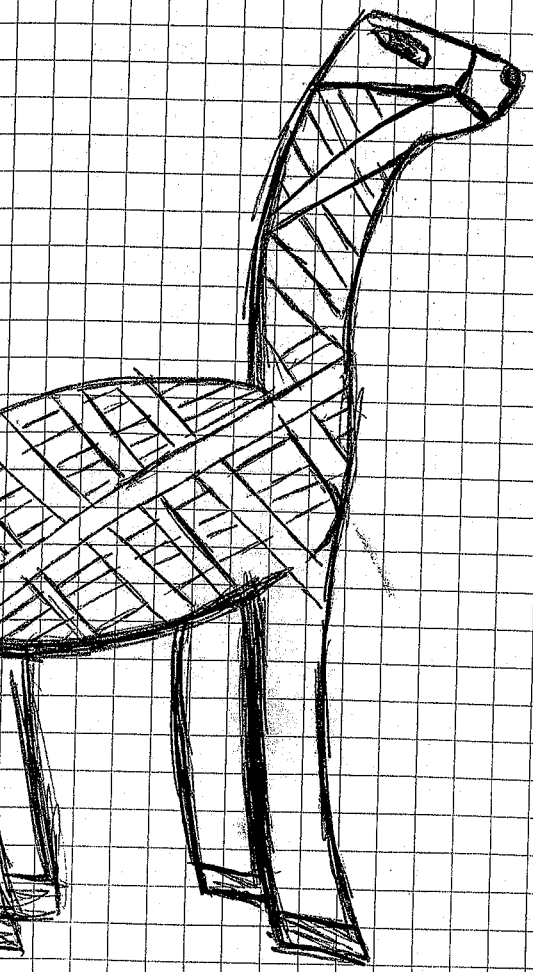
$\Sigma F_y = 0 \Rightarrow RA + (q \cdot l) + RB = 0$

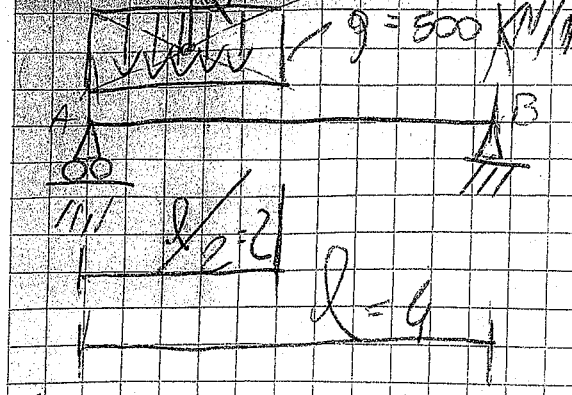
$\Sigma T_A = 0 \Rightarrow RB \cdot 0 + \left(q \cdot l \cdot \frac{l}{2}\right) - RB \cdot l$

$0 + \frac{1}{2} q l^2 - RB \cdot l = 0$

$\frac{1}{2} q l^2 = RB \cdot l$

$RB = -\frac{1}{2} q l$





$q = 500\ KN/ml$

$l = 4\ ml$

$Q = q \cdot \frac{l}{2} = 500 \cdot 2 = 1000$

$\Sigma F_x = 0 \Rightarrow = 0$

$\Sigma F_y = 0 \Rightarrow RA + (q \cdot \frac{l}{2}) + RB = 0 \longrightarrow RA + q \cdot \frac{l}{2} + (-\frac{1}{8} q l) = 0$

$\Sigma M_A = 0 \Rightarrow RA \cdot 0 + (q \cdot \frac{l}{2} \cdot \frac{3}{4}) - RB \cdot l = 0$

$0 + \frac{1}{8} q l^2 - RB \cdot l = 0$

$\frac{1}{8} q l^2 = RB \cdot l$

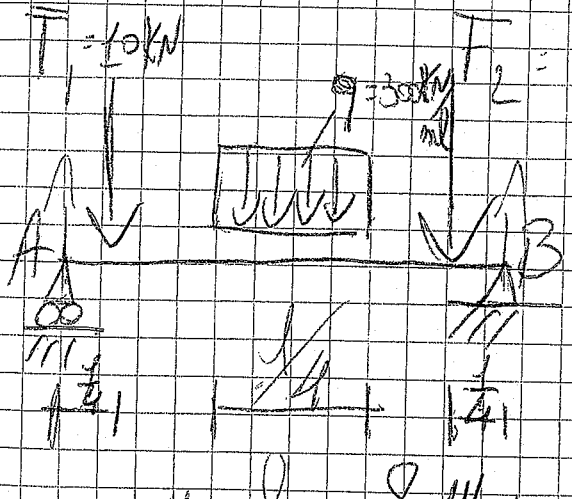
$RB = -\frac{1}{8} q l$

$RB = -250$

$RA + 1000 - 250 = 0$

$RA = -750$

$RA + RB = Q$



$l = 8$

$F_1 = 10\ KN$

$F_2 = 20\ KN$

$q = 300\ KN/ml$

$RA = ?$

$RB = ?$

$Q = q \cdot \frac{l}{4}$

$\Sigma F_x = 0 \Rightarrow \Sigma F_x = 0$

$\Sigma F_y = 0 \Rightarrow \Sigma F_y = RA + F_1 + q \cdot \frac{l}{4} + F_2 + RB = 0 \Rightarrow RA + 10 + 600 + 20 - 31$

$\Sigma M_A = 0 \Rightarrow RA \cdot 0 + F_1 \cdot 2 + q \cdot \frac{l}{4} \cdot 4 + F_2 \cdot 6 + RB \cdot 8 = 0$

$0 + 20 + ql + 120 - RB \cdot 8 = 0$

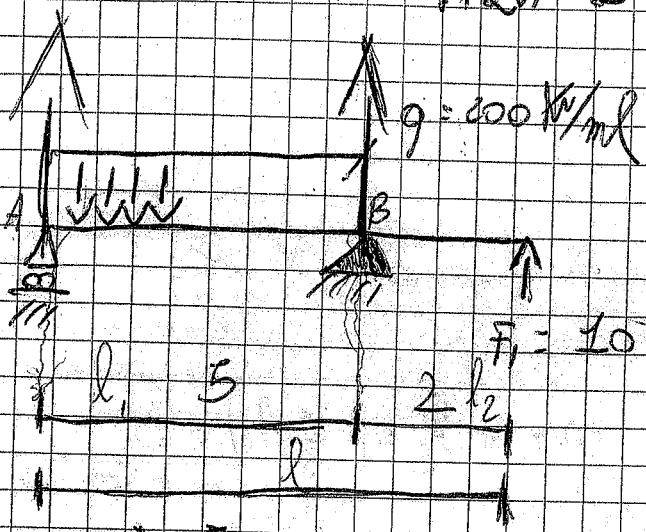
$+RB \cdot 8 = 20 + 2400 + 120$

$RB = \frac{2540}{8} = 317,5$

$RA = 312,5$

$RA + RB = F_1 + Q + F_2$

Tr2vi 2 Sbalzo



$q = 200\ kN/ml$
$F_1 = 10\ kN$
$l = 7\ m$
$R_A = ?$
$R_B = ?$
$Q = q \cdot l_1$

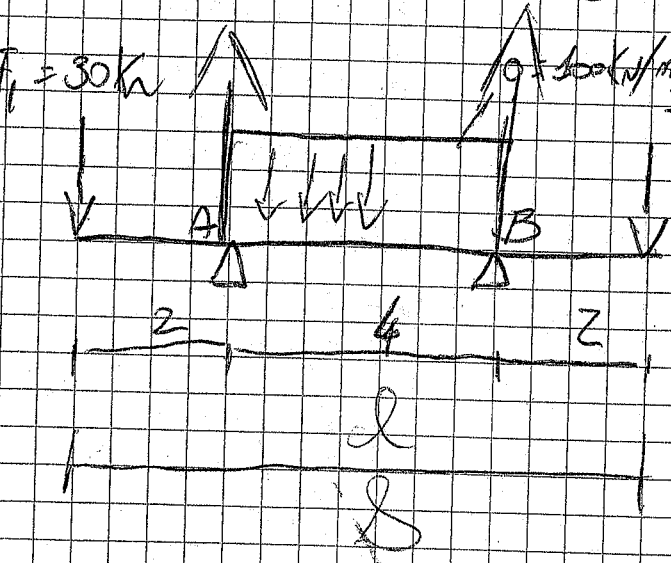
$\begin{cases} \Sigma F_x = 0 \Rightarrow = 0 \\ \Sigma F_y = 0 \Rightarrow R_A + (q \cdot l_1) + R_B + F_1 = 0 \Rightarrow R_A + 1000 - 486 - 70 = 0 \\ \Sigma M_A = 0 \Rightarrow R_A \cdot 0 + \left(q \cdot l_1 \cdot \frac{l_1}{2}\right) - R_B \cdot l_1 - 70 \cdot 7 = 0 \end{cases}$

$0 + q \cdot l_1 \cdot \frac{l_1}{2} - R_B \cdot l_1 - 70 = 0$

$-R_B \cdot l_1 = -q \cdot l_1 \cdot \frac{l_1}{2} + 70$

$R_B = -486\ kN$

$R_A = -504\ kN$



$q = 100\ kN/ml$
$F_1 = 30\ kN$
$F_2 = 5\ kN$
$R_A = ?$
$R_B = ?$
$Q = q \cdot \frac{l}{2}$

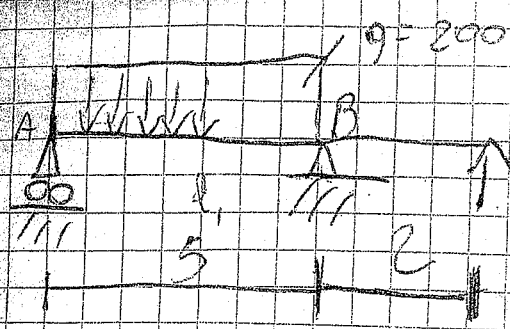
$\begin{cases} \Sigma F_x = 0 \Rightarrow = 0 \\ \Sigma F_y = 0 \Rightarrow F_1 + q \cdot \frac{l}{2} + F_2 = 0 \\ \Sigma M_A = 0 \Rightarrow -30 \cdot 2 + \left(q \cdot \frac{l}{2} \cdot \frac{l}{4}\right) + 5 \cdot 6 - R_B \cdot \frac{l}{4} = 0 \end{cases}$

$-60 + \frac{1}{8} q l^2 + 30 - R_B \cdot \frac{l}{4} = 0$

$-30 + \frac{1}{8} q l^2 = R_B \cdot \frac{l}{4}$

$-30 + 800 = R_B \cdot 4$

$R_B = \frac{770}{2} = 410$



q = 200 KN/ml
F1 = 1000 KN/m
l = 7 ml
RA = ?
RB = ?
Q = q · l

$$\begin{cases} \Sigma F_x = 0 \Rightarrow 0 \\ \Sigma F_y = 0 \Rightarrow R_A + (q \cdot l_1) + R_B + F_1 = 0 \\ \Sigma M_A = 0 \Rightarrow R_A \cdot 0 + \left(q \cdot l_1 \cdot \frac{l_1}{2}\right) + R_B \cdot l_1 - 1000 \cdot 7 = 0 \end{cases}$$

$$R_A + 1000 + 900 - 1000 = 0 \Rightarrow R_A = -900\,KN$$

$$0 + \frac{1}{2} q l_1^2 + R_B \cdot l_1 - 7000 = 0$$

~~$R_B \cdot l_1 = -\frac{1}{2} q l_1^2 + 7000$~~

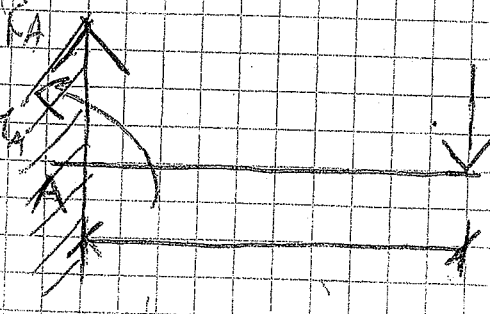
~~$R_B = -500 + 7000$~~

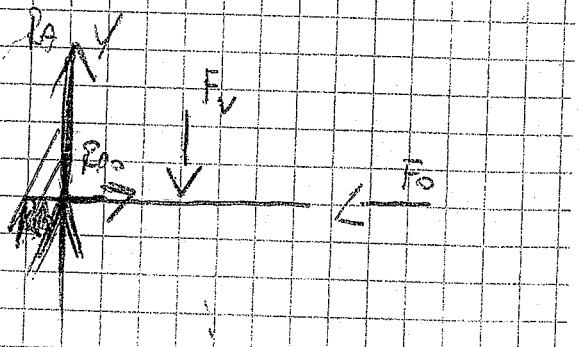
~~$R_B = 6500\ KN$~~

$$2600 + R_B \cdot 5 - 7000 = 0$$

$$-R_B \cdot 5 = -2600 + 7000$$

$$R_B = \frac{4500}{5} = +900\ KN$$





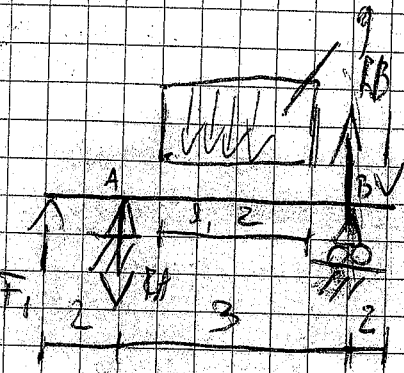
$$\Sigma M_A = F \cdot l + M_A = 0$$

$$+M_A = -F \cdot l$$

$$\Sigma F_y = R_A + F_1 = 0$$

$$R_A = -F_1$$

1



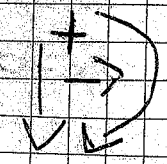
$F_1 = 100$ KN

$F_2 = 20$ KN

$q = 50$ KN/ml

$3n = 1 \cdot 2 + 2 \cdot 2 + 3 \cdot 2$

$3 = 3\ (150)$ RA? RB?



$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases}$

$\begin{cases} \Sigma F_x = \text{non ci sono forze orizzontali} \Rightarrow = 0 \\ \Sigma F_y \Rightarrow F_1 + RA + (q \cdot l_1) + RB + F_2 = 0 \Rightarrow -100 + RA + 100 + 150 + 20 \\ \Sigma M_A \Rightarrow +F_1 \cdot 2 + RA \cdot 0 + (q \cdot l_1 \cdot 1{,}5) + RB \cdot 3 + F_2 \cdot 5 \end{cases}$

$200 + 150 + RB \cdot 3 + 100$

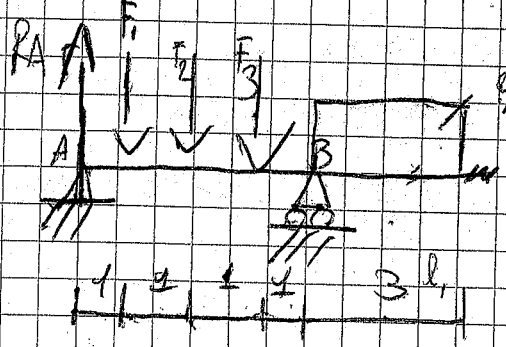
$-RB \cdot 3 = -200 - 150 + 100$

$-RB = -\frac{150}{3}$

$RB = 150$

$RA = -170$

2



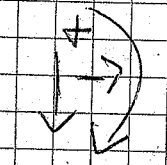
$F_1 = 10$ KN

$F_2 = 20$ KN

$F_3 = 30$ KN

$q = 25$ KN/ml

RA? RB?



$\Sigma F_x = 0$

$\Sigma F_y = 0$

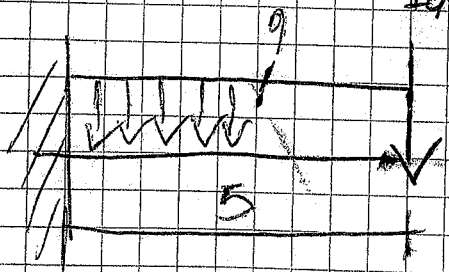
$\Sigma M = 0$

$\begin{cases} \Sigma F_x \Rightarrow 0 \\ \Sigma F_y \Rightarrow F_1 + F_2 + F_3 + RA + RB + (q \cdot l_1) = 0 \Rightarrow 60 + RA + 312{,}5 + 75 = 0 \\ \Sigma M_A \Rightarrow 10 + 60 + 90 + RA \cdot 0 + RB \cdot 4 + 612{,}5 \end{cases}$

$RA = -373{,}13$ KN

$160 - RB \cdot 4 + 612{,}5$

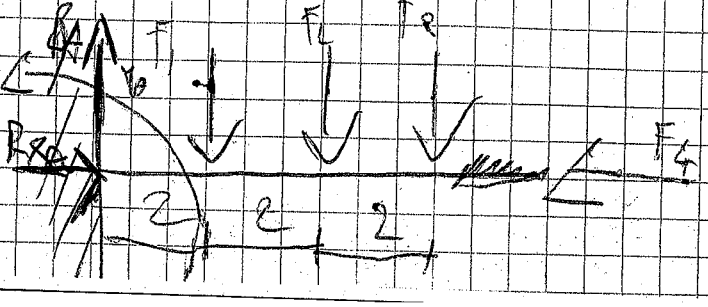
$+RB = +178{,}13$ KN



$q = 10$ KN/ml

$F_1 = 24$ KN

5



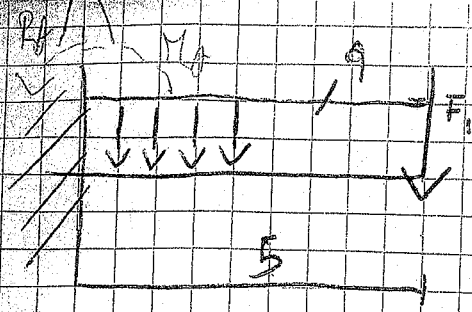
$F_1 = 15$ KN

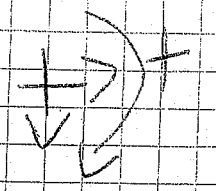
$F_2 = 10$ KN

$F_3 = 5$ KN

$F_4 = 20$ KN

③





$F_1 = 24$ kN

$q = 10$ kN/m

$l = 5$ m

$3 \cdot b = 1 \cdot a + 2 \cdot c + 3 \cdot i$

$3 = 3$ (ISTINITO)

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = R_A + q \cdot l + F_1 = 0 \\ \sum M = R_A \cdot 0 + \left(q \cdot l \cdot \frac{l}{2}\right) + F_1 \cdot 5 + M_A = 0 \end{cases}$$

$M_A =$ (—, =, ≡)

④

$\sum F_x = 0 \Rightarrow R_{xA} - F_4 = 0$

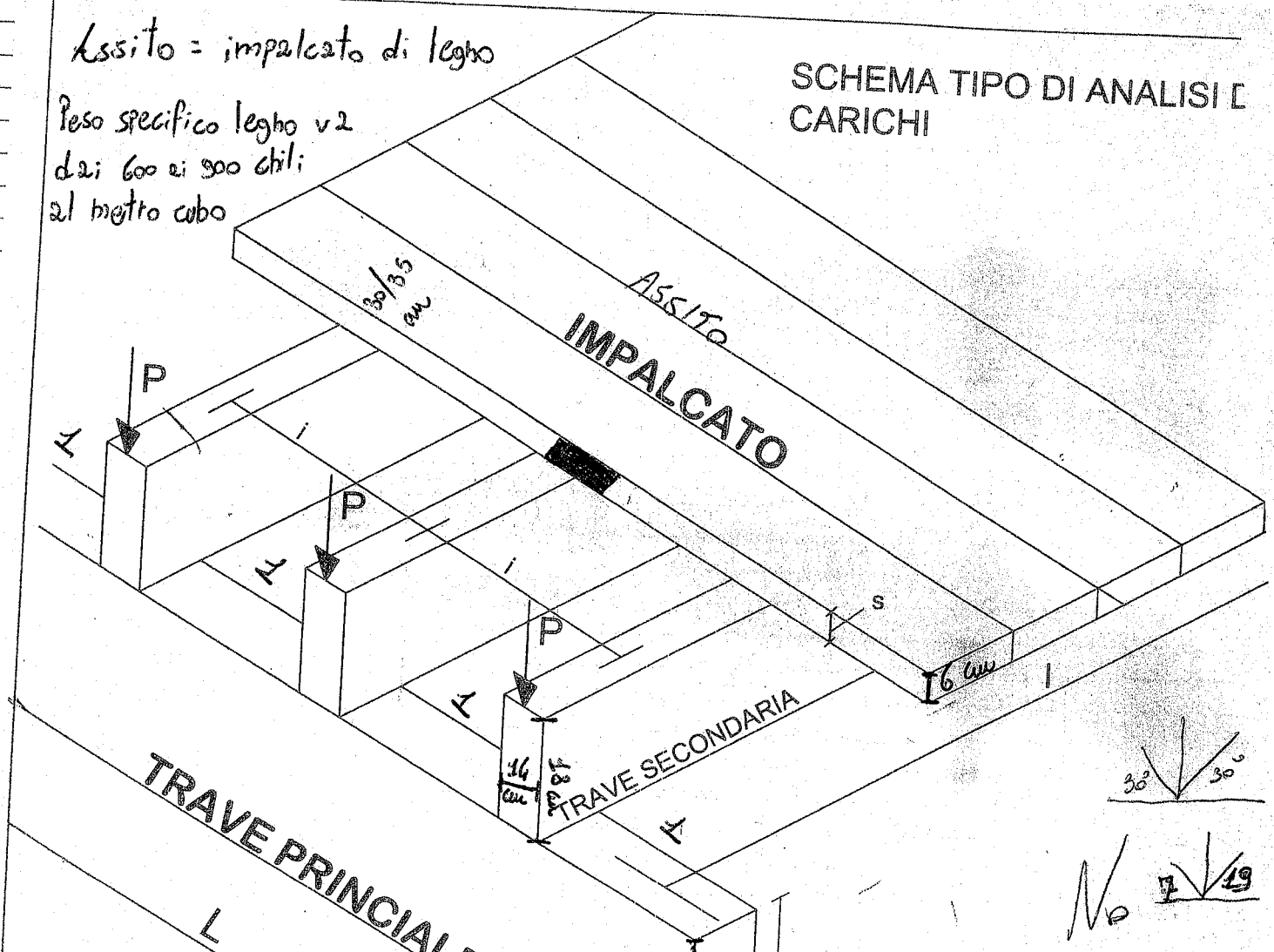
$\sum F_y = 0 \Rightarrow R_A + F_1 + F_2 + F_3$

$\sum M_A = 0 \Rightarrow R_A \cdot 0 + F_1 \cdot l_1 + F_2 \cdot l_2 + F_3 \cdot l_3 - F_4 \cdot 0 - M_A =$

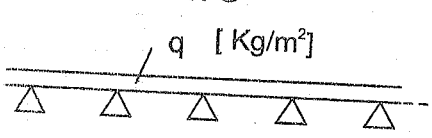
Assito = impalcato di legno

Peso specifico legno v2 da: 600 a: 900 chili al metro cubo

SCHEMA TIPO DI ANALISI D CARICHI



IMPALCATO



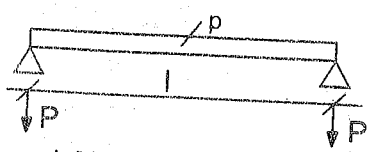
sovraccarico accidentale Kg/m²

peso proprio impalcato

$(s \cdot 1 \cdot 1)\ m^3/m^2 \times \gamma\ Kg/m^3 = Kg/m^2$

q = sovr. acc. + p.p.i [Kg/m²]

TRAVE SECONDARIA



p = q·i [Kg/m] + b·h·1·γ [Kg/m]

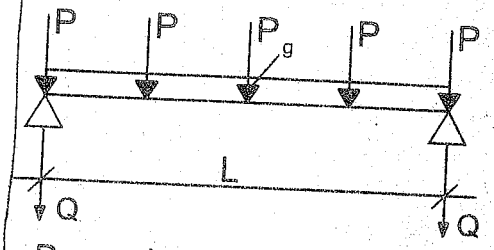
$P = \frac{p \cdot l}{2}$ [Kg]

PILASTRO

$P_p = a^2 \cdot h_1 \cdot \gamma_p$ [Kg]

$R = Q + P_p$ [Kg]

TRAVE PRINCIPALE

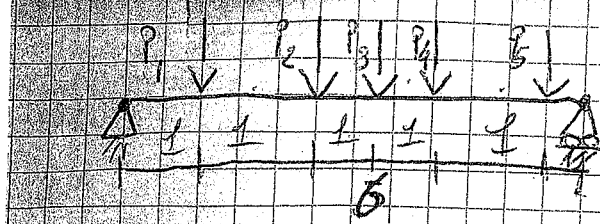


$P = \frac{p \cdot l}{2}$ [Kg]

$g = B \cdot H \cdot 1 \cdot \gamma$ [Kg/m]

$Q = \frac{\Sigma P}{2} + \frac{g \cdot L}{2}$ [Kg]

53



$l = 6\ m$

$i = 1\ m$

$\gamma_{legno} = 850\ Kg/m^3$

$P_{sbalzo} = 1 \cdot 1 \cdot 0,06 \cdot 850 = 51\ Kg$

$P_{trave\ sec} = 0,14 \cdot 0,18 \cdot 1 \cdot 850 = 21,42\ Kg$

$P_{trave\ princi} = 0,22 \cdot 0,28 \cdot 1 \cdot 850 = 52,36\ Kg$

circa $125\ Kg/ml$

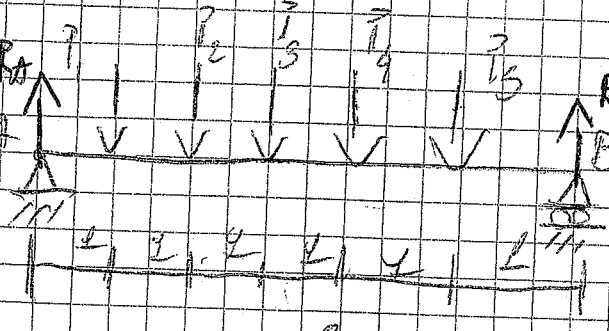
$P_{esercizio\ persone} = 200\ Kg/m^2$

$P_{tot} = 200 + 125 = 325\ Kg/ml$

$\begin{cases} P_1 = 325\ Kg \\ P_2 = 325\ Kg \\ P_3 = 325\ Kg \\ P_4 = 325\ Kg \\ P_5 = 325\ Kg \end{cases}$

$1\ Kg = 10\ N$

$325\ Kg = 3250\ N = 3,25\ KN$



$P_1 = 325\ Kg$

$P_2 = 325\ Kg$

$P_3 = 325\ Kg$

$P_4 = 325\ Kg$

$P_5 = 325\ Kg$

$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases}$

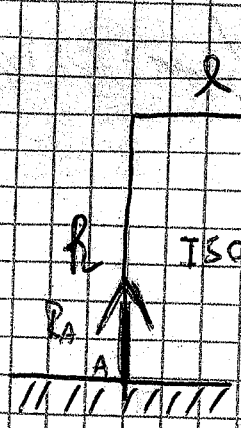
$\begin{cases} \sum F_y : R_A + P_1 + P_2 + P_3 + P_4 + P_5 + R_B = 0 \\ \sum M_A = R_A \cdot 0 + P_1 \cdot 1 + P_2 \cdot 2 + P_3 \cdot 3 + P_4 \cdot 4 + P_5 \cdot 5 + R_B \cdot 6 = 0 \end{cases}$

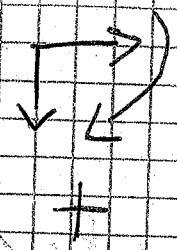
$\sum F_x = 0$

$\sum M_A = 325 + 650 + 975 + 1300 + 1625 + R_B \cdot 6 = 0$

$\sum F_y = R_A + P_1 + P_2 + P_3 + P_4 + P_5 + R_B = 0 \Rightarrow \boxed{R_A = -812,5\ Kg}$

$\sum M_A = 0 \Rightarrow R_B = \frac{4875}{6} = \boxed{-812,5\ Kg}$



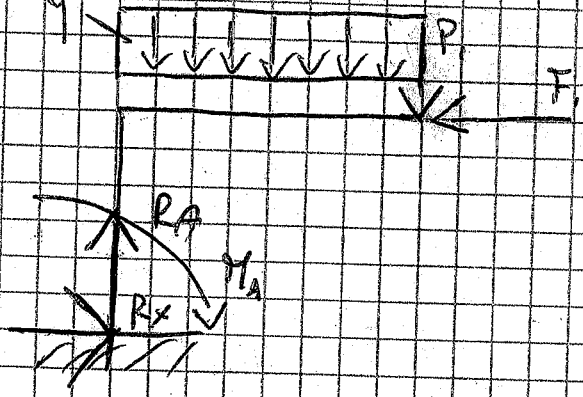


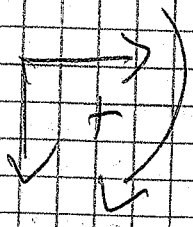
$P_1 = 10\ kN$

$h = 3\ m$

$l = 2\ m$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases} \qquad \begin{cases} \Sigma F_y = P_1 - R_A = 0 \\ \Sigma M_A = R_A \cdot 0 + P_1 \cdot l + M_A \end{cases} \qquad \begin{cases} R_A = -P_1 = -10\ kN \\ M_A = -20\ kN/m \end{cases}$$





$P_1 = 10\ kN$

$q = 5\ kN/m$

$h = 3\ m$

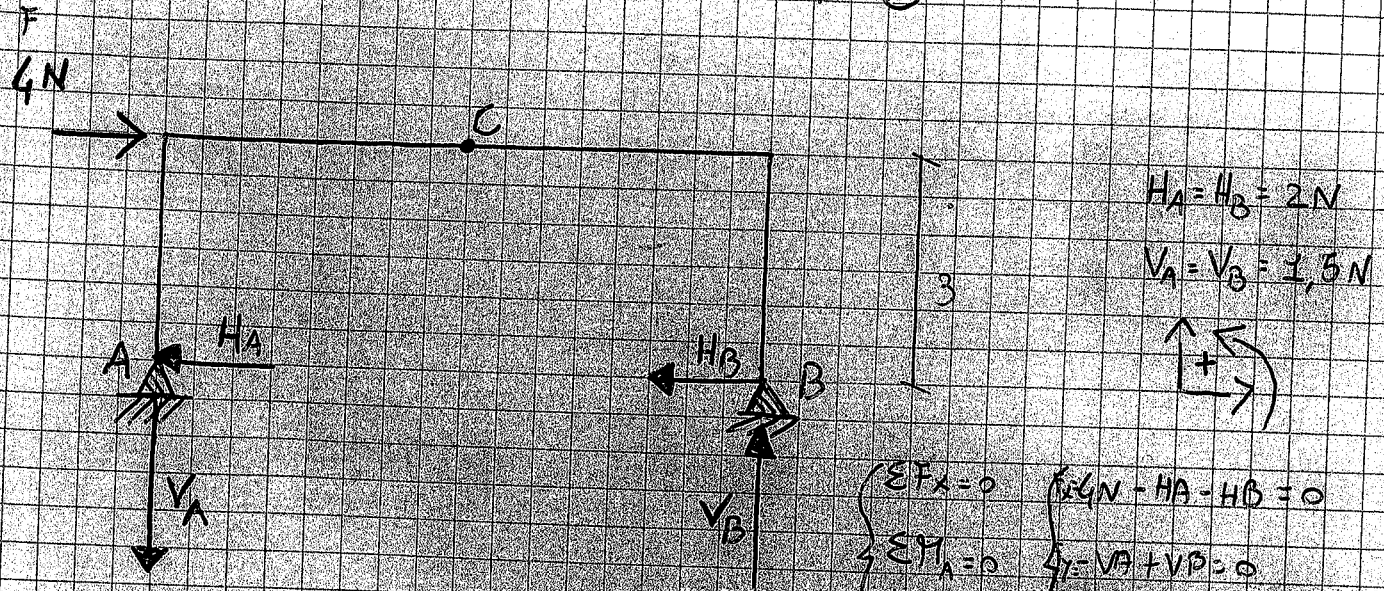
$l = 2$

$F_1 = 3\ kN$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M_A = 0 \end{cases} \qquad \begin{cases} \Sigma F_x = F_1 + R_X = 0 \\ \Sigma F_y = P_1 + q \cdot l + R_A = 0 \\ \Sigma M_A = R_A \cdot 0 + P_1 \cdot l + \left(q \cdot l \cdot \frac{l}{2}\right) + M_A - F_1 \cdot h = 0 \end{cases} \qquad \begin{cases} R_X = -F_1 \\ \Sigma F_y = 10 + 10 + R_A = 0 \\ \Sigma M_A = 20 + 10 + M_A \end{cases}$$

$$\begin{cases} R_X = -3 \\ \Sigma F_y = 10 + 10 + R_A = 0 \\ \Sigma M_A = 30 + 10 + M_A - 9 \end{cases} \qquad \begin{cases} R_X = +3\ kN \\ R_A = -20\ kN \\ M_A = -21\ kN/m \end{cases}$$

Portale

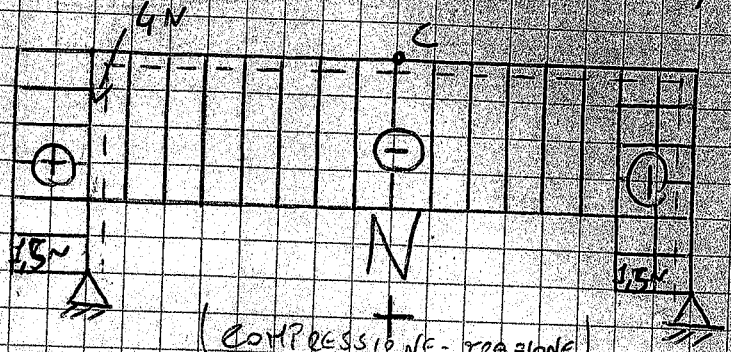


$H_A = H_B = 2N$

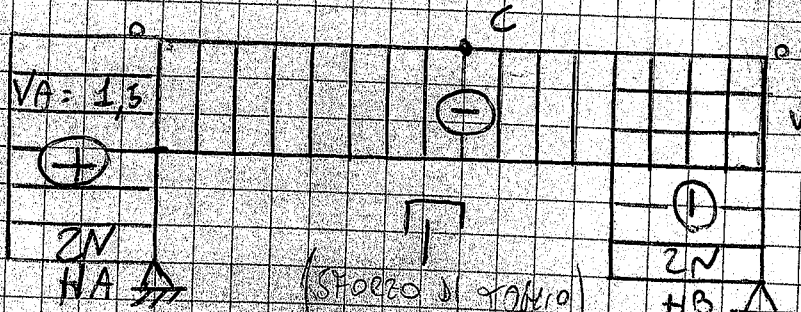
$V_A = V_B = 1,5N$

$$\begin{cases} \sum F_x = 0 \\ \sum M_A = 0 \\ \sum F_y = 0 \end{cases} \quad \begin{cases} F_x = 4N - H_A - H_B = 0 \\ 4_y = V_A + V_B = 0 \\ M_A = F \cdot 3 + V_B \cdot 8 = 0 \\ M_C = -H_A \cdot 3 + V_A \cdot 4 = 0 \end{cases}$$

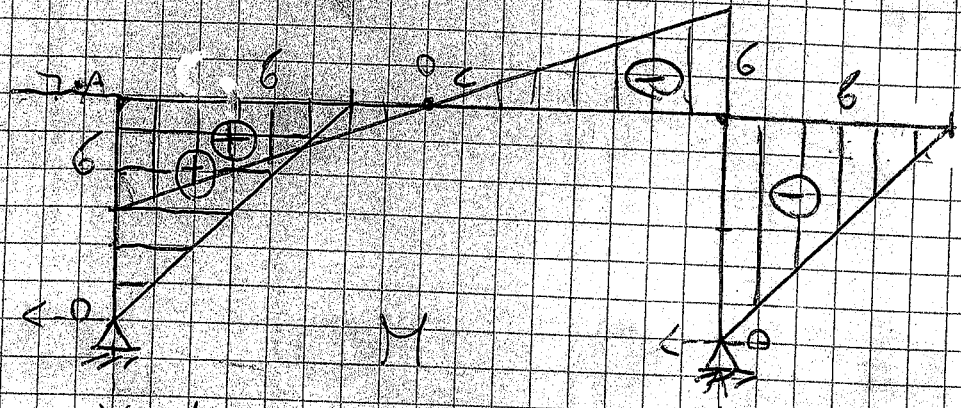
$$\begin{cases} M_A \Rightarrow V_B = \frac{12}{8}\frac{3}{2} = 1,5\,N \\ F_y \Rightarrow V_A = -1,5\,N \\ M_C \Rightarrow H_A = -\frac{6}{3} = -2\,N \\ F_x \Rightarrow -H_B 4N - 2N = -2N \end{cases}$$



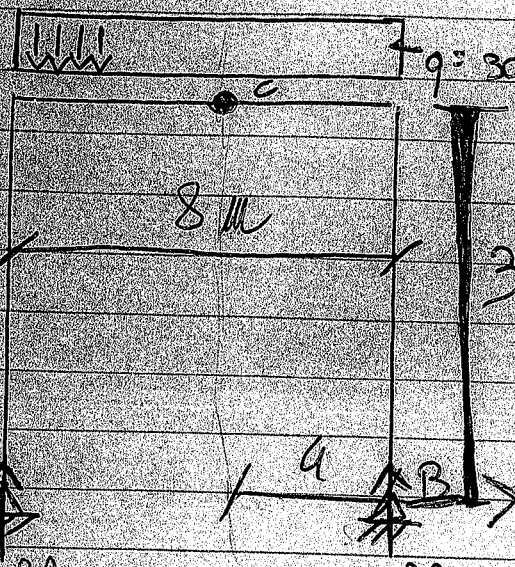
(COMPRESSIONE - TRAZIONE)



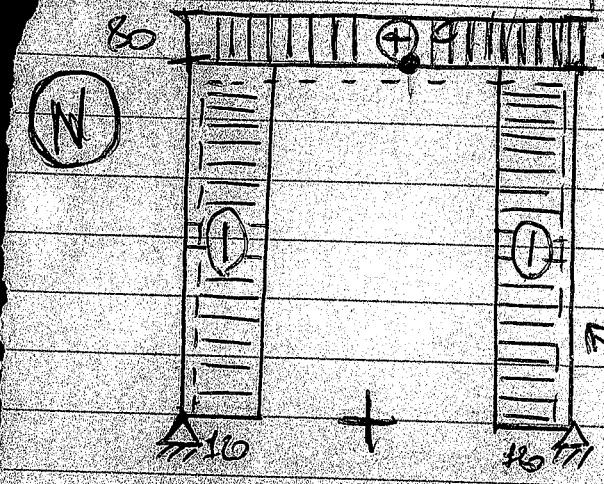
(SFORZO DI TAGLIO)

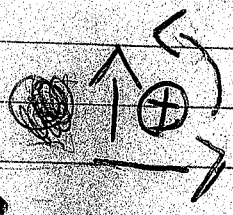


(MOMENTO FLETTENTE SE C'È)



RA = 120 KN
RB = 120 KN





MC = ●

$\Sigma F_x = \bullet$

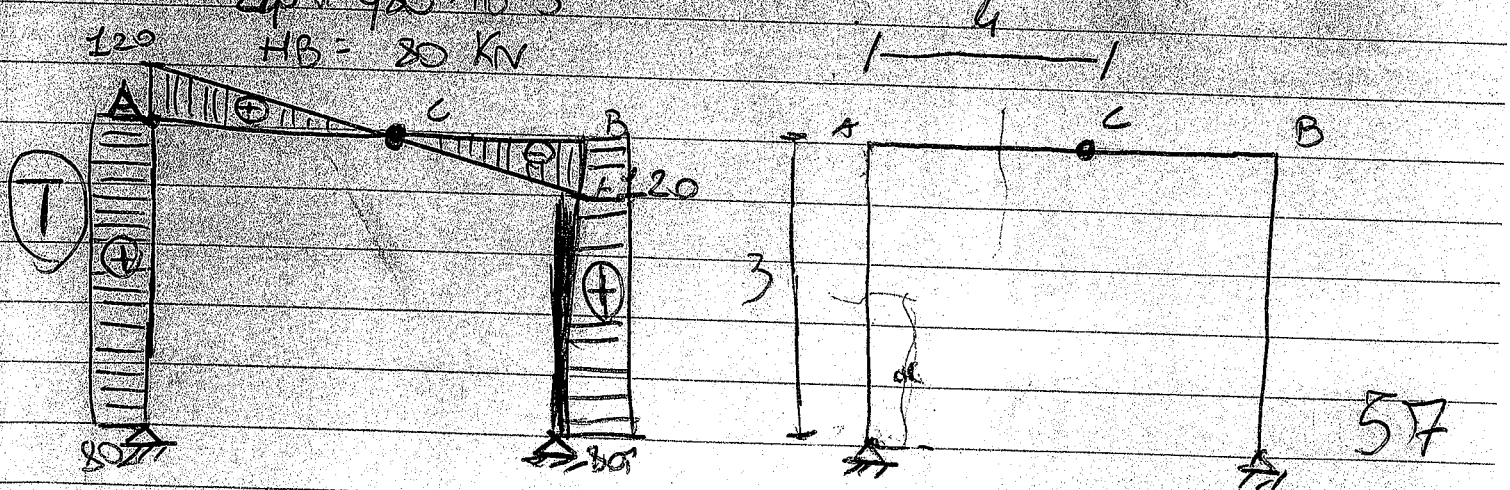
$\Sigma F_y = \bullet$

$\Sigma M_A = \bullet$

$\Sigma M_C = \bullet$

$\Sigma M_C = -(9 \cdot 4 \cdot 2) + RB \cdot 4 + HB \cdot 3$

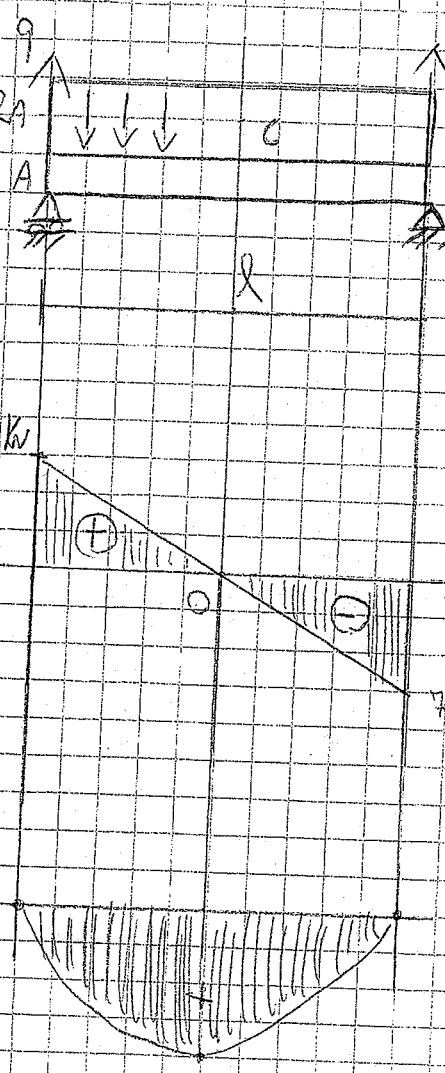
$-240 + 480 + HB \cdot 3$



57

$T_A = 120$ KN ; $T_C = 0 \Rightarrow 120 - (9 \cdot 4) = 0$; $T_{SB} = RA - (9 \cdot 8) = -120$ KN ; $T_B = 0$

$M_A = -HB \cdot 3 = -160$; M_C



$R_A = R_B = \frac{q \cdot l}{2}$

$R_A = R_B = 7{,}5\ kN$

$q = 3\ kN/ml$

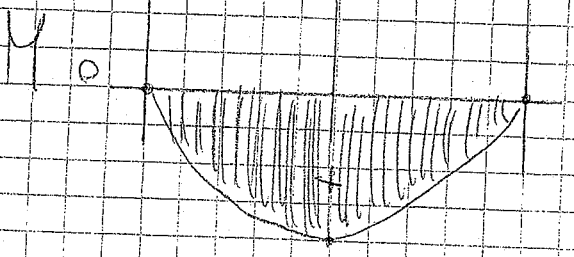
$l = 5$

$T_A = R_A = 7{,}5\ kN$

$T_{sA} = 0$

$T_C = R_A + q \cdot l_2 = 7{,}5 - 7{,}5 = 0$

$T_{sB} = R_A - q \cdot l = -7{,}5\ kN$

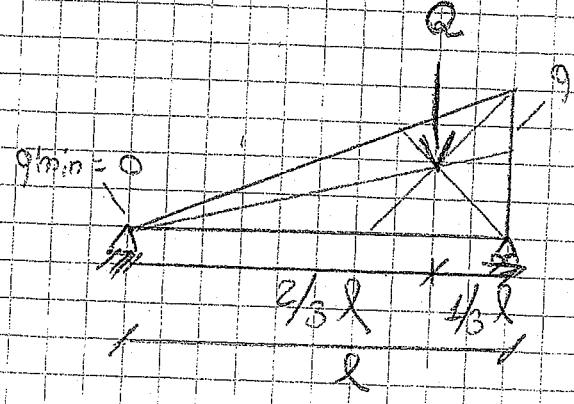


$M_A = 0$

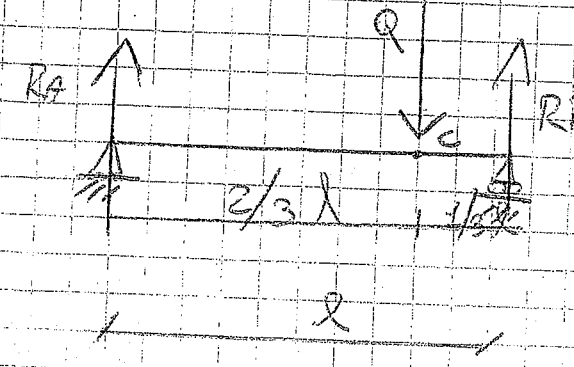
$M_B = 0$

$M_C = R_A \cdot \frac{l}{2} - q \cdot \frac{l}{2} \cdot \frac{l}{4} = 7{,}5 \cdot 2{,}5 - 3 \cdot 2{,}5 \cdot 1{,}25 = 18{,}75 - 9{,}375$

$M_C = \frac{1}{8} q \cdot l^2 = 9{,}38$



$Q = \frac{q_{MAX} \cdot l}{2}$

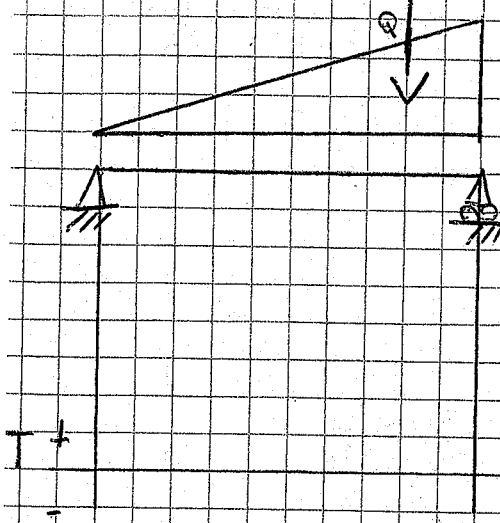


$R_B = \frac{2}{3} Q \cdot l = \frac{2}{3} \cdot \frac{q_{MAX} \cdot l}{2} \cdot l = \frac{1}{3} q_{MAX} l$

$R_A = \frac{1}{3} Q \cdot l = \frac{1}{3} \cdot \frac{q_{MAX} \cdot l}{2} \cdot l = \frac{1}{6} q_{MAX} l$

$\Sigma F_y = -R_A + Q - R_B$

$\Sigma M_A = R_A \cdot 0 + Q \cdot \frac{2}{3} l - R_B \cdot l$



$Q_{MAX} = 10\ KN/m$

$q_{min} = 0$

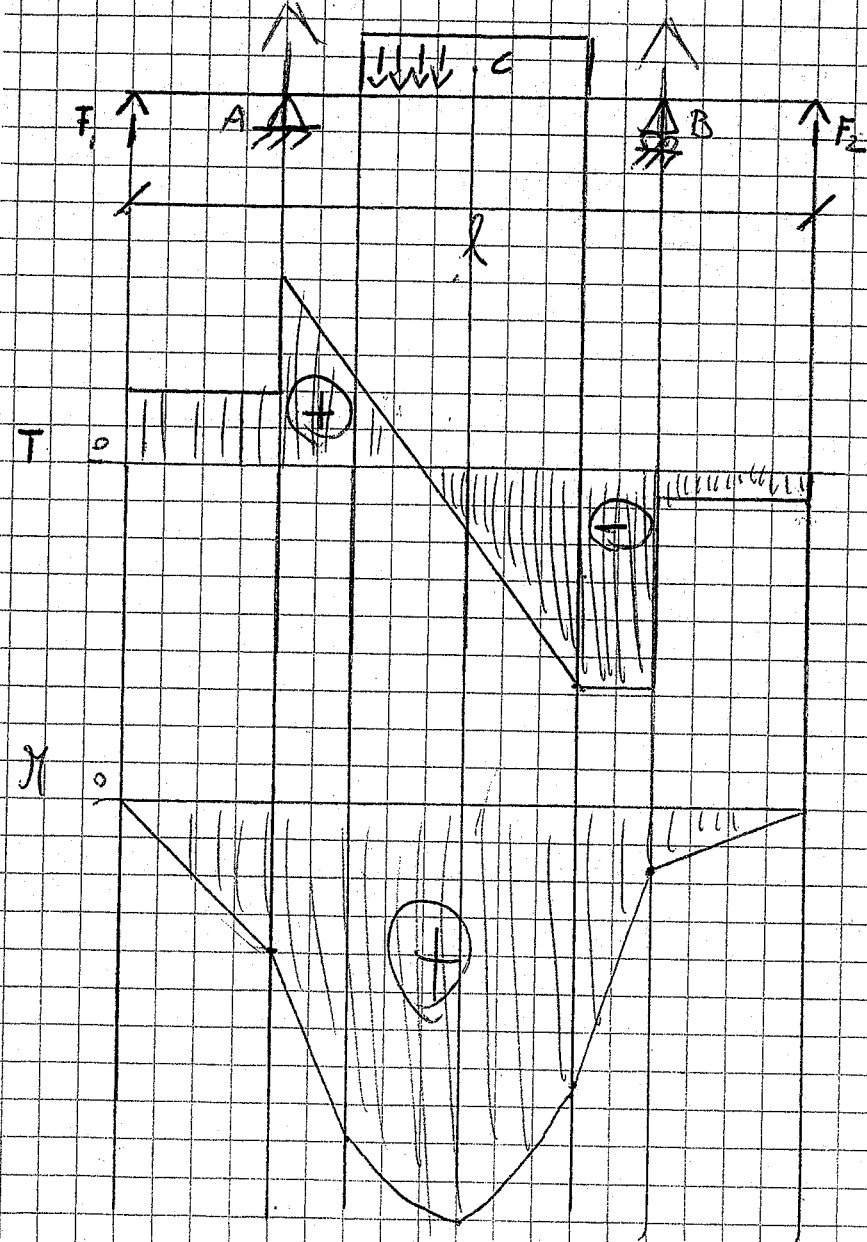
$l = 6\ m$

$X = 0,80\ m$

$T_x = R_A - \left(\frac{q_x \cdot X}{2}\right)$

T + −

(1)



$R_A = -14,60\ KN$ $F_1 = 10\ KN$

$R_B = -25,40\ KN$ $q = 13\ KN/$

$F_2 = 4\ KN$

$T_C = 0$

$T_{F_1} = 10\ KN$

$T_A = -R_A + q \cdot l_1 - R_B - F_2 = 10\ KN$

$T_D = q \cdot l - R_B - F_2 = 24,6\ KN$

$T_E = q \cdot R_B - F_2 = 11,4\ KN$

$T_B = R_B - F_2 = 21,4\ KN$

$T_F = F_2 = 4\ KN$

$M_{F_1} = \left(-R_A \cdot 2\right) + \left(q \cdot l \cdot \frac{l}{2}\right) + \left(-R_B \cdot$

$= -29,2 + 243 - 177,8 - 36 = 0$

$M_{RA} = F_1 \cdot 2 = 20\ KN$

$M_D = F_1 \cdot 3 + R_A \cdot 1 = 44,6\ KN$

$M_4 = F_1 \cdot 4,5 + R_A \cdot 2,5 - \frac{q}{2} = 54,$

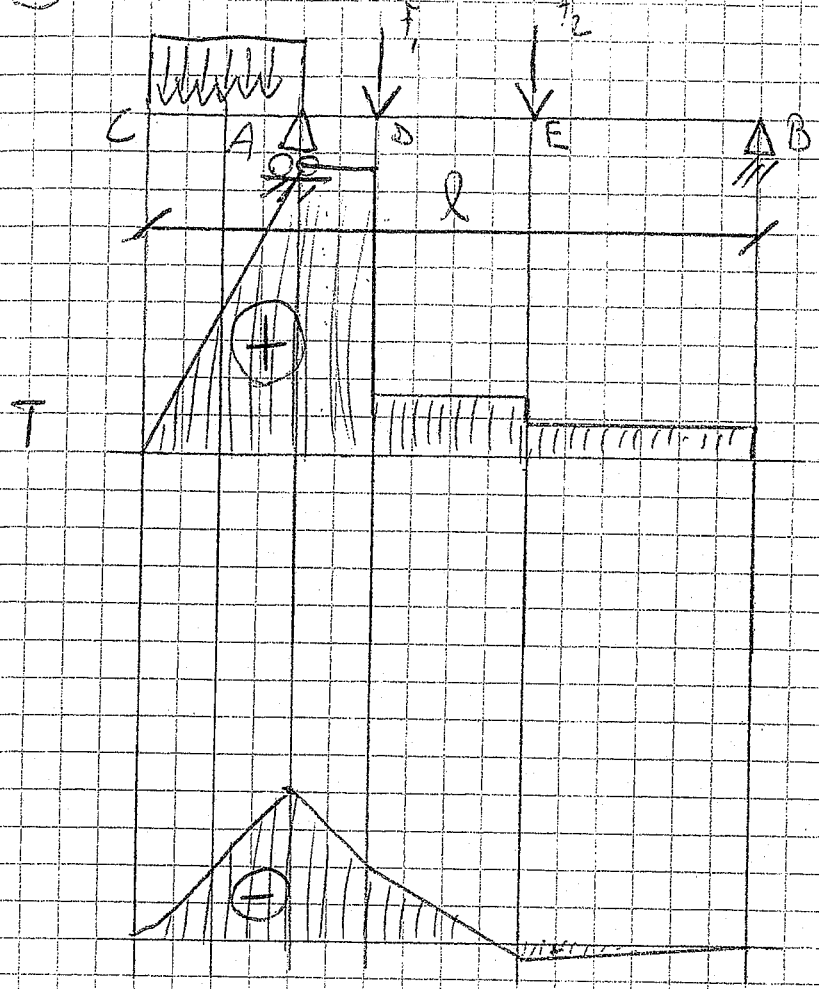
$M_E = -\left(10 \cdot 6\right) - \left(R_A \cdot 4\right) + \left(q \cdot l \cdot \frac{}{2}\right)$

$= -37,4\ KN$

$M_{RB} = -\left(F_1 \cdot 7\right) - \left(R_A \cdot 5\right) + \left(q \cdot$

$= -8\ KN$

②



$R_A = -50{,}88$ KN
$F_1 = 30$ KN
$R_B = -4{,}17$ KN
$q = 10$ KN/ml
$F_2 = 5$ KN

$T_C = q \cdot l - R_A + F_1 + F_2 - R_B = 0$

$M_C = 0$

$M_A = -\left(q \cdot l \cdot \frac{l}{2}\right) = -20$

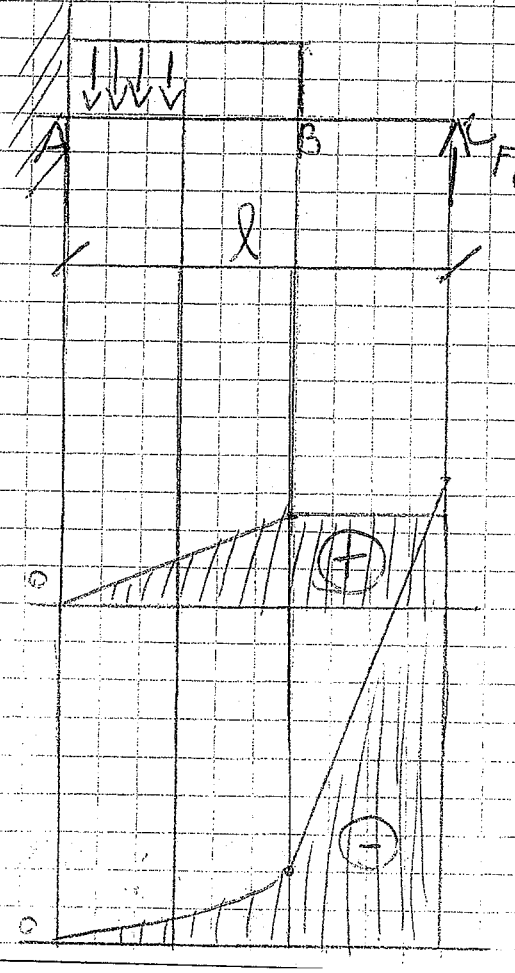
$M_D = -\left(q \cdot l \cdot \frac{l}{2} + 1\right) + (R_A \cdot 1) = 10{,}8$

$M_E = -\left(q \cdot l \cdot \frac{l}{2} + 3\right) + (R_A \cdot 3) - (F_1 \cdot 2)$

$M_B = -\left(q \cdot l \cdot \frac{l}{2} + 6\right) + (R_A \cdot 6) - (F_1 \cdot 5) - (F_2$

$= -140 + 304{,}98 - 150 - 15 = -0{,}$

③



$R_A = -48$ KN

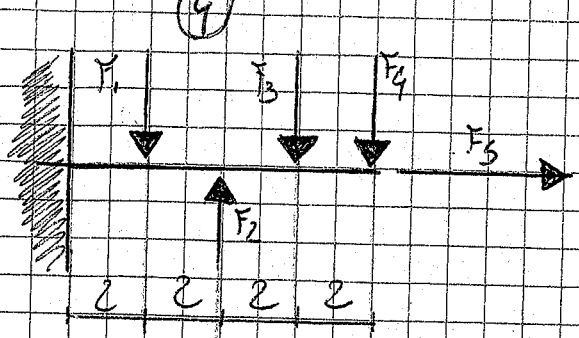
$M_A = -30$ KN

$M_A = -30$ KN

$M_B = -\left(q \cdot l \cdot \frac{l}{2}\right) - 30 \cdot 3 + (48 \cdot 3) = -3$

$M_C = -\left(q \cdot l \cdot \frac{l}{2} + 2\right) - (30 \cdot 5) + (48 \cdot 3)$

60

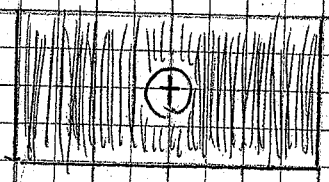


(N) 1 cm = 6 KN

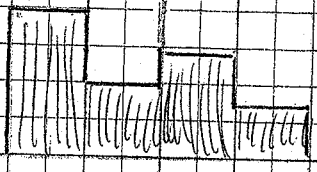
(F) 1 cm = 10 KN

$F_1 = 10$ KN
$F_2 = 4$ KN
$F_3 = 6$ KN
$F_4 = 8$ KN
$F_5 = 12$ KN
$R_A = -20$ KN
$R_x = -12$ KN
$M_A = -104$ KN

N



T



$T_{AS} = 0$

$T_A = R_A = -20$ KN

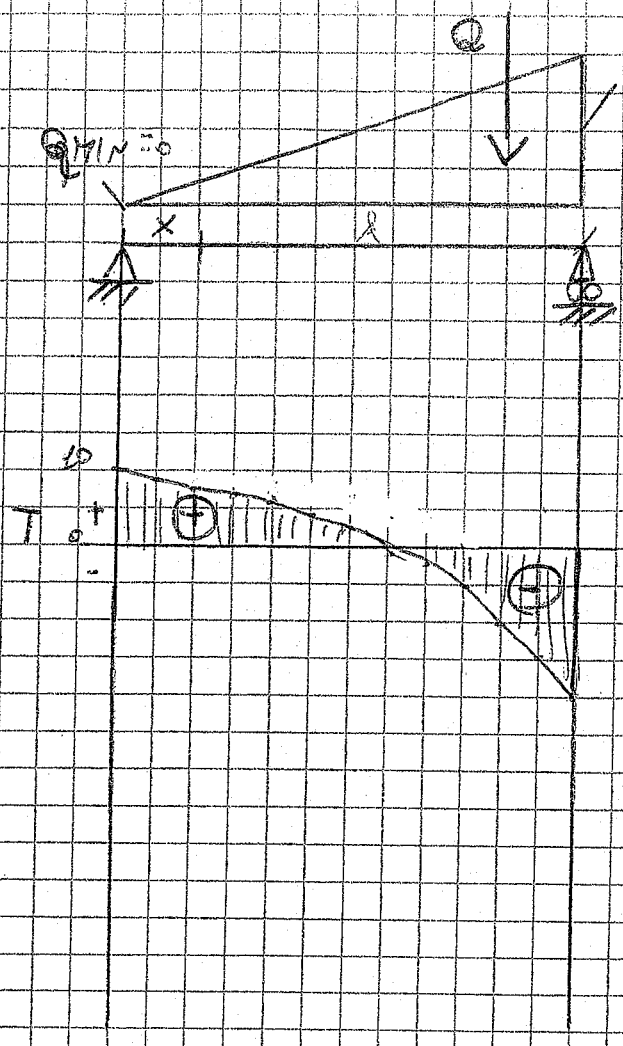
$T_{F_1} = F_1 - R_A = -10$ KN

$T_{F_2} = F_2 - F_1 - R_A = -14$ KN

$T_{F_3} = F_3 - F_2 - F_1 - R_A = 8$ KN

$T_{F_4} =$

61



qMAX = 10 KN/ml

l = 6 m

$RA = \frac{1}{6} qMAX \cdot l$ $\quad x = 0,60$ m

$RB = \frac{1}{3} qMAX \cdot l$ $\quad RA = 10$ KN

$RB = 20$ KN

$Q = \frac{qMAX \cdot l}{2} = \frac{10 \cdot 6}{2} = 30$ KN

10 : 6 = X : 0,50

X = 0,83 KN

$T(0,50) = RA - \left(\frac{0,83 \cdot 0,50}{2}\right) = \boxed{9,8 \text{ KN}}$

$T(1) = RA - \left(\frac{1,67 \cdot 1}{2}\right) = \boxed{9,2 \text{ KN}}$

10 : 6 = X : 1

X = 1,67 KN

$T(1,50) = RA - \left(\frac{2,5 \cdot 1,5}{2}\right) = \boxed{8,1 \text{ KN}}$

10 : 6 = X : 1,50

X = 2,5

$T(2,00) = RA - \left(\frac{3,33 \cdot 2}{2}\right) = \boxed{6,67 \text{ KN}}$

10 : 6 = X : 2

X = 3,33

$T(2,50) = RA - \left(\frac{4,17 \cdot 2,50}{2}\right) = \boxed{4,8 \text{ KN}}$

qMAX 10 : 6 = X : 2,50

X = 4,17

$T(3,00) = RA - \left(\frac{5 \cdot 3}{2}\right) = \boxed{2,5 \text{ KN}}$

10 : 6 = X : 3

X = 5

$T(3,50) = RA - \left(\frac{5,83 \cdot 3,5}{2}\right) = \boxed{-0,2 \text{ KN}}$

10 : 6 = X : 3,50

X = 5,83

62

$T(4,00) = RA - \left(\frac{6,67 \cdot 4}{2}\right) = \boxed{-3,34\ K}$

$10 : 6 = X : 4$

$X = 6,67$

$T(4,50) = RA - \left(\frac{7,5 \cdot 4,5}{2}\right) = \boxed{-5\ K}$

$10 : 6 = X : 4,50$

$X = 7,5$

$T(5,00) = RA - \left(\frac{8,33 \cdot 5}{2}\right) = \boxed{-10,8}$

$10 : 6 = X : 5$

$X = 8,33$

$T(5,50) = RA - \left(\frac{9,17 \cdot 5,5}{2}\right) = \boxed{-15,2}$

$10 : 6 = X : 5,50$

$X = 9,17$

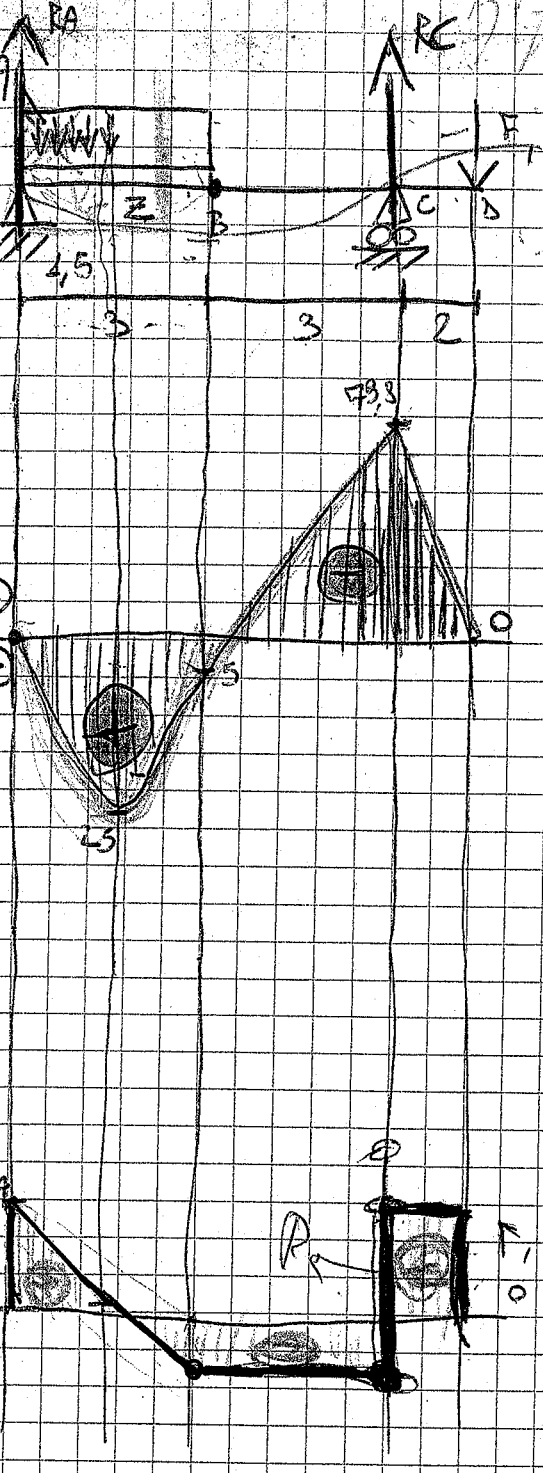
$T(6,00) = RA - \left(\frac{10 \cdot 6}{2}\right) = \boxed{-20\ K}$

$10 : 6 = X : 6$

$X = 10$

$$X = \frac{l}{\sqrt{3}} = 3,47\ m$$

IL TAGLIO È NULLO



$q = 20\ KN/ml$

$F_1 = 40\ KN$

$R_A = 31,67\ KN$

$R_C = 68,33\ KN$ (↑ +)

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases} \qquad \begin{cases} \Sigma F_y = R_A + R_C + q \cdot l + F_1 = 0 \\ \Sigma M_A = q \cdot l \cdot \frac{l}{2} + F_1 \cdot l + R_C \cdot 6 = \end{cases}$$

$$\begin{cases} \Sigma F_y = R_A + R_C + 60 + 40 \Rightarrow R_A = -31,67 \\ \Sigma M_A = 90 + 320 + R_C \cdot 6 \Rightarrow R_C = -68,33 \end{cases}$$

$M_A = 0$

$M_Z = R_A \cdot 1,5 - \left(q \cdot \frac{l}{2} \cdot 1,5 \cdot 0,75\right) = 47,5 - 22,5 =$

$= 25\ KN \cdot m$

$M_D = R_A \cdot 3 - \left(q \cdot \frac{l}{2} \cdot \frac{l}{3}\right) = 95,1 - 90 = 5,1$

$M_C = R_A \cdot 6 - \left(q \cdot \frac{l}{2} \cdot 4,5\right) = 190,2 - 270 = -79,8$

$M_D = R_A \cdot 8 - \left(q \cdot 3 \cdot 6,5\right) + R_C \cdot 2 =$

$253,32 - 390 + 136,6 = 0$

$T_A = R_A$

$T_{SZ} = R_A - q \cdot \frac{l}{2} = 1,7\ KN$

$T_{SB} = R_A - q \cdot l = -28,3\ KN$

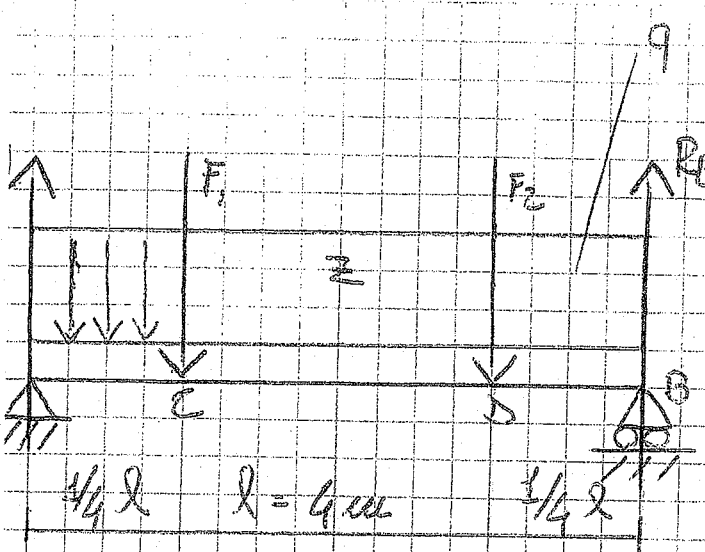
$T_{SC} = R_A - q \cdot l = -28,3\ KN$

$T_C = R_A - q \cdot l + R_C = 40\ KN$

$T_D = F_1$

E SAPERE DOVE IL TAGLIO È NULLO SI FA:

$T = 0 \Rightarrow R_A + q \cdot l_2 = 0 \Rightarrow l_2 = \frac{R_A}{q}$



1 cm = 5 KN

1 cm = 0,5 m

$q = 10$ KN/ml

$F_1 = F_2 = 5$ KN

$l = 4$ m

$R_A = -25$ KN

$R_B = -25$ KN

$T = ?$

$M = ?$

$T_{SA} = 0$

$T_A = R_A = 25$ KN

$T_{SC} = R_A - q \cdot \frac{1}{4} l = 25 - 10 = 15$ KN

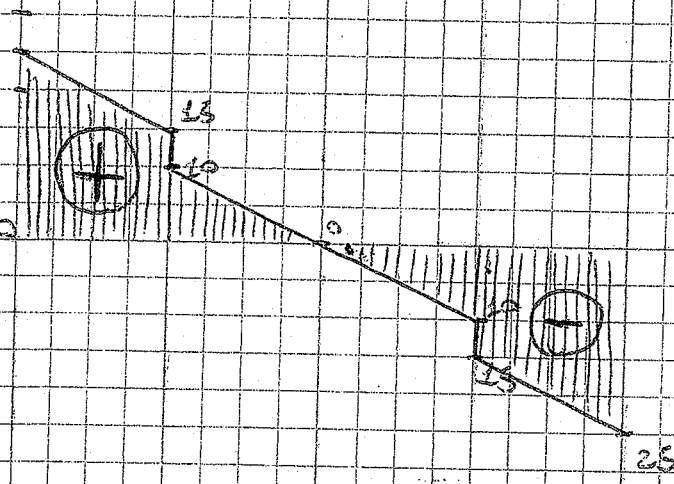
$T_C = R_A - q \cdot \frac{1}{4} l - F_1 = 25 - 10 - 5 = 10$ KN

$T_{SE} = R_A - q \cdot \frac{l}{2} - F_1 = 25 - 20 - 5 = 0$

$T_E = 0$

$T_{SD} = F_2 - q \cdot \frac{1}{4} l - R_B = -5 - 10 + 25 = 10$ KN

$T_D = R_B - q \cdot 1 = 25 - 10 = 15$



$M_A = 0$

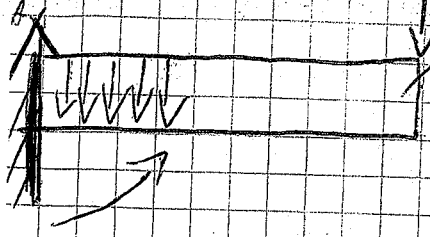
$M_C = +R_A \cdot \frac{1}{4} - q \frac{1}{4} l \cdot \frac{1}{8} l - F_1 \cdot 0 = 25 - 5 = 20$ KN

$M_E = R_A \cdot \frac{l}{2} - q \cdot \frac{l}{2} \cdot \frac{l}{4} - F_1 \cdot \frac{l}{4} = +50 - 20 - 5 = 25$ KN

$M_D = R_A \cdot \frac{l}{3} - q \cdot \frac{l}{3} \cdot \frac{l}{6} - F_1 \cdot \frac{l}{2} - F_2 \cdot 0 =$

$75 - 45 - 10 = +20$

$M_B = 0$



$F_1 = 5\ KN$

$q = 6\ KN/ml$

$l = 5$

$RA = ?$

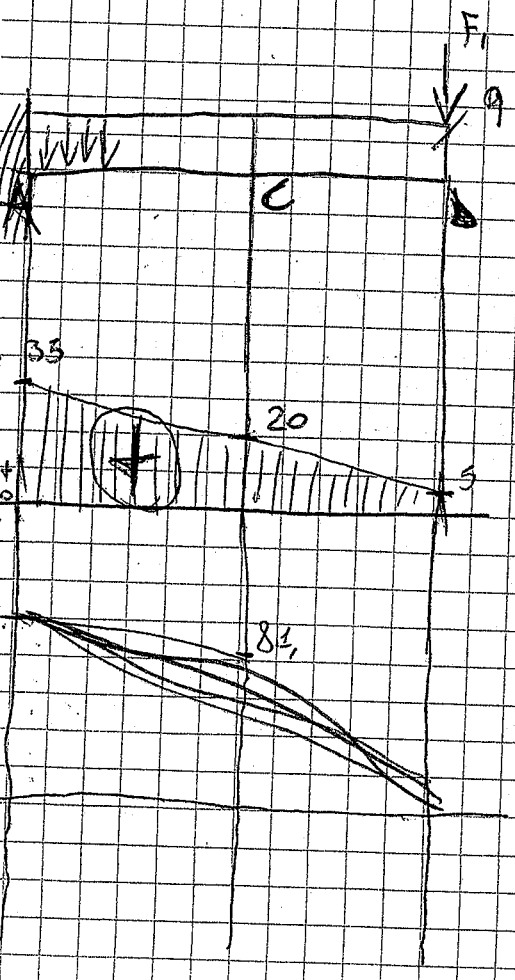
DIAGRAMME $\begin{cases} T \\ M \end{cases}$

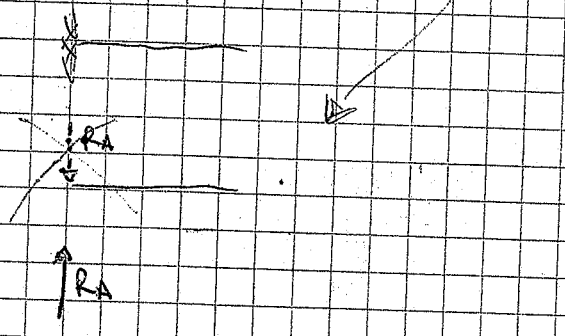
$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases} \qquad \begin{cases} \Sigma F_y = RA + q \cdot l + F_1 = 0 \\ \Sigma M_A = q \cdot l \cdot \frac{l}{2} + F_1 \cdot l + \cancel{RA \cdot 0} + M \end{cases}$$

$$\begin{cases} \Sigma F_y = CA + q \cdot l + F_1 = 0 \\ \Sigma M_A = 75 + 25 + M \end{cases} \qquad \begin{cases} \Sigma F_y = +RA + 35 = 0 \\ M = -75 - 25 \end{cases}$$

$RA = -35$

$M = -100$





$1\square = 10\ KN$

$T_A = RA$

$T_{SC} = RA - q \cdot \frac{l}{2} = 35 - 15 = 20\ KN$

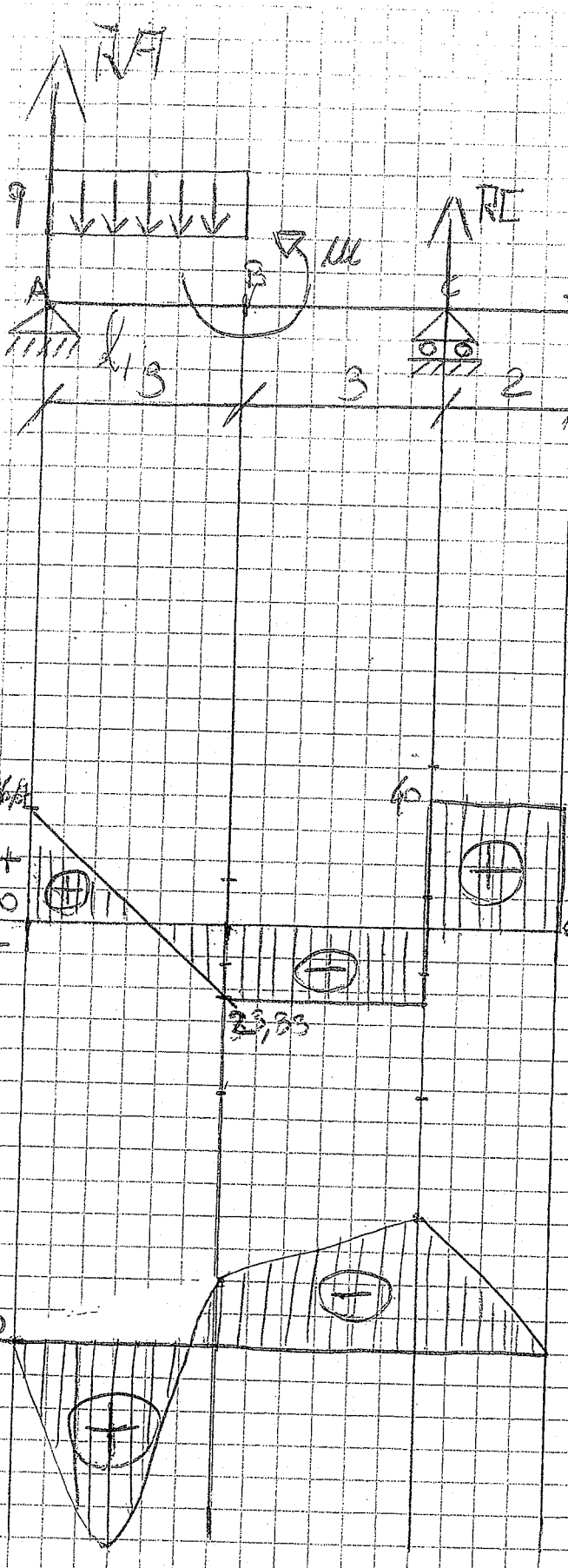
$T_{SD} = RA - q \cdot l = 35 - 30 = 5\ KN$

$T_D = F_1$

$1\square = 20\ KNm$

$M_A = \gamma_A$

$M_C = 100 - q\left(\frac{l}{2}\right)\left(\frac{l}{4}\right) = 100 - 18{,}75 = 81{,}25\ KNm$



Dati

$q = 20\ KN/ml$

$III. = 30\ KN/m$

$F = 40\ KN$

$T = ?$ $RA = 36,67$

$M = ?$ $RC = 63,33$

$1\ \square = 10\ KN$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases} \qquad \begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = q \cdot l + R_A + R_C + F = 0 \\ \Sigma M_A = q \cdot l \cdot \frac{l}{2} - III + R_C \cdot 6 + F \cdot 8 \end{cases}$$

$$\begin{cases} \Sigma F_y = R_A + q \cdot l + R_C + F = 0 \Rightarrow R_A = -36,67\ KN \\ \Sigma M_A \Rightarrow R_C = \frac{-90 + 30 - 320}{6} = -63,33\ KN \end{cases}$$

$T_{SA} = 0$

$T_A = R_A = 36,67\ KN$

$T_{SB} = R_A - q \cdot l = -23,33\ KN$

$T_B = R_A - q \cdot l = -23,33\ KN$

$T_{SC} = R_A - q \cdot l = -23,33\ KN$

$T_C = R_A - q \cdot l + R_C = 40\ KN$

T_{AD}

$M_2 = R_A \cdot 1,5 - q \cdot 1,5 \cdot 0,75 = 32,5$

$M_B = R_A \cdot 3 - q \cdot l \cdot \frac{l}{2} - M = -9,99\ KN/m$

$M_C = R_A \cdot 6 - q \cdot l \cdot 4,5 - M = -19,98\ KN/m$

$M_D = R_A \cdot 8 - q \cdot l \cdot 6,5 - M + R_C \cdot 2 = 0$

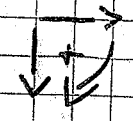
67

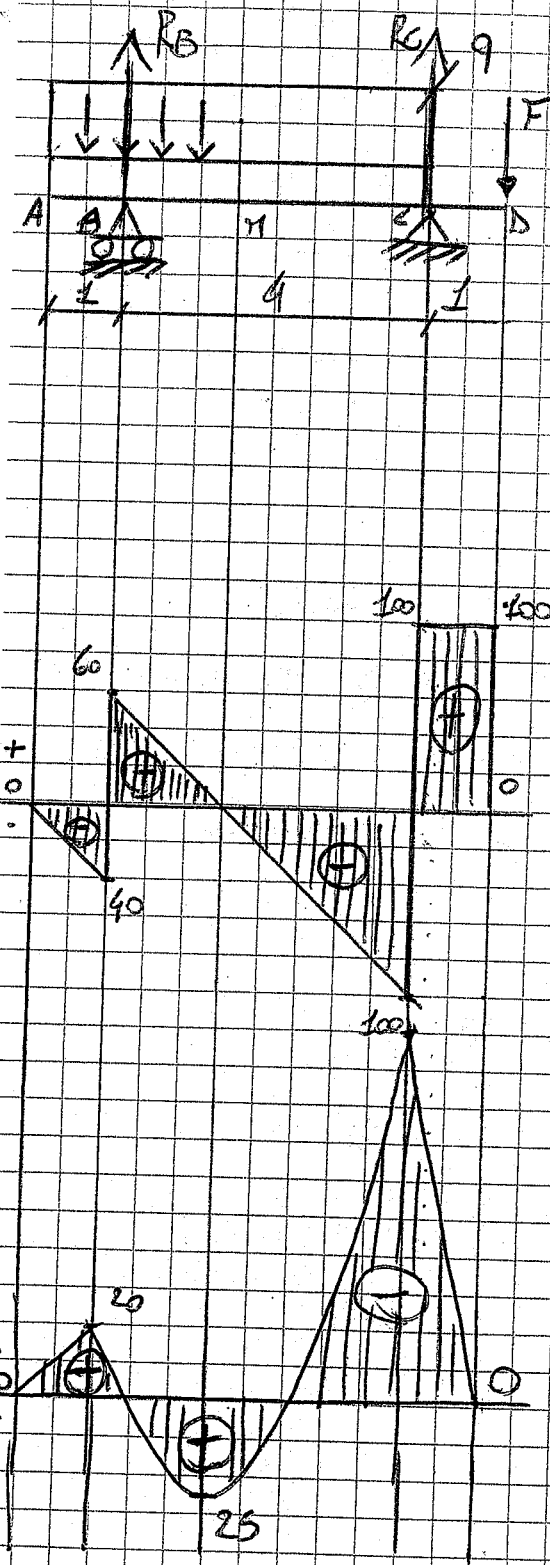
$q = 40\ KN/ml$

$F = 100\ KN$

$R_B = 100\ KN$

$R_C = 200\ KN$





Scala 1

1 ■ = 20 KN

$T_A = 0\ KN$

$T_{B_S} = -q \cdot 1 = -40\ KN$

$T_B = -q \cdot 1 + R_B = 60\ KN$

$T_C = -q \cdot 5 + R_B = -100\ KN$

$T_C = -q \cdot 5 + R_B + R_C = 100\ KN$

$T_D = -q \cdot 5 + R_B + R_C - F = 0\ KN$

$M_A = 0\ KN$

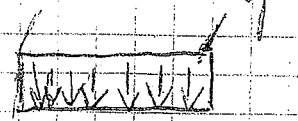
$M_B = -(q \cdot 1 \cdot 0{,}5) = -20\ KN \cdot m$

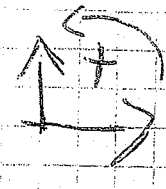
$M_C = -(q \cdot 5 \cdot 2{,}5) + (R_B \cdot 4) = -100\ KN \cdot m$

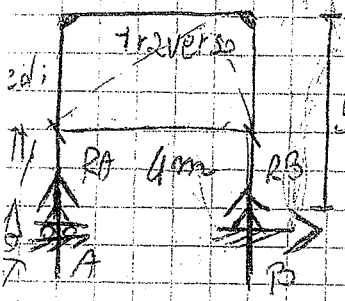
$M_D = -(q \cdot 5 \cdot 3{,}5) + (R_B \cdot 5) + (R_C \cdot 1) = 0$

$M_M = (q \cdot 2{,}5 \cdot 1{,}25) + (R_B \cdot 1{,}5) = -125 + 150 = 25\ KN$

$q = 20\ KN/ml$

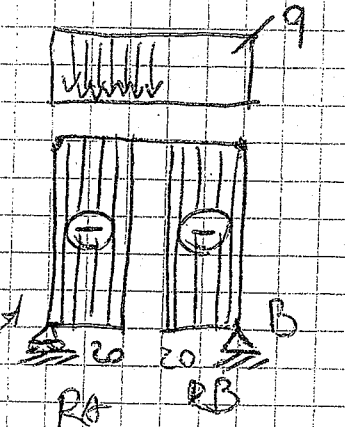


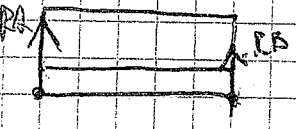


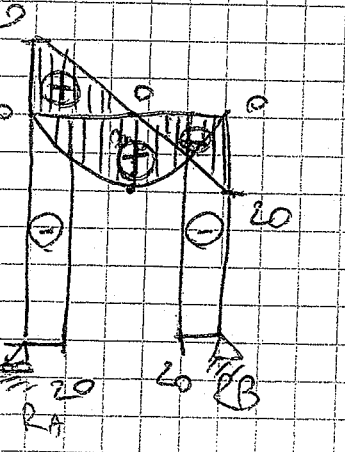


$$\begin{cases} \cancel{\Sigma F_x = 0} \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases} \qquad \begin{cases} \Sigma F_y = R_A - (q \cdot 4) + R_B \\ \Sigma M_B = R_B \cdot 0 + (q \cdot 4 \cdot 2) - R_A \cdot 4 \end{cases}$$

$$\begin{cases} \Sigma F_y = R_A - 40 + R_B \Rightarrow R_B = 20 \\ \Sigma M_B \Rightarrow -R_A = +\frac{80}{4} \Rightarrow +20\ KN \end{cases}$$

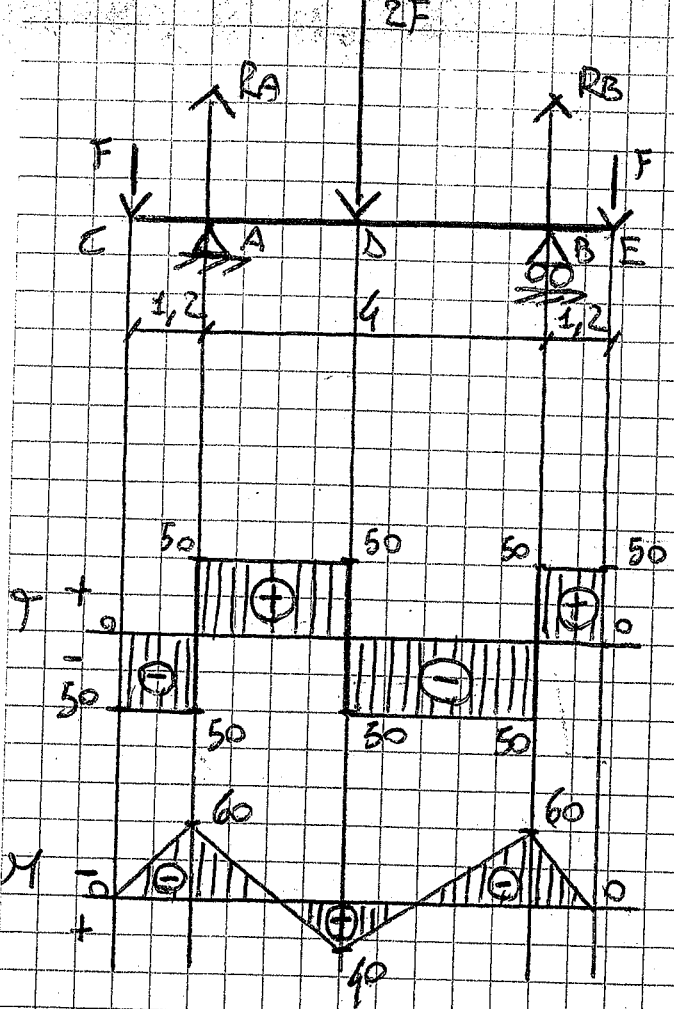






$$T_A = R_A$$
$$T_M = R_A - q \cdot \frac{l}{2} = 0$$

$$M_A = 0$$
$$M_C = R_A \cdot 2 + q \cdot \frac{l}{2} \cdot \frac{l}{4} = -40 + 20 = -20\ KN \cdot m$$

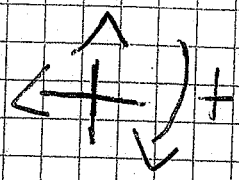


$F = 50\ KN$

$R_A = 100\ KN$

$R_B = 100\ KN$

$2F = 100\ KN$



$T_{SC} = 0\ KN$

$T_C = F = -50\ KN$

$T_{SA} = F = -50\ KN$

$T_A = -F + R_A = 50\ KN$

$T_{SD} = -F + R_A = 50\ KN$

$T_D = -F + R_A - 2F = -50\ KN$

$T_{SB} = -F + R_A - 2F = -50\ KN$

$T_B = -F + R_A - 2F + R_B = 50\ KN$

$T_{SE} = -F + R_A - 2F + R_B = 50\ KN$

$T_E = -F + R_A - 2F + R_B - F = 0\ KN$

$M_C = 0$

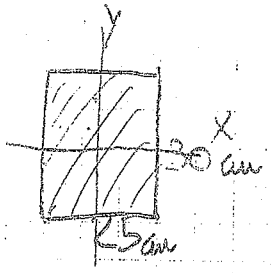
$M_A = -F \cdot 1,2 = -60\ KN \cdot m$

$M_D = -F \cdot 3,2 + R_A \cdot 2 = 40\ KN \cdot m$

$M_B = -F \cdot 5,2 + R_A \cdot 4 - 2F \cdot 2 = 60\ KN \cdot m$

$M_E = 0$

Carico Di Punta



$N = 50\ kN$ $\qquad \sigma = \frac{N}{A}$

$\lambda = \frac{l_0}{\rho(\min)}$ => LUNGHEZZA LIBERA DI INFLESSIONE

↙ LAMBDA

$l = 3\ m$ $\qquad \boxed{\sigma_{amm} = 5{,}2\ N/mm^2}$ $\qquad N = 50\ KN$

$\lambda = \frac{l_0}{\rho} \Rightarrow l_0 = l$ $\qquad \rho = \sqrt{\frac{I_{min}}{A}}$

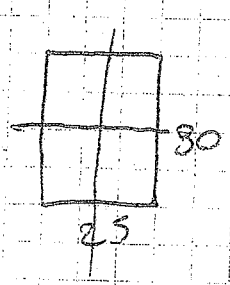
$I_x = \frac{1}{12} \cdot 25 \cdot 30^3 = 56250\ cm^4$

$I_y = \frac{1}{12} \cdot 30 \cdot 25^3 = 39062{,}5\ cm^4$

$\rho = \sqrt{\frac{39062{,}5}{30 \cdot 25}} = \sqrt{52{,}08} = 7{,}22\ cm$

$\lambda = \frac{300\ cm}{7{,}22\ cm} = 41{,}55 \Rightarrow \omega = 0$

$\tau = \frac{N}{A} = \frac{50\ kN}{25 \cdot 30\ cm} = \frac{50000\ N}{75000\ mm} = 0{,}7$ $\qquad \tau < \tau_{AMM}$ È VERIFICATO



$N = 50\ kN$ $\qquad \tau = \frac{N}{A}$

$l = 4\ m$ $\qquad \sigma_{amm} = 5{,}2\ N/mm^2$

$\rho = \sqrt{\frac{I}{A}} = \sqrt{\frac{39062{,}5}{30 \cdot 25}} = 7{,}22\ cm$

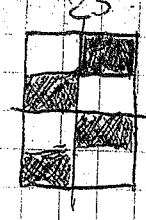
$\lambda = \frac{400}{7{,}22} = 56 \Rightarrow \omega = 1{,}02$

$\tau = \frac{N}{A} \cdot \omega = \frac{50000\ N}{75000\ mm} \cdot 1{,}02 = 0{,}74$

71

$\tau < \tau_{AMM}$ È VERIFICATO

PILASTRO IN LEGNO MASSICCIO



$N = 50\ KN$ $\quad \sigma_{AMM} = 8$

$l = 4\ m$

$\lambda = \frac{l_0}{\rho} \Rightarrow \quad \rho = \sqrt{\frac{I}{A}} = 7{,}22\ cm$

$\lambda = \frac{4\ m}{7{,}22} = \frac{400\ cm}{7{,}22} = 56 \Rightarrow \omega \Rightarrow X = \frac{(\omega_{MAG} - \omega_{MIN}) \cdot (\lambda_{CAL} - \lambda_{MIN})}{(\lambda_{MAG} - \lambda_{MIN})}$

$X = \frac{(60-50)\cdot(56-50)}{(60-50)} = 0{,}12 \qquad \omega = \omega(50) + 0{,}12 = 1{,}54$

$\tau = \frac{N}{A} \cdot 1{,}54 = 1{,}02 < \sigma_{AMM}$

ACCIAI DA CARPENTERIA (PILASTRO)

$HE = 160\ mm$

$l = 6{,}00\ m \Rightarrow \lambda$

$N = 10\ KN$

$S235 \Rightarrow \sigma_{AMM}$

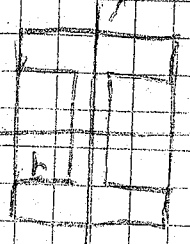
$\sigma_{AMM} = \frac{235}{1{,}1} = 213{,}64\ N/mm^2$

$\tau = \frac{N}{A} \le \tau_{AMM} \qquad \tau = \frac{10000\ N}{5430\ mm^2} = 1{,}8\ N/mm^2$

$I = \frac{1}{12}b^4 - 2\left[\frac{1}{12}b\cdot h^3\right]$

$\lambda = \frac{l_0}{\rho} = \frac{\omega \cdot \lambda}{\rho} \Rightarrow$

$\rho = \sqrt{\frac{I_{MIN}}{A}} = \sqrt{\frac{288}{54{,}3}} = \sqrt{16{,}27} = 4{,}04$



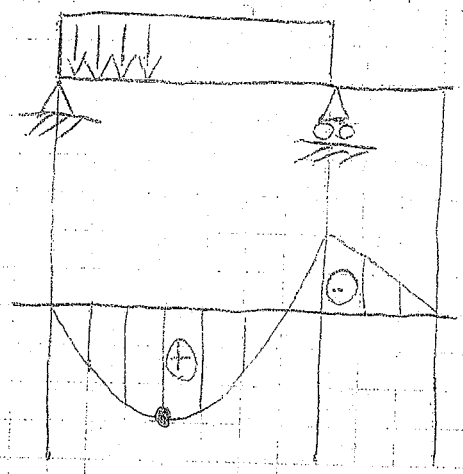
$\lambda = \frac{420\ cm}{4{,}04\ cm} = 103{,}80 \qquad \omega = \text{TABELLA}(b) = 1{,}90$

$I_x = 2492\ cm^4$

$I_y = 835\ cm^4$

Legno (FLESSIONE)

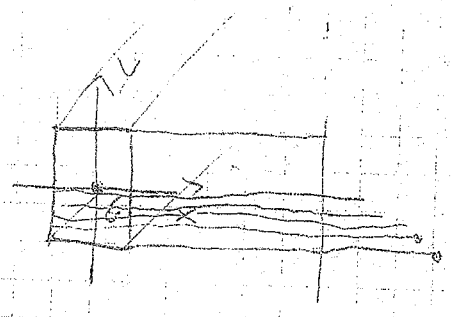
$\sigma_{amm} = 10\ N/mm^2$
(Flessione)



$\frac{b}{h} = 0{,}7$

$b : 0{,}7 = h$

$\sigma_R = \frac{M}{I} \cdot y$



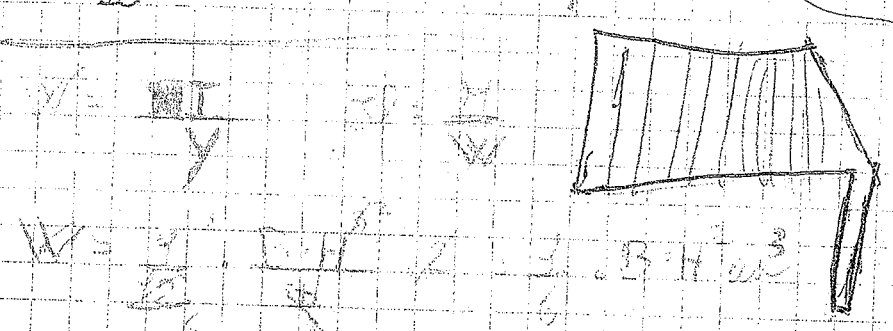
$I = \frac{1}{12} b \cdot h^3$

$M = 18\ KN \cdot m \Rightarrow 18 \cdot 10^6\ N/mm^2$

$\sigma = \frac{18 \cdot 10^6}{\frac{1}{12} 160 \cdot 240^3} \cdot 120 = 79{,}43\ N/mm^2$

$I = \frac{1}{12} 160 \cdot 240^3$ □

$I = \frac{1}{12} 240 \cdot 160^3$ ▭



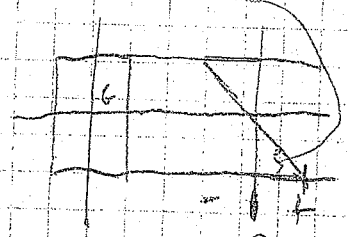
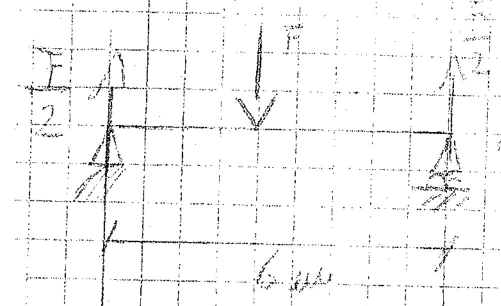
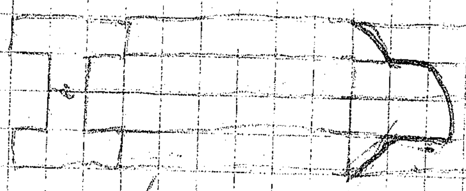


DIAGRAMMA DELLE TENSIONI





(TAGLIO)

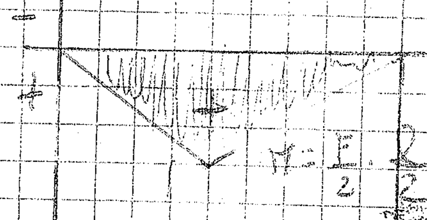
F = 80 KN

N = 100 KN

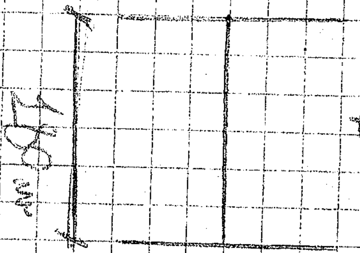
$\sigma = 275$

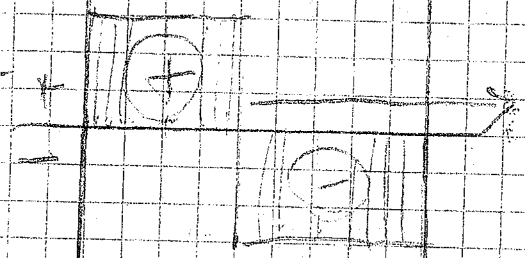
$\sigma_{amm} = \frac{275}{1{,}16} = 239\ N/mm^2$

$\tau_{amm} = 239 \cdot 0{,}577 = 138\ N/mm^2$



$M = \frac{F}{2} \cdot \frac{l}{2} = 120\ KN$

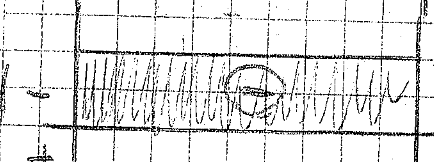




$T = F = 80\ KN$

180 mm

(COMPRESSIONE)



$N = 100\ KN$

$\sigma = \frac{N}{A} \cdot \omega \leq \sigma_{amm}$

$\lambda = \frac{l_0}{\rho} = \frac{600\ mm}{7{,}4\ mm} = 81{,}08$

$\rho = \sqrt{\frac{I}{A}} = \sqrt{\frac{2510 (TAB)}{45{,}3 (TAB)}} = 7{,}4$

$\sigma = \frac{100 \cdot 10^3}{4530\ mm} \cdot 1{,}47 = 32{,}45 \leq \sigma_{amm}$

VERIFICATO

TAGLIO

$\tau = \frac{T \cdot S}{I \cdot b} = \frac{80 \cdot 10^3 \cdot 372 \cdot 10^3 (TAB)}{7763 \cdot 10^4 \cdot 240} = 1{,}6$ (TAB)

$\tau = \frac{T}{A}$

$\sigma_T \leq \tau_{amm}$

$1{,}6 \leq 138$

(FLESSIONE)

$\sigma_f = \frac{M}{I} \cdot y$

$W = \frac{I}{y}$

$\sigma = \frac{M}{W} \leq \sigma_{amm}$

$\sigma_f = \frac{120 \cdot 10^6}{294 \cdot 10^3} = 408$ NON È VERIFICATO

(MEMORO HE 220)

$\frac{120 \cdot 10^6}{475 \cdot 10^3}$ (TAB) $\boxed{253}$

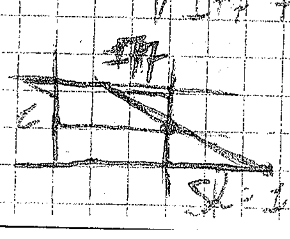
(MEMORO HE 260)

VON MISES

$\sigma_{IDEALE} = \sqrt{\sigma_f^2 + 3\tau^2} \leq \sigma_{amm}$

$\sqrt{177^2 + 3 \cdot 1{,}6^2} = \boxed{177}$

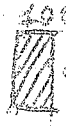
74

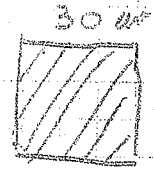


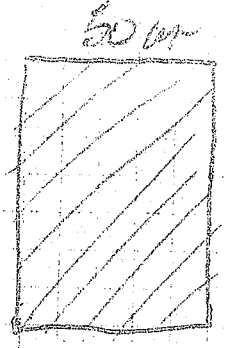
Esercizio

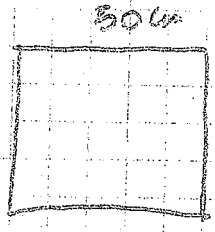
CALCOLARE LA STRUTTURA DI UN PILASTRO IN ACCIAIO, LEGNO, MURATURE IN TUFO CHE DEVE RESISTERE A UN CARICO DI 2500 KN ALLA COMPRESSIONE.

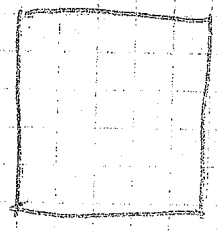
SI CONSIDERI INIZIALMENTE LE SEGUENTI DIMENSIONI.

ACCIAIO SEZ. RETT 

LEGNO SEZ. QUADRATO 



TUFO 

=> 

ACCIAIO: $\sigma = \frac{N}{A} \leq \sigma_{AMM}$

$\sigma = \frac{2500\,KN}{10 \cdot 20\,cm} = \frac{2500000\,N}{2000\,mm^2} = 125\,N/mm^2$

S235 $\sigma_{AMM} = \frac{235}{1,1} = 213,64\,N/mm^2$

$125 \leq 213,64$ È VERIFICATO

LEGNO: $\sigma = \frac{N}{A} \leq \sigma_{AMM}$

$\sigma = \frac{2500\,KN}{900\,cm} = \frac{2500000\,N}{90000\,mm^2} = 27,78$

$\sigma = 8\,N/mm^2$

$27,78 \leq 8$ NON È VERIFICATO

=> $\sigma = \frac{2500\,KN}{50 \cdot 70\,cm} = \frac{2500000\,N}{350000\,mm^2} = 7,14$

$7,14 \leq 8$ VERIFICATO

TUFO: $\sigma = \frac{N}{A} \leq \sigma_{AMM}$

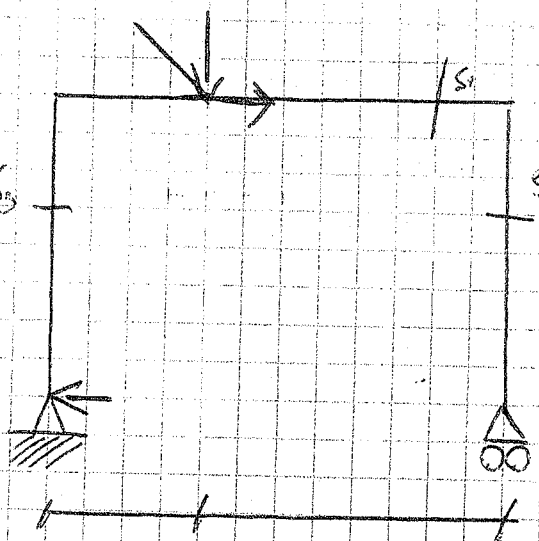
50·45 $\sigma = \frac{2500\,kN}{2250\,cm^2} = \frac{2500000\,N}{225000\,mm^2} = 11,11\,N/mm^2$

$11,11 \leq 0,8$ NON È VERIFICATO

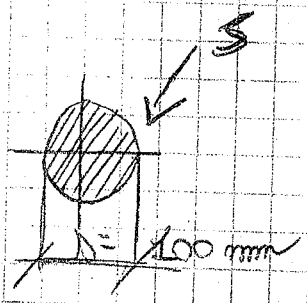
60·65 $\sigma = \frac{2500\,kN}{3900\,cm^2} = \frac{2500000\,N}{39000}$

250·250 $\sigma = \frac{2500\,kN}{62500\,cm^2} = \frac{2500000\,N}{6250000\,mm^2} = 0,4$

$0,4 \leq 0,8$ È VERIFICATO



$S_1 = S_2 = S_3$



$$\tau = \frac{N}{A} \leq \tau_{AMM}$$

$V_f = 200\ KN$

$V_{P1} = -50\ KN$

$V_{P2} = -180\ KN$

S235 S355 S450 => FERRO MENO COSTOSO S275 (RESISTE A 23,5 Kg/mm² ALLO SNERVAMENTO) $25\ \frac{N}{mm^2}$

σ = TENSIONE UNITARIA DEL MATERIALE DI TRAZIONE O COMPRESSIONE $= \frac{N}{A} \rightarrow \frac{200\ KN}{7850} = 0,025\ \frac{KN}{mm^2}$

$\sigma_{AMMISSIBILE} = \frac{\sigma_R\ (\text{SIGMA ROTTURA})}{g_s\ (\text{FATTORE DI SICUREZZA})}$

1,05	10
1,25	

$\frac{S}{1,5}$

ACCIAIO ($\tau_{AMMISSIBILE}$ [N/mm²])

S235	223,8
S275	261,9

LEGNO = LEGNO 1

MURATURE (COMPRESSIONE)

Cls (COMPRESSIONE)

$\tau_{AMMISSIBILE} = \frac{235}{1,05}$

ACCIAIO (τ = 0,59 S_{AMM}) $A = \frac{\pi}{4} D^2 = 7850\ mm^2$

τ = 0,59 S_{AMM} (TAU = 0,59 S AMMISSIBILE)

$\tau \leq \tau_{AMMISSIBILE}$

LEGNO (TAGLIO) = 0,9 · S_{AMM}

$S_{AMMISSIBILE}$

γ = 0,9 LEGNO

- τ COMP $\perp$ => 8 N/mm²
- τ COMP // => 2 N/mm²
- τ FLESSIONE => 10 N/mm²
- τ TRAZIONE // => 9 N/mm²

$S_{AMMISSIBILE}$

MURATURE

- $S_{PIETRAM}$ => 1,6 N/mm²
- S_{TUFO} => 0,8 N/mm²

(COMPRESSIONE)

CALCESTRUZZO (Cls) 5,6 N/mm² (COMPRESSIONE)

CALCESTRUZZO (TAGLIO) = 0,5 · S_{AMM}

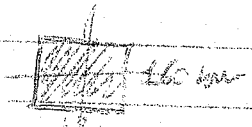
MURATURE (TAGLIO) = 0,1 · S_{AMM}

77

ACCIAIO

T = 156,25 KN

T = 125 KN



$\frac{1}{2}$ h =

$\tau_M = \frac{156,25 \cdot 10^3}{54613,333} = 2,86 \ N/mm^2$

$\tau_{amm} = 213 \ N/mm^2$ $\quad \tau_M < \tau_{amm}$

$2,86 < 213$ È VERIFICATA

$T = \frac{2}{3} \cdot \frac{T}{A} = \frac{3}{2} \cdot \frac{125000}{25600} = 7,32$

$T_{amm} = 0,577 \cdot 213 = 122,90$

$T \ 7,32 < T_{amm} \ 122,90$

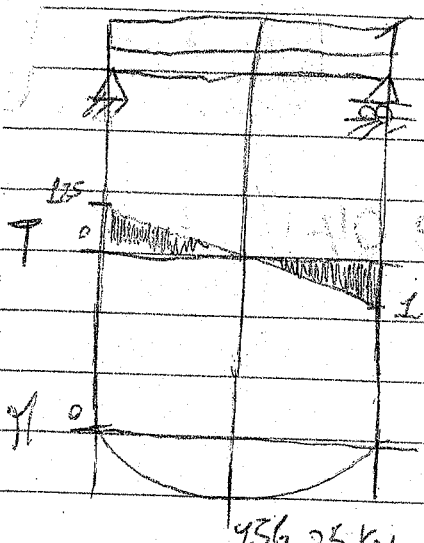
È VERIFICATA

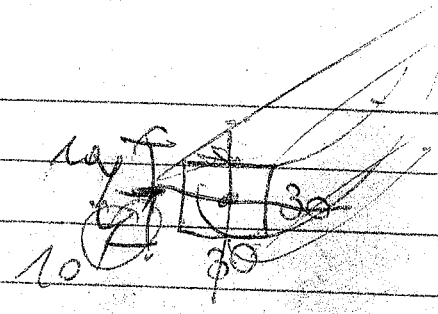
S275 $\quad \sigma_M = \frac{15625 \cdot 10^3}{54613,333} \cdot 30 = 22,88 \ N/mm^2$

$\sigma_{amm} = 262,5 \ N/mm^2$

$\sigma_M < \sigma_{amm}$ È VERIFICATA

$q = 50\ kN/ml$ **Legno**

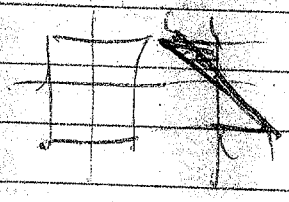




$l = 5$ $\quad M = \frac{1}{8} q l^2$

$\sigma_{amm} = 10\ N/mm^2$

$\sigma_f = \frac{M}{I} y$ $\quad I = \frac{1}{12} b^4 = 67500\ mm^4$



$\sigma_f = \frac{156 \cdot 10^6}{67500 \cdot 10^4} \cdot 150 \leq \sigma_{amm}$

$\sigma_f = 3,47\ N/mm^2 \leq \sigma_{amm}$ $\quad 3,47 \leq 10\ N/mm^2$ VERIFICATO

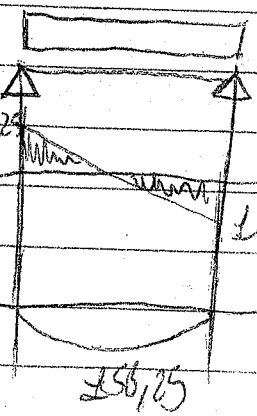
TAGLIO

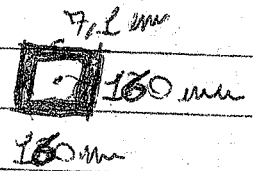
$\tau_{amm} = 0,9 \cdot \sigma_{amm}$ $\quad \tau = \frac{T \cdot S}{I \cdot b} \Rightarrow \frac{3}{2} \frac{T}{A} = \frac{3}{2} \frac{125 \cdot 10^3}{900 \cdot 10^2} = 2,08$

NON È VERIFICATO

CAMBIO MATERIALE

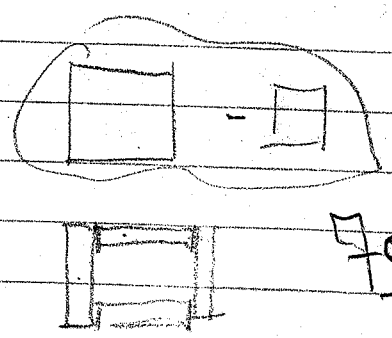
S235 $\quad \sigma_{amm} = 213,63$





$\sigma_f = \frac{M}{I} \cdot y = \frac{156,25 \cdot 10^6}{1696 \cdot 10^4} \cdot$

$\sigma_f = \frac{M}{W} = \frac{156,25 \cdot 10^6}{212 \cdot 10^3}$



S275

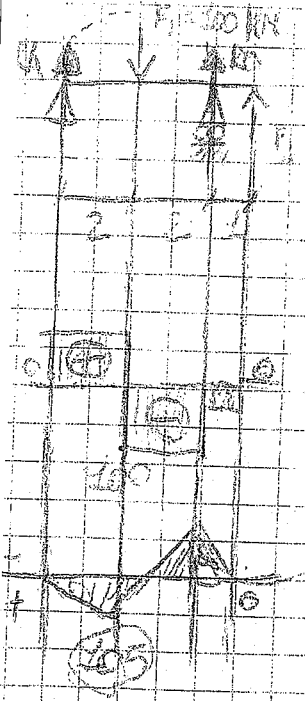
IPE

$S = 1,05$

$S_{adm} = 220\ MPa$

$T = 100\ kN$

$M = 105\ kN \cdot m$



$\tau_{xy} = \frac{T}{I} \cdot V$

$\tau_{xy} = \frac{100 \cdot 10^3}{3239 \cdot 10^4} \cdot 325,7 \cdot 10^{-3} = 10,05\ MPa$

$\sigma_x = \frac{M}{I} \cdot y$

$\sigma_x = \frac{105 \cdot 10^6}{5790 \cdot 10^4} \cdot 135 = 244,82\ MPa$

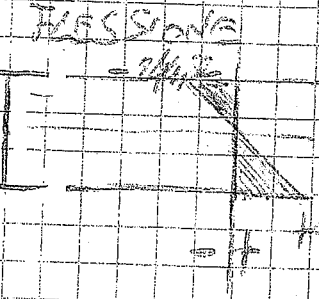
$\sigma_{adm} = 0,577 \cdot S_{adm} = 150,15\ MPa < S_{adm}$

$\tau = \frac{T \cdot S}{I \cdot b} = \frac{100 \cdot 10^3 \cdot 212 \cdot 10^3}{5790 \cdot 10^4 \cdot 15} = 30,9\ MPa$

$\tau < S_{adm}$ È VERIFICATO

VON MISES

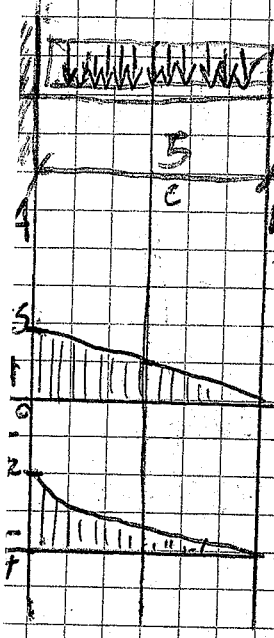
$\sigma_{id} = \sqrt{244,37^2 - 244,37 \cdot 3,09 + 3 \cdot 30,9^2} = 254,87\ MPa$

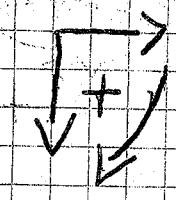


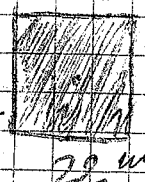
$A = 45,9$

$S = 212\ mm^3$

LEGNO MASSICCIO







$\sigma = 10\ N/mm^2$

$I = \frac{1}{12} b \cdot h^3 = 78604\ cm^4$

$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases}$ $\begin{cases} \Sigma F_y = R_A + q \cdot l = 0 \\ \Sigma M_A = q \cdot l \cdot \frac{l}{2} + M \end{cases}$

$\begin{cases} +M = -312,5\ KN \cdot m \\ R_A = 125\ KN \end{cases}$

$T_{MAX} = 125\ KN$

$M = 312,5\ KN/m$

$cm^2 - mm^2 \quad 10^4 = cm^4 - mm^4$

$\sigma = KN - N \quad 10^6 = KN \cdot m^2 - N/m^2$

$T_A = R_A = 125\ KN$

$T_C = R_A + q \cdot \frac{l}{2} = 62,5\ KN$

$T_B = 0$

$M_A = 312,5\ KN \cdot m$

$$\sigma_f = \frac{M}{I} \cdot y = \frac{312,5}{78604} \cdot 17,5$$

$$\sigma_f = \frac{312,5 \cdot 10^6}{78604 \cdot 10^4} \cdot 17,5 = 6,96\ N/mm^2$$

$\sigma_f < \sigma_{amm} \Rightarrow$ È VERIFICATO

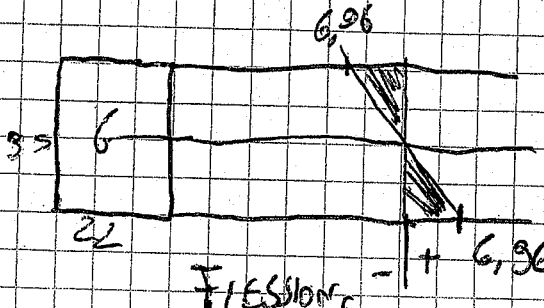
$$\tau = \frac{T \cdot S}{I \cdot b} \Rightarrow \frac{3}{2} \cdot \frac{T}{A} = \frac{3}{2} \frac{125 \cdot 10^3}{770 \cdot b^2} = 2,44\ N/mm^2$$

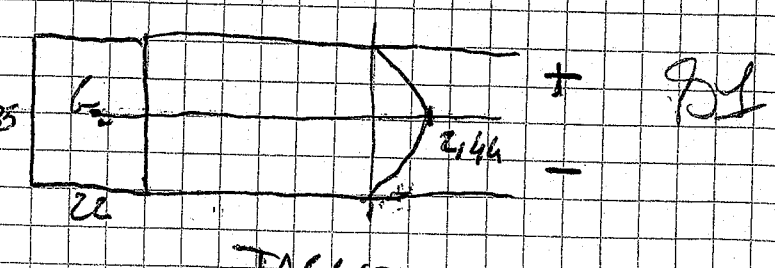
$\tau_{amm} = 0,9 \cdot \sigma_{amm} = 9\ N/mm^2$

$\tau < \tau_{amm} \Rightarrow$ È VERIFICATO

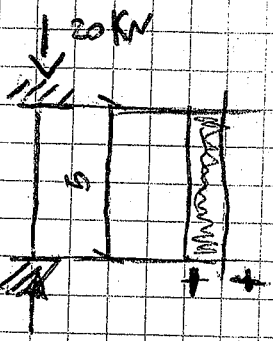
VON MISES

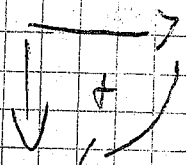
$$\sigma_{IDEALE} = \sqrt{\sigma_f^2 + 3\tau^2} = \sqrt{48,44 + 17,86} = \sqrt{66,3} = 8,14\ N/mm^2$$



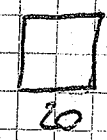


(3)





LEGNO MASSICCIO



$N = 20\ KN$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases} \begin{cases} R_A = F_x = -20 \end{cases}$$

$$\sigma = \frac{N}{A} \cdot \omega$$

$$\sigma_{amm} = 8\ N$$

$$\sigma = \frac{20 \cdot 10^3}{400 \cdot 10^2} \cdot \omega$$

$$\omega \Rightarrow \lambda = \frac{l_0}{\rho} = \frac{\beta}{\rho}$$

$$I = \frac{1}{12} \cdot b^4 = 13333{,}33\ cm^4$$

$$\rho = \sqrt{\frac{I}{A}} = \sqrt{\frac{13333}{400}} = \sqrt{33{,}33} = 5{,}77\ cm$$

$$\lambda = \frac{5 \cdot 10^2}{5{,}77} = 86{,}65 \Rightarrow 2{,}45$$

$$\omega_{int} = \frac{(2{,}58 - 2{,}20) \cdot (86{,}65 - 80)}{(90 - 80)} = 0{,}25$$

$$\omega_{FINALE} = \omega_{MIN} + \omega_{INT} = 2{,}20 + 0{,}25 = 2{,}45$$

$$\sigma = \frac{20 \cdot 10^3}{400 \cdot 10^2} \cdot 2{,}45 = 1{,}23$$

$$\sigma < \sigma_{amm} \Rightarrow \text{È VERIFICATO}$$

82

160 KN

3,50

S235

HEA 100

$\sigma_c = \frac{N}{A} \cdot \gamma_{M0} \leq \sigma_{adm}$

$\sigma_{adm} = \frac{235}{1,05} = 223,8 \text{ N/mm}^2$

$\lambda = \frac{l_0}{i} = \frac{1400}{4,05} = 172,52$

$i = \sqrt{\frac{I}{A}} = \sqrt{\frac{760}{30,6}} = 4,05$

$\omega_{(\lambda)} = 4,24$

$\sigma_c = \frac{160.000 \cdot 4,24}{21,2^2} = 360 \text{ N/mm}^2$

1) $q = 25 \text{ KN/m}$

5

2) 120 KN

2 | 2 | 4

20 KN

3) 20 KN

L

L6xM0

3,5 L6xM0 108,1cm²

20

200

IPE 200

5217,5

$\gamma_{M0} = 1,2$

$I_x = 194,3$

$A = 28,5$

L8xM0

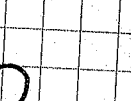
20

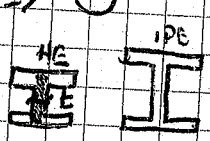
1,5

83

$\tau = \frac{T \cdot S_x}{I_x \cdot b}$ JOURAWSKI

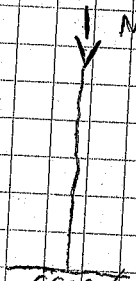
$\tau = \frac{3}{2} \cdot \frac{T}{A} \Rightarrow$ 

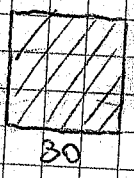
$\tau = \frac{4}{3} \cdot \frac{T}{A} \Rightarrow$ 

$\tau = \frac{T}{A} \Rightarrow$ 

$A = b \cdot h$

LEGNO





$\sigma_{com} = 8\ N/mm^2$

$\sigma_c = \frac{N}{A} \cdot \omega \leq \sigma_{com}$

$\lambda = \frac{l_0}{\rho} = \frac{l \cdot 2}{\varphi} = \frac{12\ m}{\varphi} = \frac{1200\ cm}{8,66} = 138,57$

$\varphi = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{1}{12} h^4}{900}} = \sqrt{75} = 8,66$

$\omega_{(138)} = \frac{(\omega_{MAG} - \omega_{MIN}) \cdot (\lambda_{COL} - \lambda_{MIN})}{(\lambda_{MAG} - \lambda_{MIN})} = \frac{(5,88 - 5,07) \cdot (138,57 - 130)}{(140 - 130)} = 5,76$

$\sigma_c = \frac{100000}{90000} \cdot 5,76 = 6,4\ N/mm^2 \leq 8\ N/mm^2$ È VERIFICATO

86

IL LEGNO COME MATERIALE DA COSTRUZIONE

CONFRONTO TRA LE PROPRIETÀ DEL LEGNO E DEGLI ALTRI MATERIALI DA COSTRUZIONE

PROPRIETA'	LEGNO	ACCIAIO	CLS
Tensione ammiss. (compr.) σ_{adm} [MPa]	10	160	10
Peso specifico γ_m [daN/m^3]	600	7850	2400
Rapporto γ_m / σ_{adm}	60	50	240
Modulo elastico [GPa]	8	210	30
Duttilita'	No	Si	No
Resistenza a trazione	Si	Si	No
Comportamento reologico	Si	No	Si
Comportamento igroscopico	Si	No	No
Isotropia	No	Si	Si
Omogeneita'	No	Si	Si
Combustibilita'	Si	No	No

IL LEGNO COME MATERIALE DA COSTRUZIONE

FATTORI CHE INFLUENZANO LA RESISTENZA MECCANICA DEL LEGNO

DIREZIONE DEL CARICO RISPETTO ALLA DIREZIONE DELLE FIBRE

PRESENZA DI DIFETTI

COMPORTAMENTO IGROSCOPICO – REAZIONE CON L'ACQUA

DURATA DI APPLICAZIONE DEL CARICO – EFFETTI VISCOSI

L'influenza di tutti questi fattori può essere analizzata osservando le particolari caratteristiche della microstruttura e della macrostruttura del legno