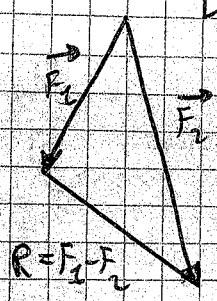
LA DIFFERENZA DI VETTORI



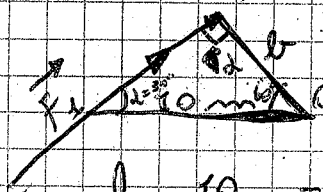
10/10/2016 IL MOMENTO

Il momento è una grandezza vettoriale definito come il prodotto tra una forza per il braccio

$$\Pi = \vec{F} \cdot b = N \cdot m$$

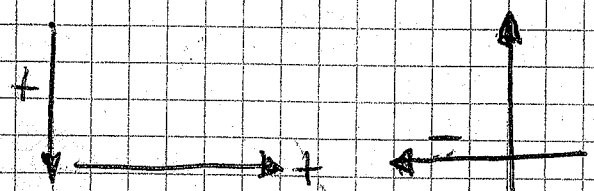
b = distanza = m

La distanza è quella perpendicolare alla forza.



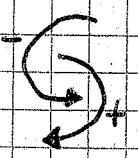
$$b = \frac{10}{\sin 30°} = 20\,m$$

~~Per convenzione definiamo le forze che vanno verso l'alto~~



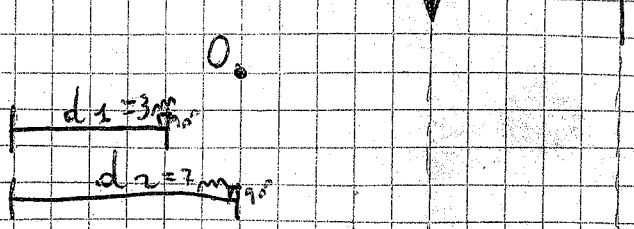
MOMENTO ORARIO + (POSITIVO)

MOMENTO ANTIORARIO − (NEGATIVO)



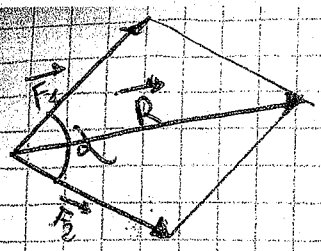
$\Pi = \Sigma M$

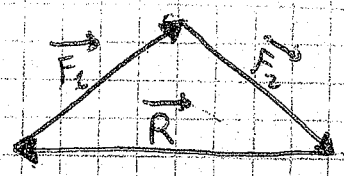
Σ = sommatoria



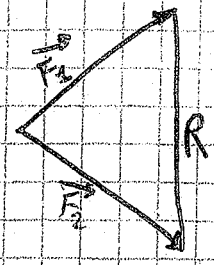
$$\Pi = F_1 \cdot d_1 + (-F_2 \cdot d_2)$$

$$\Pi = 10 \cdot 3 - 5 \cdot 7 = -5\,N \cdot m$$





$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2F_1 \cdot F_2 \cdot \cos\alpha}$$



$$\vec{R} = |\vec{F_1}| - |\vec{F_2}|$$

CARNOT

$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2F_1 \cdot F_2 \cdot \cos\alpha}$$

$26,372 \cdot 0,42$

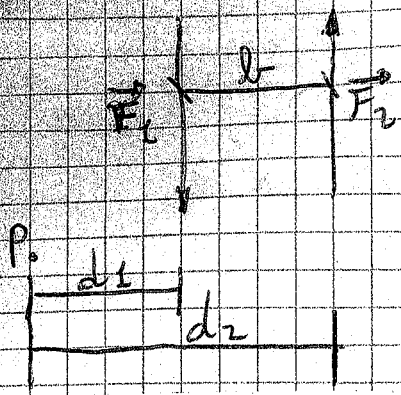
$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\alpha} = \sqrt{100 + 784 + 20 \cdot 28 \cdot \cos 60}$

$= \sqrt{884 + 560 \cdot 0,421} = \sqrt{884 + 235,2} = \sqrt{1119,2}$

$\sqrt{884 - 235,2} = \sqrt{648,8}$

$F = 25,47\ N$

9



$$\Pi_P = F_1 \cdot d_1 - F_2 \cdot d_2$$

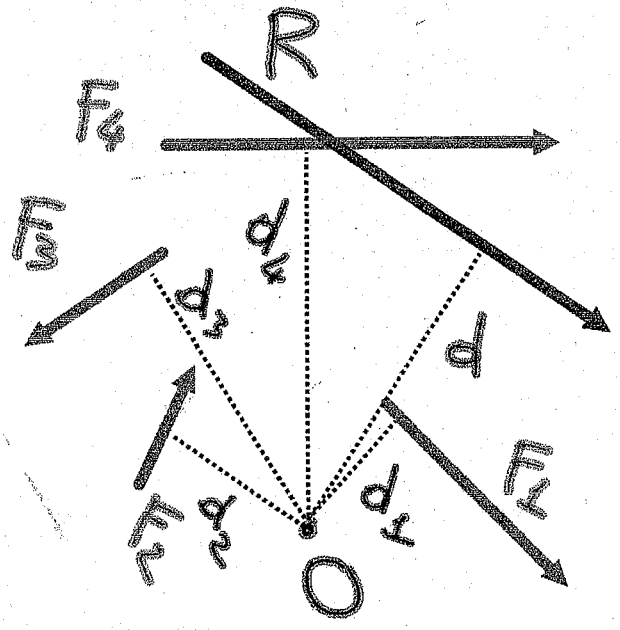
$$\Pi_P = F(d_2 - d_1)$$

Teorema di Varignon

3 BV

Dato un sistema di forze complanari e scelto un punto nel piano, si può calcolare il momento di ciascuna forza e determinare il momento risultante ; ma i singoli momenti e il momento risultante devono soddisfare il teorema di Varignon, per il quale:
"In un sistema di forze complanari il momento della risultante , rispetto a un punto "O" qualsiasi nel piano, è uguale alla somma algebrica dei momenti delle singole forze rispetto al piano stesso."

es. $F_1 \times d_1$ ....



$$F_1 \times d_1 + F_2 \times d_2 + F_3 \times d_3 + F_4 \times d_4 = R \times d$$

10

$$d = \frac{F_1 \times d_1 + F_2 \times d_2 + F_3 \times d_3 + F_4 \times d_4}{}$$

EQUAZIONI CARDINALI DELLA STATICA 3°B

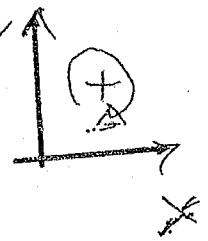
SISTEMA LINEARE

$$\begin{cases} \sum \bar{F} = 0 \Rightarrow \text{NO TRASLAZIONI} \\ \sum \bar{M}_p = 0 \Rightarrow \text{NO ROTAZIONI} \end{cases}$$

Condizione <u>necessaria</u> e <u>sufficiente</u> affinché un corpo sia in quiete (STATICA) è che la $\sum \bar{F} = 0$ e che la $\sum \bar{M}_p = 0$

2D

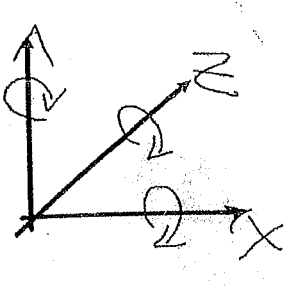
NEL PIANO



$$\begin{cases} \sum \bar{F} = 0 \\ \sum \bar{M}_p = 0 \end{cases} \Rightarrow \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_p = 0 \end{cases}$$

3D

NELLO SPAZIO



$$\begin{cases} \sum \bar{F} = 0 \\ \sum \bar{M}_p = 0 \end{cases} \Rightarrow \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \\ \sum M_{(xy)p} = 0 \\ \sum M_{(yz)p} = 0 \\ \sum M_{(zx)p} = 0 \end{cases}$$

*

$$\alpha = \text{sen}^{-1}\left(\frac{a}{c} \cdot \text{sen}\,\gamma\right)$$

$$\alpha = \text{sen}^{-1}\left(\frac{20}{61,88} \cdot \text{sen}\,50\right)$$

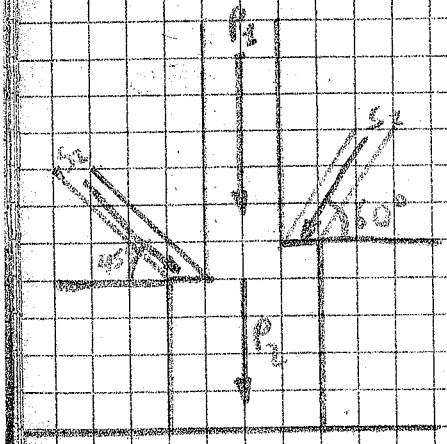
$$\alpha = \text{sen}^{-1}(1,29 \cdot 0,77)$$

$$\alpha = \text{sen}^{-1}\,0,99 = 81,89$$

$$130 - 48,11 = 81,89$$

$$180 + 81,89 = 261,89$$

3)



DATI

$P_1 = 200\,KN$

$P_2 = 60\,KN$

$S_1 = 80\,KN$

$S_2 = 120\,KN$

INCOGITE

?

CALCOLARE GRAFICAMENTE

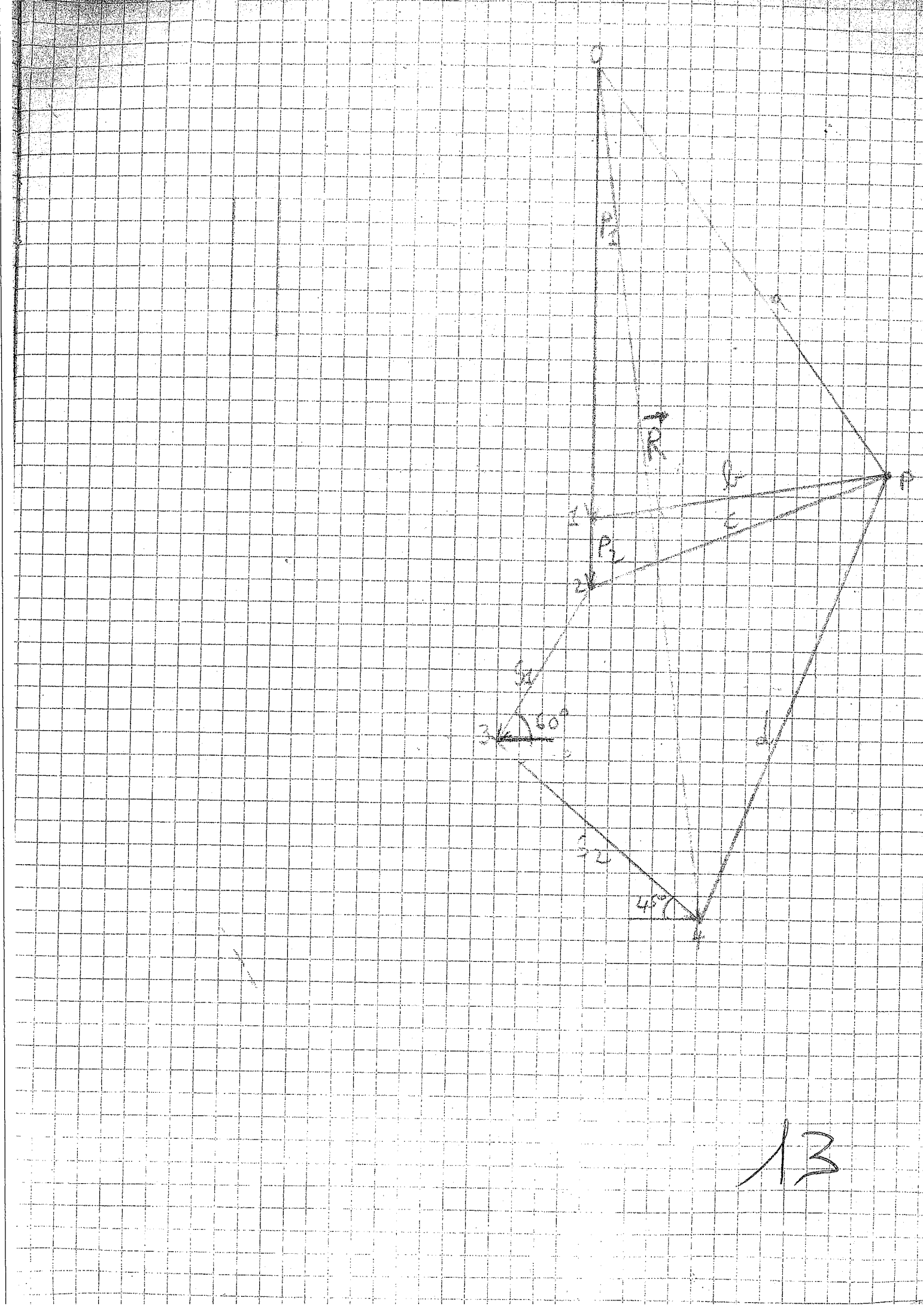
la $\vec{R}$

SCALA

1cm = 20K

$\vec{R} = \sqrt{}$

12

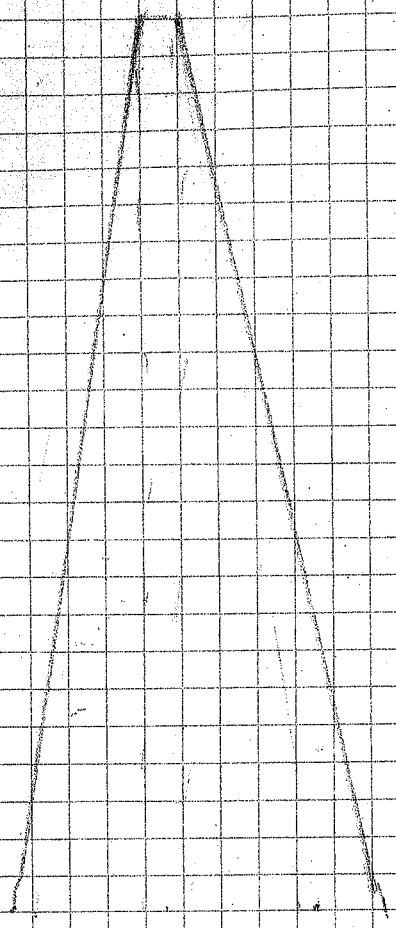


13

5

SCALA

1cm = 30cm



$$P_{P1} = \frac{0,60 \cdot 3,60}{2} \times 1,00\,m \times 24\,KN/m^3 = 25,92\,KN$$

$$P_{P2} = \text{~~~~~} 25,92\,KN$$

$$P_{P3} = \frac{0,39 \cdot 3,60}{2} \times 1,00\,m \times 24\,KN/m^3 = 16,84\,KN$$

I 4

Determinare mediante procedimento grafico analitico la risultante del sistema di 4 forze.

$F_1 = 80N \longrightarrow A(25;25)$ $\quad \alpha_1 = 225°$

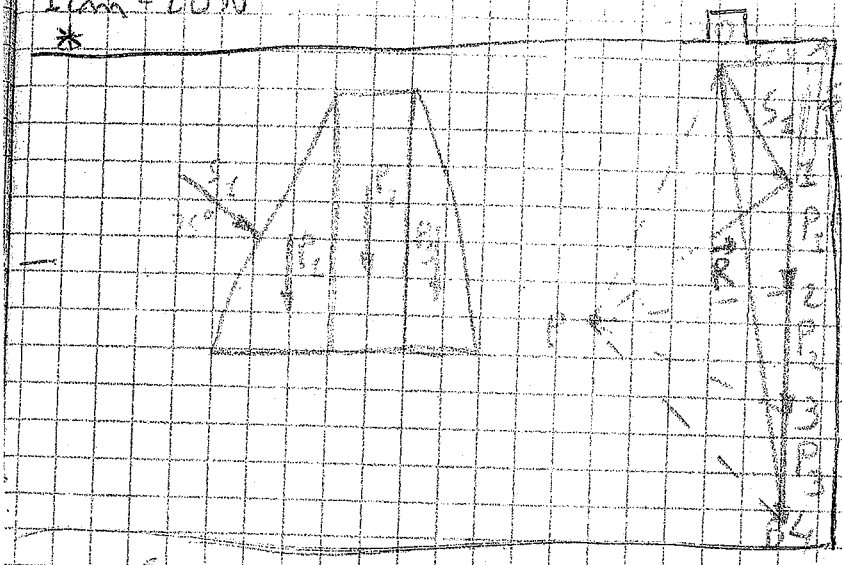
$F_2 = 50N \longrightarrow B(40;35)$ $\quad \alpha_2 = 300°$

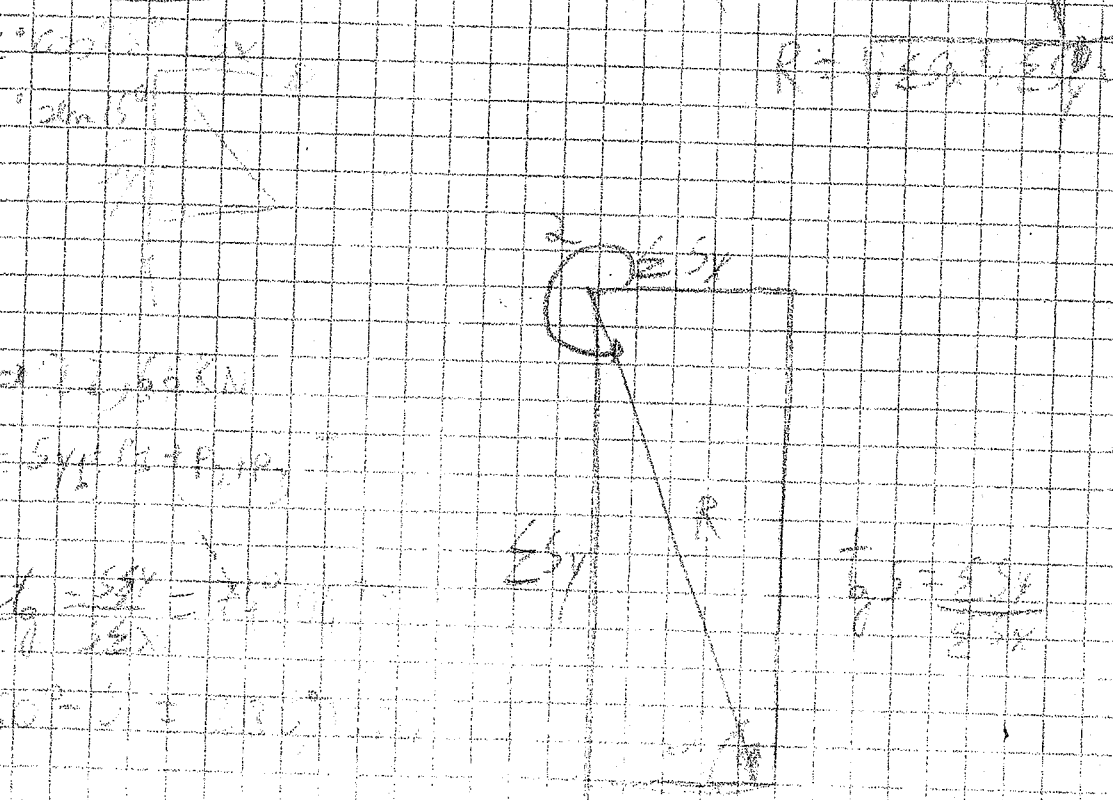
$F_3 = 30N \longrightarrow C(25;50)$ $\quad \alpha_3 = 330°$

$F_4 = 60N \longrightarrow D(50;60)$ $\quad \alpha_4 = 30°$

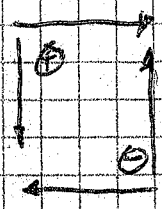
SCALA

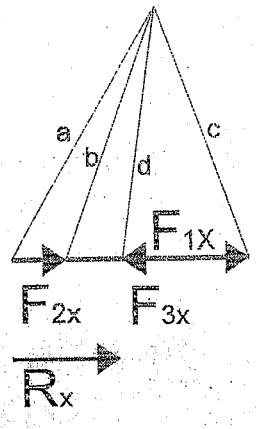
1 cm = 20 N

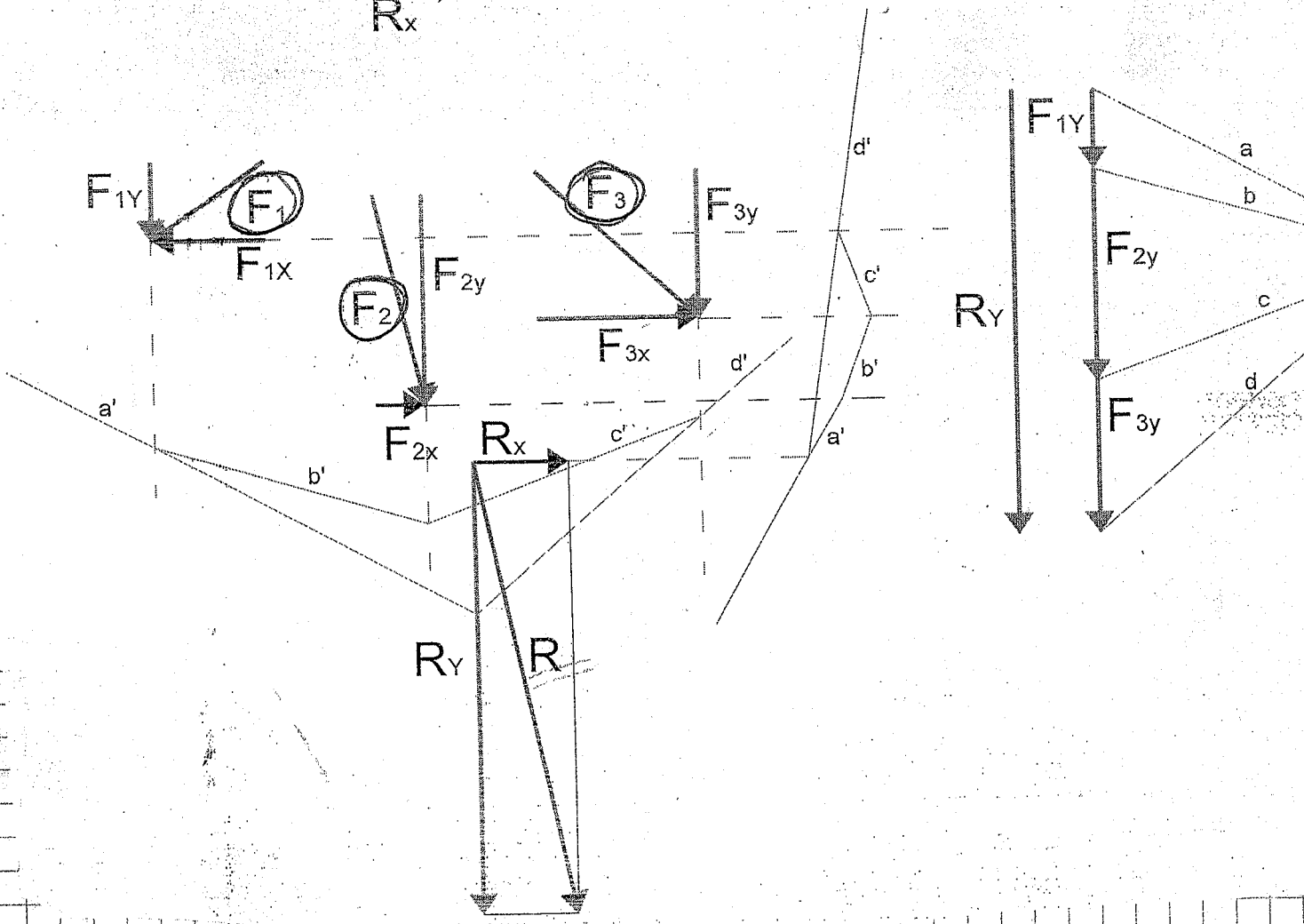




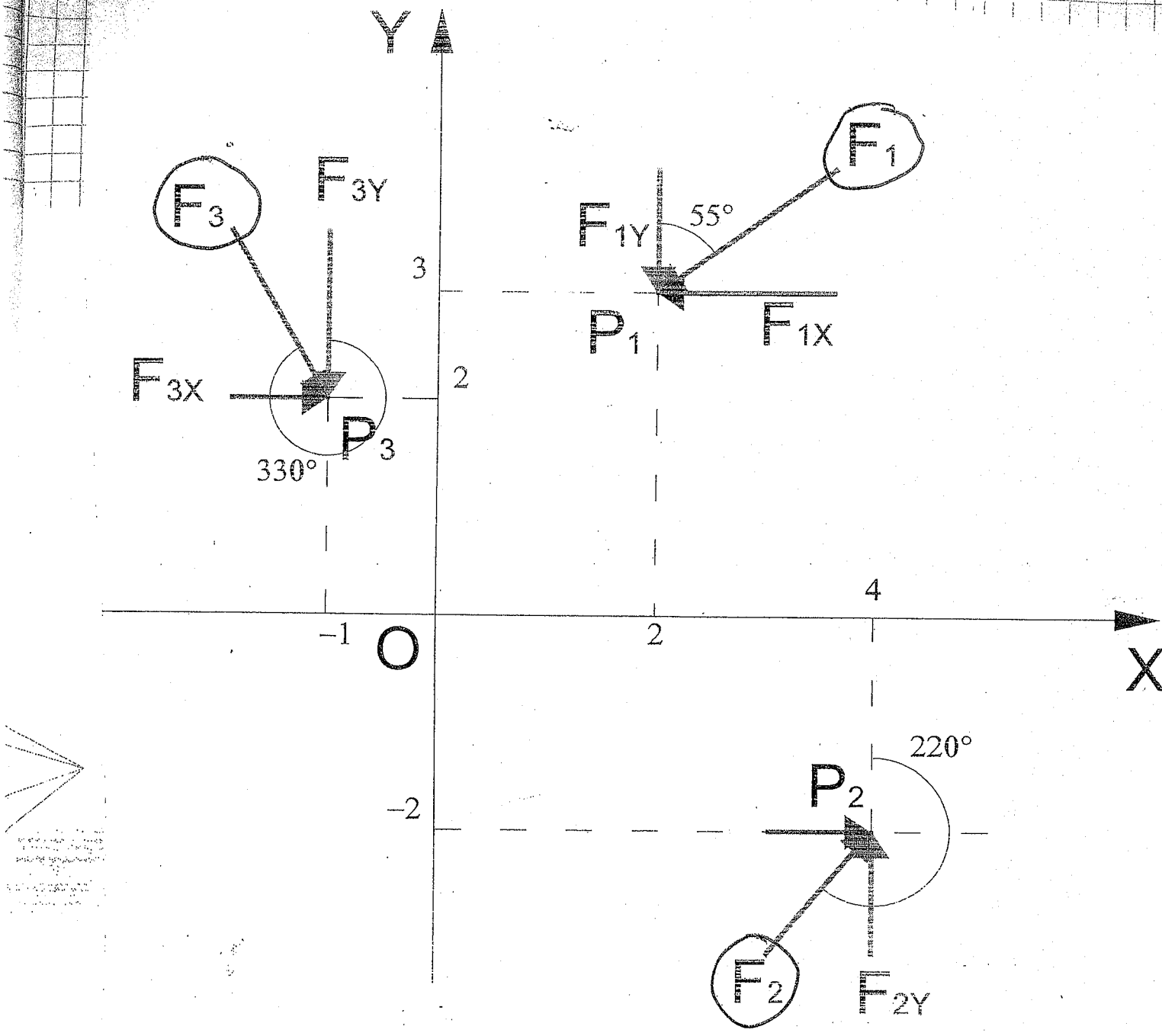
CONVENZIONE

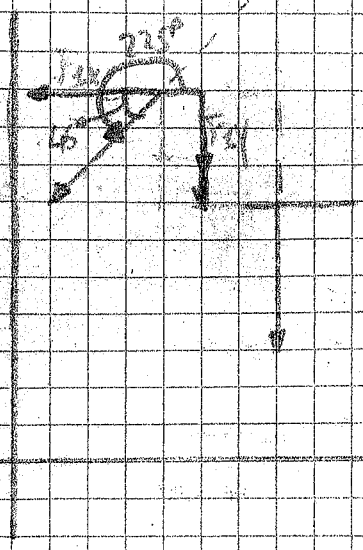


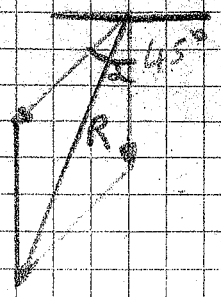




16


Y
X
O
F3
F3Y
F3X
P3
330°
F1
F1Y
55°
P1
F1X
P2
220°
F2
F2Y
3
2
4
−1
2
−2





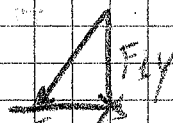
angolo $\alpha = 270 - 225 = 45°$

asse $x = \frac{\sqrt{2}}{2}$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\alpha}$

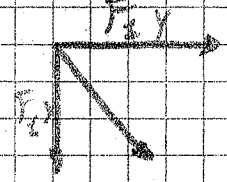
$F_{1x} = F_1 \cdot \text{asse}\,\alpha =$

$F_{1y} = F_1 \cdot \cos\alpha$



$F_{1x} = F_1 \cos\alpha$

$F_{1y} = F_1 \text{asse}\,\alpha$

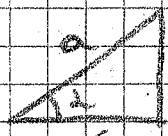


$F_{1y} = F_1 \cdot \cos\alpha$

$F_{1x} = F_1 \cdot \text{asse}\,\alpha$

$F_{1x} = F_1 \cdot \text{asse}\,\alpha$

$F_{1y} = F_1 \cdot \cos\alpha$

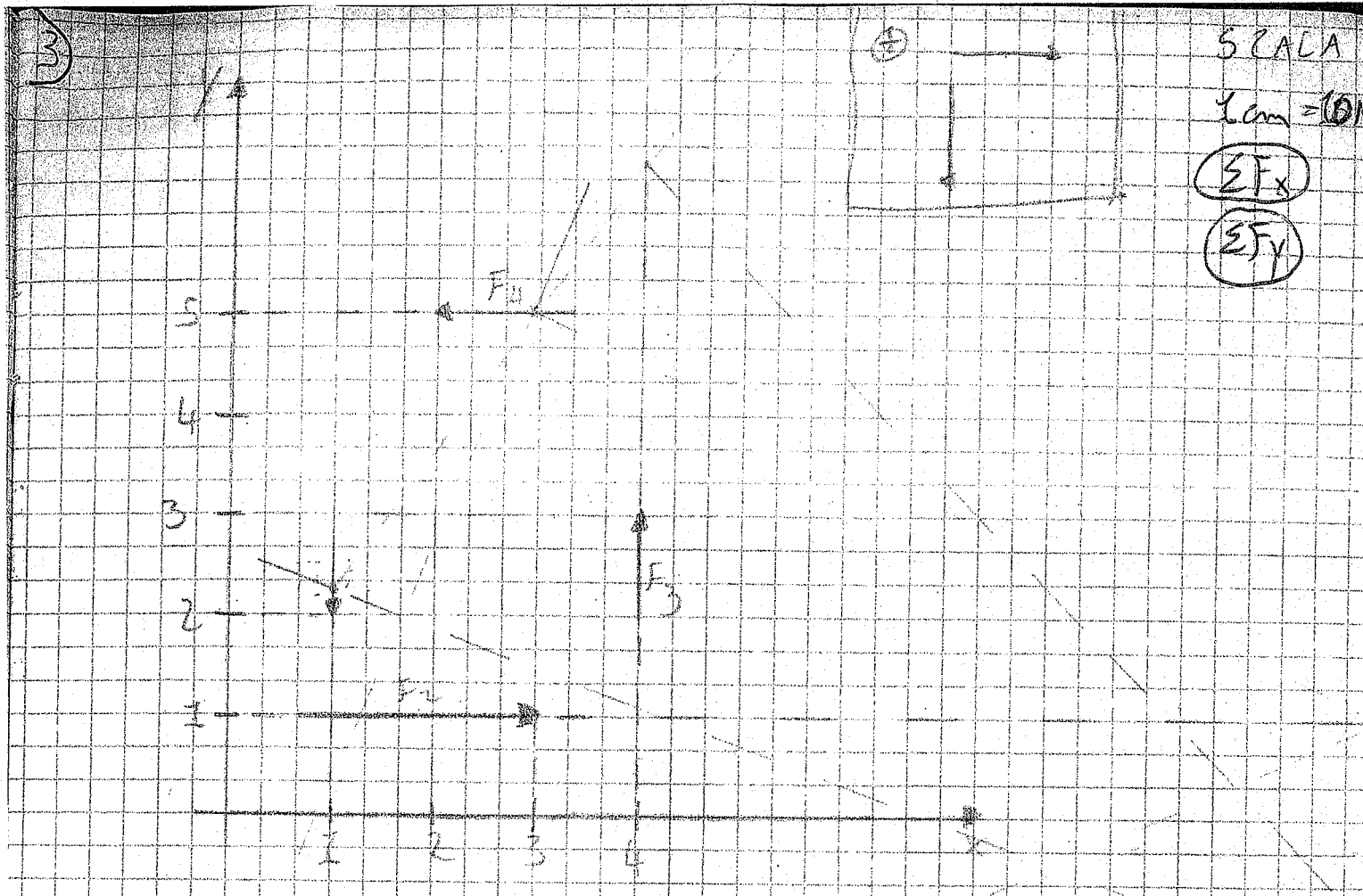


$\frac{b}{c} = 2$	$b = a \cdot \text{asse}\,\alpha$
$a = \frac{b}{2}$	$c = a \cdot \cos\alpha$
$b = c \cdot 2$	$\frac{b}{c} = \tg\alpha$

$b = a \cdot c$

$c = \frac{b}{a}$

3)

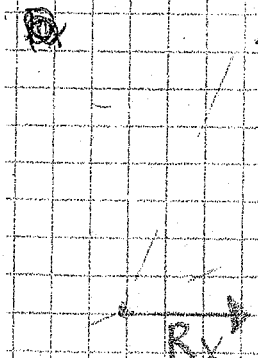


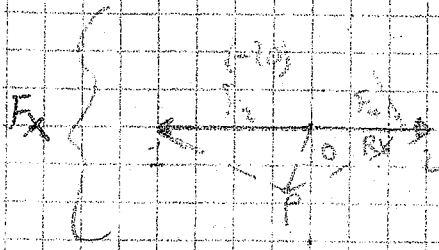
SCALA

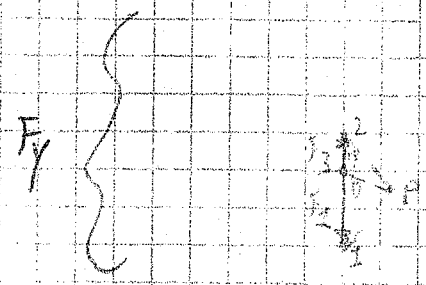
1 cm = 10

ΣF_x

ΣF_y



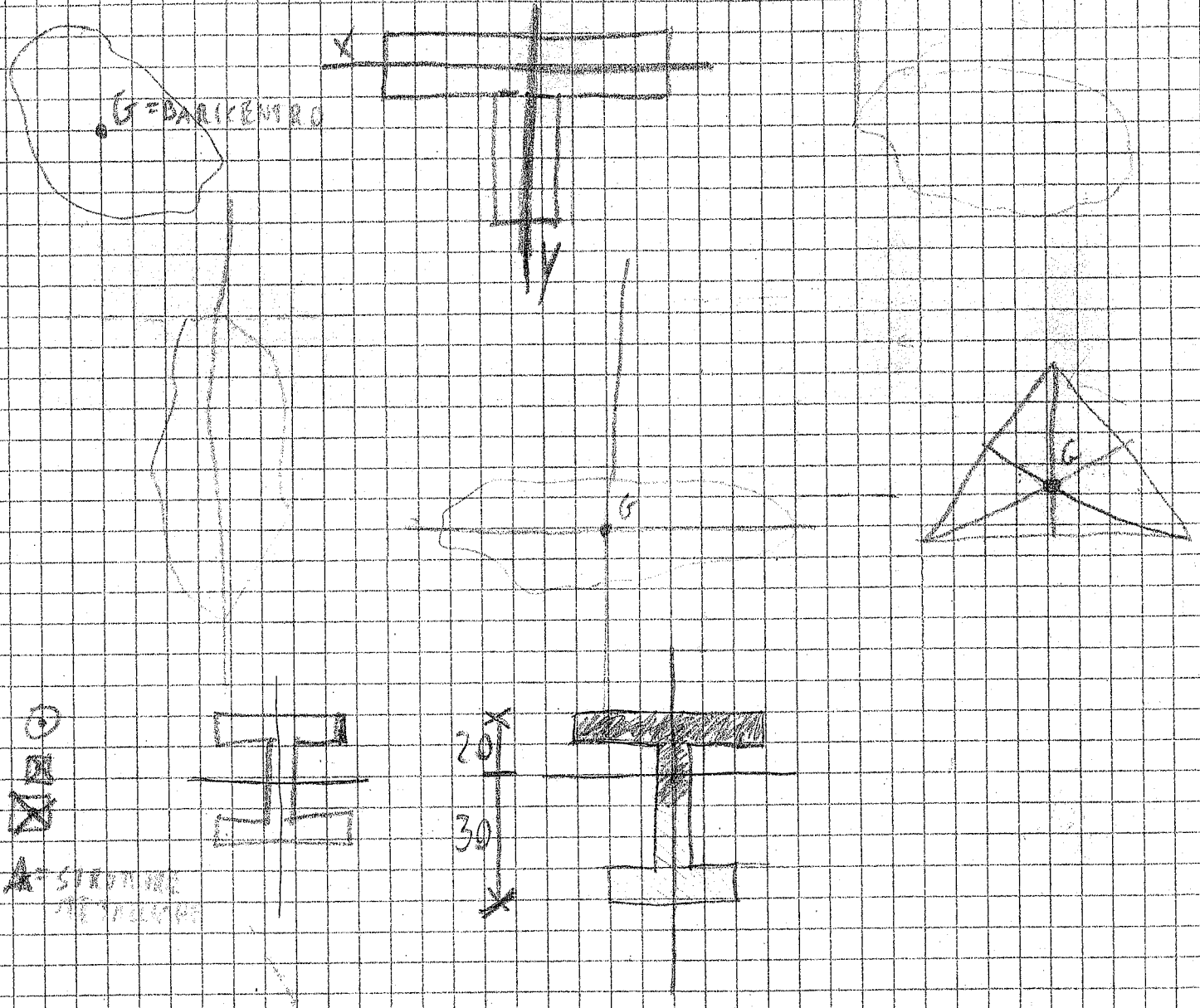




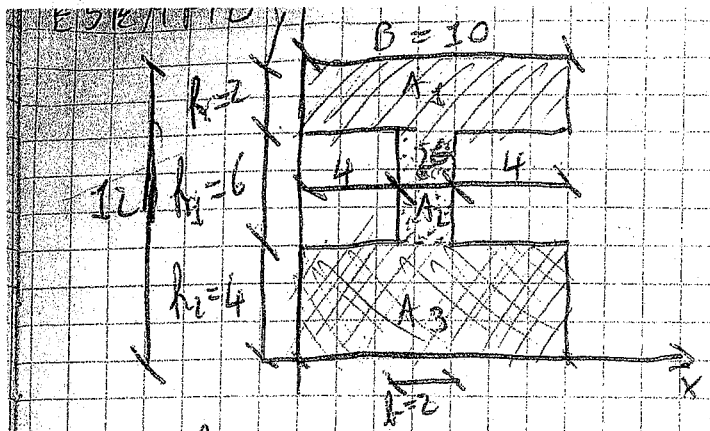
BARICENTRO

- MOMENTO STATICO
- MOMENTO DI INERZIA

Il baricentro è il punto di equilibrio di tutto il materiale che forma quell'oggetto.



S = MOMENTO STATICO



$S_x = A_t \cdot y_G = cm^3$

$S_y = A_t \cdot x_G$

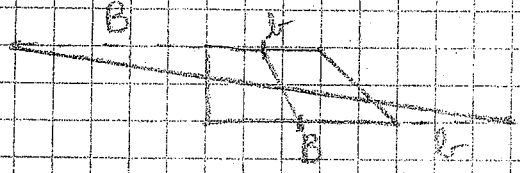
$A_1 = B \cdot h = 10 \cdot 2 = 20\ cm$

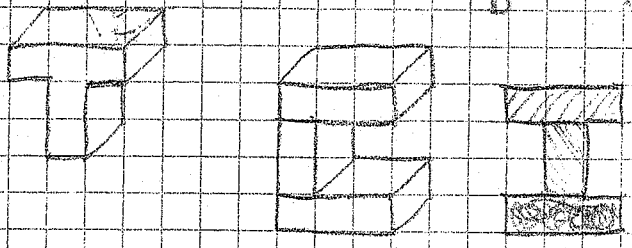
$A_2 = b_1 \cdot h_1 = 2 \cdot 6 = 12\ cm$

$A_3 = B \cdot h_2 = 10 \cdot 4 = 40\ cm$

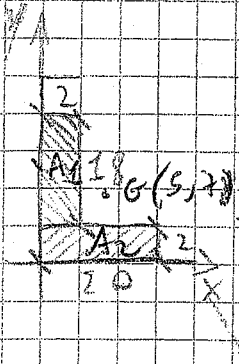
$S_x = (20 \cdot 11) + (12 \cdot 7) + (40 \cdot 2) = 220 + 84 + 80 = 384\ cm^3$

$y_G = \frac{384}{72} = 5{,}3\ cm$





$$S_x = A_t \cdot y_G$$
$$S_y = A_t \cdot x_G$$



$G = ?$

$x_G = ?$

$y_G = ?$

~~$A = A_1 + A_2$~~

~~$S_x = (2 \cdot 18 + \dots) \cdot 7 + (\dots) \cdot 1 = 4\dots$~~

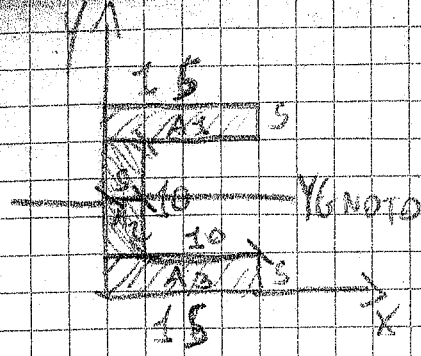
~~$S_x = A_1 \cdot y_1 + A_2 \cdot y_2$~~

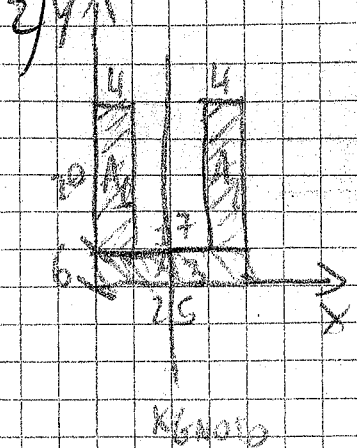
$S_x = 36 \cdot 11 + 40 \cdot 1 = 436\ cm^3$

$y_G = \frac{436\ cm^3}{76\ cm^2} = 5{,}7\ cm$

$S_y = 36 \cdot 1 + 40 \cdot 10 = 436\ cm^3$

$x_G = \frac{436\ cm^3}{\dots} = 5{,}7\ cm$





Risolvo

1) $A_1 = 15 \cdot 5 = 75\ cm$

$A_2 = 10 \cdot 5 = 50\ cm$

$A_3 = 15 \cdot 5 = 75\ cm$

$S_x = A_t \cdot y_G$

$S_y = A_t \cdot x_G$

$S_x = 75 \cdot 17,5 + 50 \cdot 10 + 75 \cdot 2,5 = 1312,5 + 187,5 + 500 = 2000\ cm^3$

$y_G = \frac{2000}{200} = 10\ cm$

$S_y = 75 \cdot 7,5 + 50 \cdot 15 + 75 \cdot 27,5 = 562,5 + 750 + 2062,5 = 3375\ cm^3$

$x_G = \frac{3375}{200} = 16,9\ cm$

2) $A_1 = 20 \cdot 4 = 80\ cm$

$A_2 = 25 \cdot 6 = 150\ cm$

$A_3 = 20 \cdot 4 = 80\ cm$

$S_x = A_t \cdot y_G$

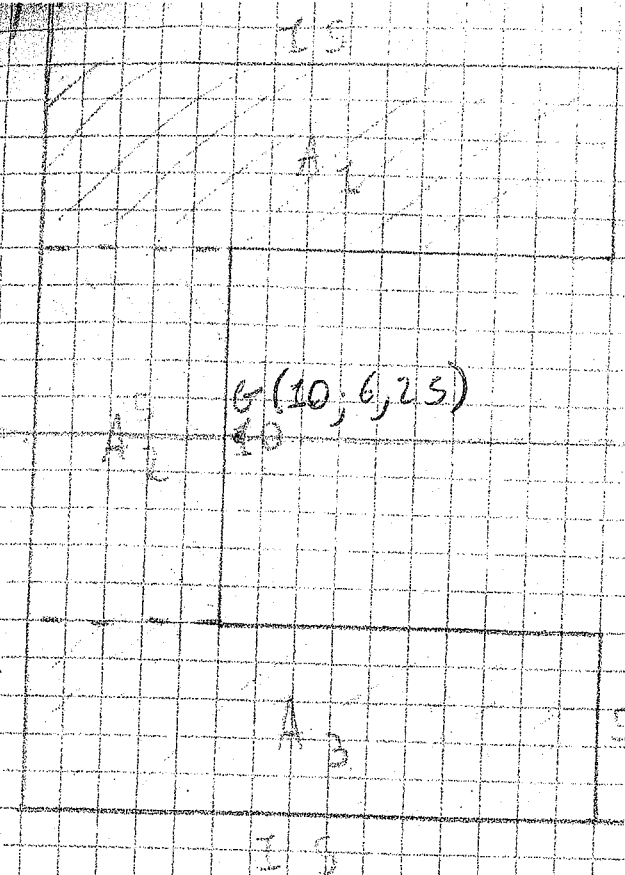
$S_y = A_t \cdot x_G$

$S_x = 80 \cdot 16 + 150 \cdot 3 + 80 \cdot 16 = 1280 + 450 + 1280 = 3010$

$y_G = \frac{3010}{310} = 9,8\ cm$

$S_y = 80 \cdot 2 + 150 \cdot 12,5 + 80 \cdot 2 = 160 + 1875 + 160 = 2195\ cm^3$

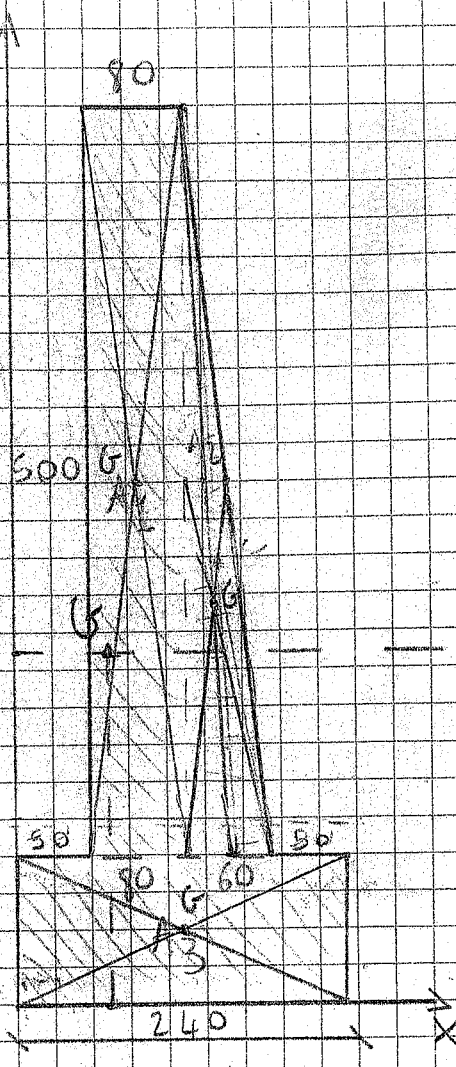
$x_G = \frac{2195}{310} = 7,1\ cm$



$S_y = (5 \cdot 15) \cdot 7,5 + (15 \cdot 5) \cdot 7,5 + (5 \cdot 10) \cdot 2,5 = 562,5 + 562,5 + 125 = 1250$ cm

$x_G = \frac{1250}{200} = 6,25$ cm

3)



SCALA 1□ = 25 cm

$A_1 = 80 \cdot 500 = 40000 cm^2$

$A_2 = \frac{60 \cdot 500}{2} = 15000 cm^2$

$A_3 = 240 \cdot 100 = 24000 cm^2$

$S_x = A_t \cdot y_G$

$S_y = A_t \cdot x_G$

$S_x = 40000 \cdot 350 + 16000 \cdot 218 + 24000 \cdot 50 =$

$14000000 + 3270000 + 1200000 = 18470000$

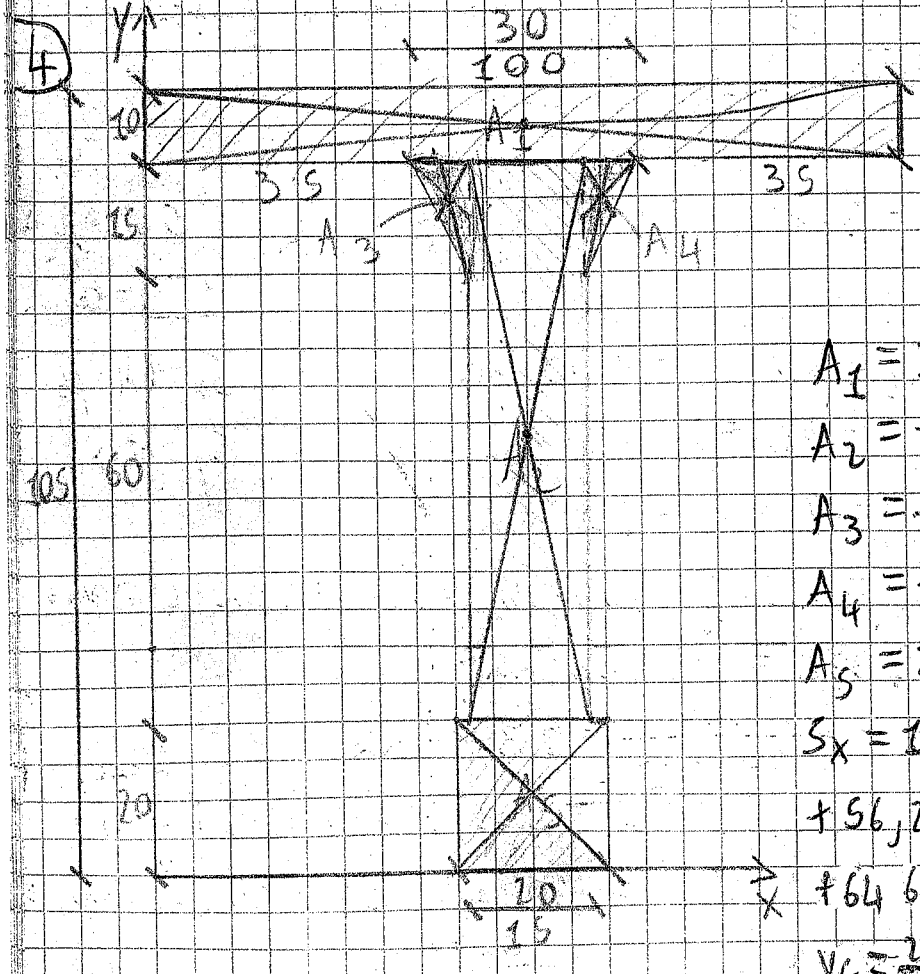
$y_G = \frac{18470000 cm^3}{79000 cm^2} = 233,80 cm$

$S_y = 4000 \cdot 90 + 15000 \cdot 150 + 24000 \cdot 120 =$

$= 360000 + 2250000 + 2880000 = 5490000$

$x_G = \frac{5490000 cm^3}{79000 cm^2} = 69,49 cm$

4)



SCALA 1□ = 5 cm

$S_x = A_t \cdot y_G$

$S_y = A_t \cdot x_G$

$A_1 = 100 \cdot 10 = 1000 cm$

$A_2 = 15 \cdot 75 = 1125 cm$

$A_3 = \frac{7,5 \cdot 15}{2} = 56,25 cm$

$A_4 = \frac{7,5 \cdot 15}{2} = 56,25 cm$

$A_5 = 20 \cdot 20 = 400 cm$

$S_x = 1000 \cdot 100 + 1125 \cdot 57,5 + 56,25 \cdot 90$

$+ 56,25 \cdot 90 + 400 \cdot 10 = 100000$

$+ 64687,5 + 64687,5 + 4000 = 233375$

$y_G = \frac{233375 cm^3}{2637,5 cm^2} = 88,48 cm$

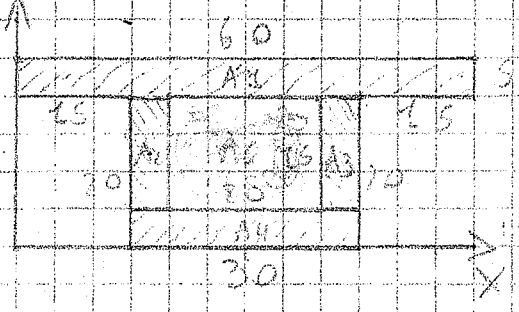
26

$S_y = 1000 \cdot 50 + 1125 \cdot 42,5 + 56,25 \cdot 38,75 + 56,25 \cdot 38,75 + 400 \cdot 45 =$

$= 120171,875 \text{ cm}^3$

$x_G = \frac{120171,875 \text{ cm}^3}{2637,5 \text{ cm}} = 45,56 \text{ cm}$

(5)



SCALA
1□ = 5cm
$S_x = At \cdot y_G$
$S_y = At \cdot x_G$

$A_1 = 60 \cdot 5 = 300 \text{ cm}$

$A_2 = 5 \cdot 15 = 75 \text{ cm}$

$A_3 = 5 \cdot 15 = 75 \text{ cm}$ $\quad At = A_1 + A_2 + A_3 + A_4 = 300 + 75 + 75 + 150 = 600c$

$A_4 = 30 \cdot 5 = 150 \text{ cm}$

$A_5 = 20 \cdot 15 = 300 \text{ cm}$

~~$S_x = 300 \cdot 30 + 75 \cdot 17,5 + 75 \cdot 42,5 + 150 \cdot 30 = 18000 \text{ cm}^3$~~

~~$y_G = \frac{}{900} = 20 \text{ cm}$~~

$S_x = 300 \cdot 22,5 + 75 \cdot 12,5 + 75 \cdot 12,5 + 150 \cdot 2,5 = 9000 \text{ cm}^3$

$y_G = \frac{9000 \text{ cm}^3}{600} = 15 \text{ cm}$

$S_y = 300 \cdot 30 + 75 \cdot 17,5 + 75 \cdot 17,5 + 150 \cdot 30 = 9000 + 1312,5 + 3137,5 + 4500 = 17950 \text{ cm}^3$

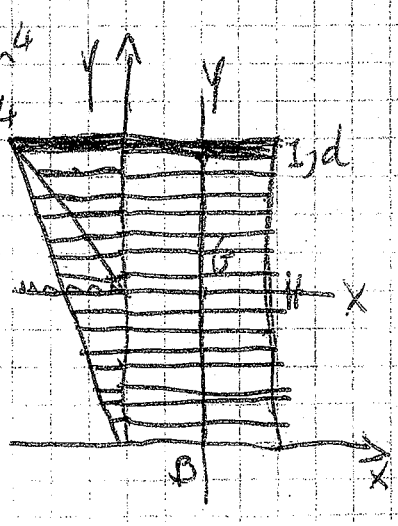
$x_G = \frac{17950 \text{ cm}^3}{600} = 29,91 \text{ cm} \approx 30 \text{ cm}$

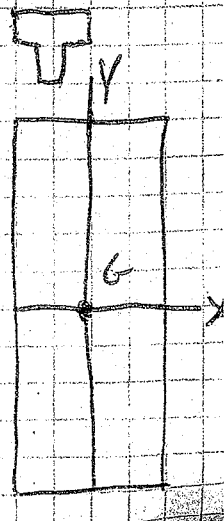
MOMENTO DI INERZIA

$I_{xG} = A_t \cdot d_y^2 = cm^2 \cdot cm^2 = cm^4$

$I_{yG} = A_t \cdot d_x^2 = cm^2 \cdot cm^2 = cm^4$

(27)





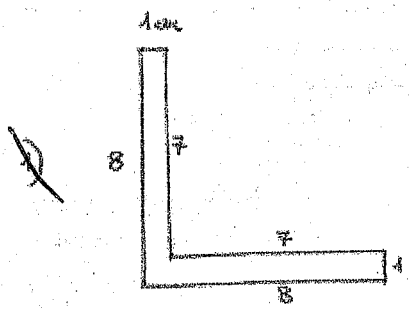
$I_x = \sum A_i \cdot d_i^2$ INFINITESIMALE

$I_x = \frac{1}{3} B \cdot H^3$

$I = \frac{1}{3} \cdot \ldots \cdot a^3$

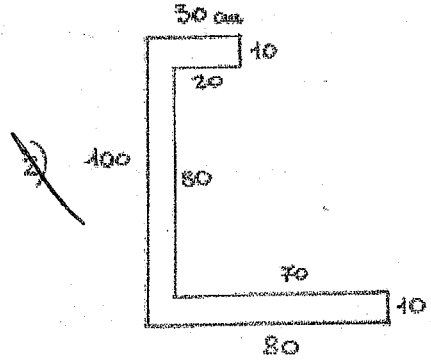
ESERCIZI DI RIEPILOGO

1)

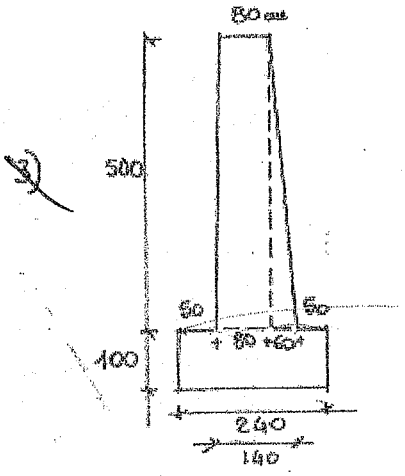


DETERMINARE LE COORDINATE DEI BARICENTRI DELLE SEGUENTI SEZIONI.

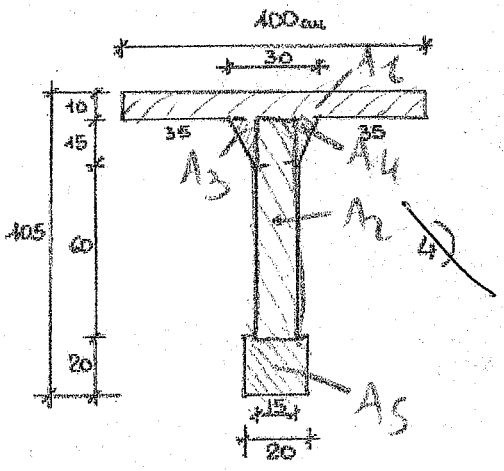
2)



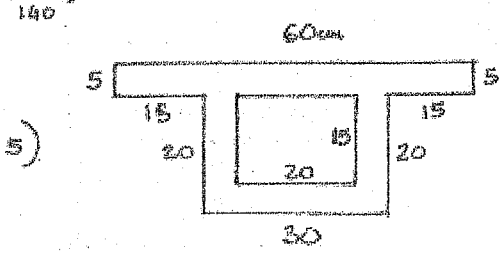
3)



4)



5)



ALFABETO GRECO

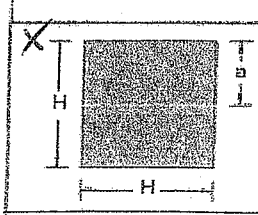
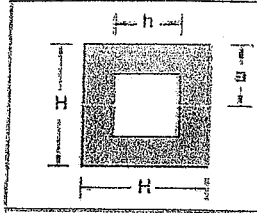
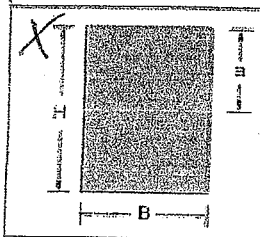
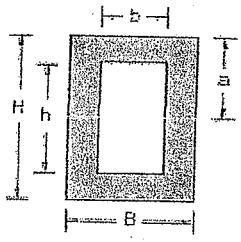
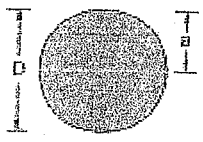
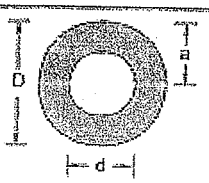
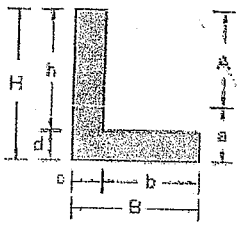
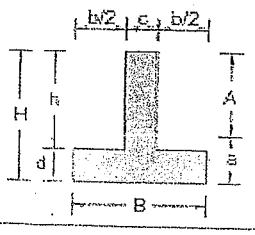
Maiuscole	minuscole	pronuncia	traslitterazione
A	α	alfa	a
B	β	beta	b
Γ	γ	gamma	g *(dura)*
Δ	δ	delta	d
E	ϵ	epsilon	é *(e chiusa)*
Z	ζ	zeta	z
H	η	eta	è *(e aperta)*
Θ	θ	theta	th
I	ι	iota	i
K	κ	cappa	c *(dura)*
Λ	λ	lambda	l
M	μ	mi *o* mu	m
N	ν	ni *o* nu	n
Ξ	ξ	csi	x
O	o	òmicron	ò *(o aperta)*
Π	π	pi	p
P	ρ	ro	r *(aspirata)*
Σ	σ	sigma	s
T	τ	tau	t
Y	υ	üpsilon	y $\rightarrow$ ü *(ue tedesca)*
Φ	ϕ	phi	ph $\rightarrow$ f
X	χ	chi	ch *(aspirata)*
Ψ	ψ	psi	ps
Ω	ω	oméga	ó *(o chiusa)*

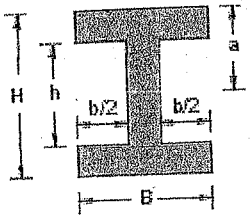
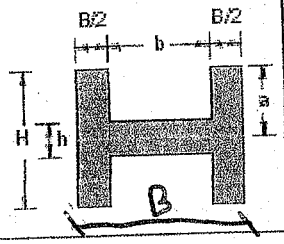
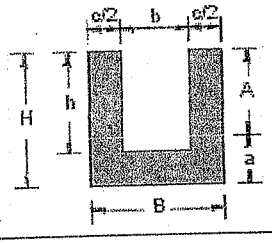
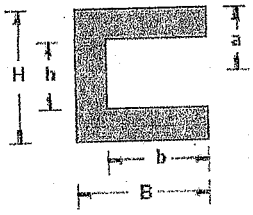
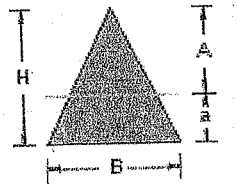
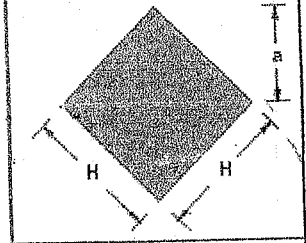
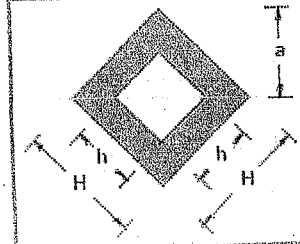
Tutti i simboli matematici che vengono utilizzati frequentemente con affianco il significato

<	Minore	>	Maggiore	≤	Minore o uguale
≥	Maggiore o uguale	±	Più o meno	≠	Diverso
÷	Diviso	×	Per	−	Meno
+	Più	/	Diviso	∑	Sommatoria
=	Uguale	∫	Funzione	σ	Sigma
%	Percentuale	‰	Per mille	°	Gradi
∛	Radice cubica	√	Radice quadrata	∞	Infinito
∠	Angolo	∀	Per ogni	⊥	Perpendicolare
∃	Esiste	∄	Non esiste	⅞	Sette ottavi
∇	Gradiente	∈	Appartiene	∉	Non appartiene
∏	Produttoria	∝	Proporzionale a	Δ	Delta
ℕ	Numeri naturali	ℂ	Numeri complessi	ℙ	Numeri primi
ℚ	Numeri razionali	ℤ	Numeri interi	ℝ	Numeri reali
∅	Insieme vuoto	∩	Intersezione	∪	Unione
≅	Congruente	≈	Circa	∫	Integrale
⊂	Sottoinsieme proprio	⊃	Soprainsieme proprio	⊆	Sottoinsieme di
⊻	Xor	∧	And	∨	Or
α	Alfa	β	Beta	γ	Gamma
π	Pi Greco	¼	Un quarto	½	Mezzo
⅔	Due terzi	⅕	Un quinto	⅗	Tre quinti
⅚	Cinque sesti	⅜	Tre ottavi	⅘	Quattro quinti

{ }	insieme	( )	insieme delle parti
∈	appartiene	∉	non appartiene
ℕ	insieme dei numeri naturali	ℤ	insieme dei numeri interi
ℚ	insieme dei numeri razionali	ℝ	insieme dei numeri reali
ℂ	insieme dei numeri complessi	∅	insieme vuoto
⊂	sottoinsieme proprio di	⊆	sottoinsieme di
⊄	non sottoinsieme	⊃	soprainsieme proprio di
⊇	soprainsieme di	⊅	non soprainsieme

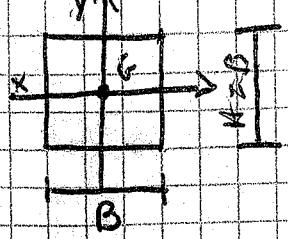
Tabella figure geometriche piane

Sezione	Area della sezione A	Distanza dal baricentro a	Momento di inerzia J	Modulo di resistenza W
	cm^2	cm	cm^4	cm^3
	H^2	$\frac{H}{2}$	$\frac{H^4}{12}$	$\frac{H^3}{6}$
	$H^2 - h^2$	$\frac{H}{2}$	$\frac{H^4 - h^4}{12}$	$\frac{H^4 - h^4}{6\,H}$
	$B \cdot H$	$\frac{H}{2}$	$\frac{B \cdot H^3}{12}$	$\frac{B \cdot H^2}{6}$
	$BH - bh$	$\frac{H}{2}$	$\frac{1}{12} \cdot (BH^3 - bh^3)$	$\frac{1}{6H} \cdot (BH^3 - bh^3)$
	$\frac{\pi \cdot D^2}{4}$	$\frac{D}{2}$	$\frac{\pi \cdot D^4}{64}$	$\frac{\pi \cdot D^3}{32}$
	$\frac{\pi \cdot (D^2 - d^2)}{4}$	$\frac{D}{2}$	$\frac{\pi \cdot (D^4 - d^4)}{64}$	$\frac{\pi \cdot (D^4 - d^4)}{32 \cdot D}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{I}{A}$ $W_a = \frac{I}{a}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{J}{A}$ $W_a = \frac{J}{a}$

Sezione	Area della sezione A	Distanza dal baricentro a	Momento di inerzia J	Modulo di resistenza W
	cm^2	cm	cm^4	cm^3
	BH - bh	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$ b/2+b/2=b
	BH - bh	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$
	BH - bh	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h-A)^3 + cA^3}{3}$	$W_A = \frac{I}{A}$ $W_a = \frac{I}{a}$
	BH - bh	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$
	$\frac{B \cdot H}{2}$	$A = \frac{2H}{3}$ $a = \frac{H}{3}$	$\frac{B \cdot H^3}{36}$	$W_A = \frac{B \cdot H^2}{24}$ $W_a = \frac{B \cdot H^2}{12}$
	H^2	$\frac{H}{2} \cdot \sqrt{2}$	$\frac{H^4}{12}$	$\frac{H^3}{6\sqrt{2}}$
	$H^2 - h^2$	$\frac{H}{2} \cdot \sqrt{2}$	$\frac{H^4 - h^4}{12}$	$\frac{H^4 - h^4}{6H\sqrt{2}}$

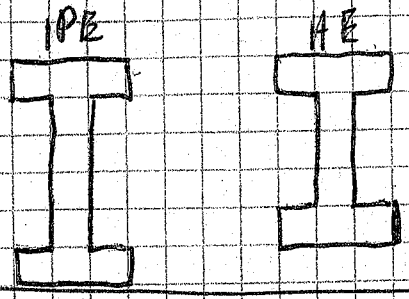
I = raggio di inerzia = $\sqrt{\frac{\text{Momento di inerzia}}{\text{Area della sezione}}} = \sqrt{\frac{J}{A}}$

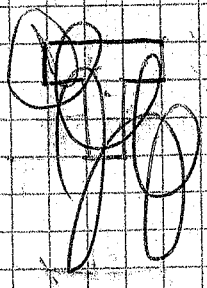
FIGURA	AREA (cm^2)	dy dist. asse y (cm)	Sy M.Statico (cm^3)	dx dist. asse x (cm)	Sx M.Statico (cm^3)	Jx M.Inerzia (cm^4)	Jy M.Inerzia (cm^4)	dy_G dist. asse y_G (cm)	dx_G dist. asse x_G (cm)	A x Dy_G^2 (cm^4)	A x Dx_G^2 (cm^4)	Jx_G M.Inerzia (cm^4)	Jy_G M.Inerzia (cm^4)
1													
2													
3													
4													
5													
6													
7													
8													
9													
10													
	A. TOTALE		Sy TOTALE		Sx TOTALE	Jx TOTALE	Jy TOTALE					Jx_G TOTALE	Jy_G TOTALE

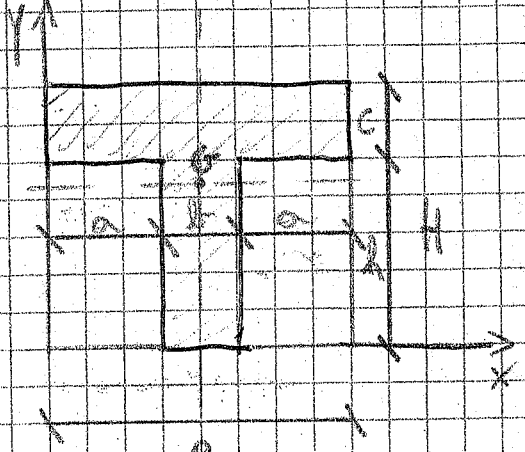


$$I_{xG} = \frac{1}{12} \cdot B \cdot H^3 = \frac{1}{12} \cdot B^4$$

$$I_{yG} = \frac{1}{12} \cdot H \cdot B^3 = \frac{1}{12} \cdot B^4$$

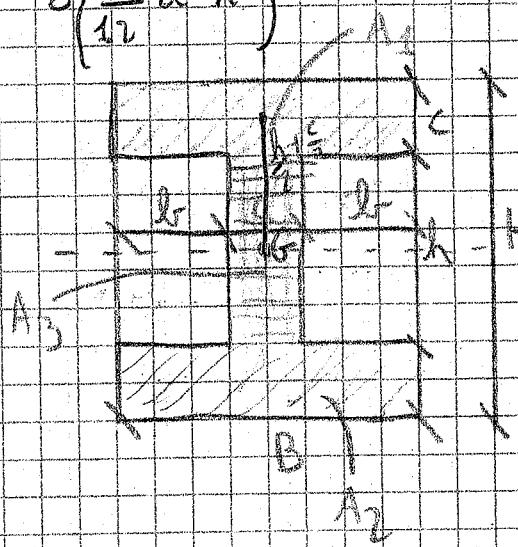






$G(x,y)$

$$I_{xG} = \left(\frac{1}{12} \cdot B \cdot h^3\right) - 2\left(\frac{1}{12} a \cdot h^3\right)$$



$$I_{xG} = \left(\frac{1}{12} \cdot B \cdot H^3\right) - 2\left(\frac{1}{12} \cdot B \cdot c^3 + A_1 \cdot \left(\frac{h}{2} + \frac{c}{2}\right)^2 + \frac{1}{12} \cdot c \cdot h^3\right)$$

$A_1 = c \cdot B$

$A_2 = c \cdot B$

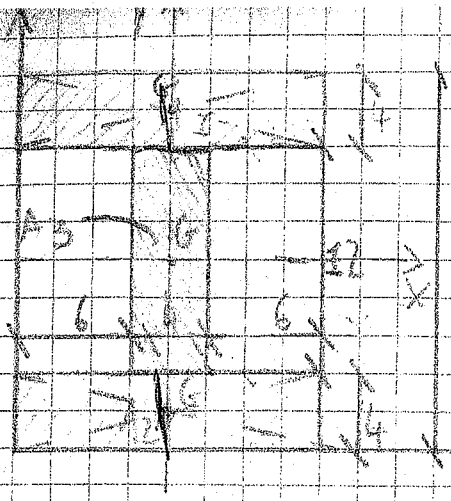
$A_3 = c \cdot h$

$* = 1706,67 + 1472,323$

$+80 + 1956,246 = 5215,24$

$\oplus = 1706,66667 + 80 = 1786,$

34



SCALA

$$G(8;10)$$

$$I_{xG} = \left[\left(\frac{1}{12}\cdot 16\cdot 4^3\right) + \left(\frac{\pi}{64}\cdot 8^2\right)\right]\cdot 4 + \frac{1}{12}\cdot 4\cdot 6$$

$$20 + 48 \cdot 0^2$$

$$= \left[\left(\frac{1\cdot 16\cdot 64}{12}\right) + (64\cdot 64)\right]^2 + \frac{1\cdot 4}{12}$$

$$= \left[\left(\frac{1024}{12}\right) + 4096\right]^2 + 6912 =$$

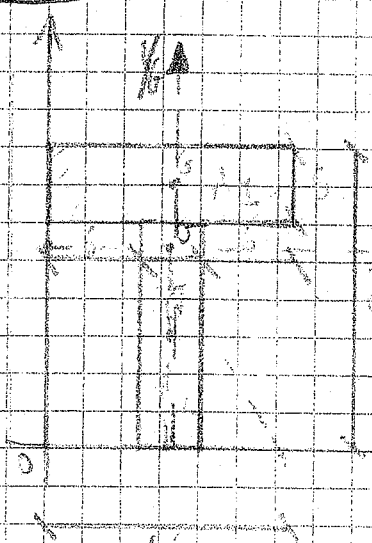
$$= \left[\frac{5120}{12}\right]^2 + 6912 = \frac{26214400}{144} +$$

$$+ \frac{26214400 + 6912}{144} =$$

$$I_{xG} = \left[\left(\frac{1}{12}\cdot 16\cdot 4^3\right) + 64\left(2+6\right)^2\right]\cdot 2 + \frac{1}{12}\cdot 4\cdot 12^3$$

$$\left[85{,}3 + 4096\right]\cdot 2 + 576$$

$$8362{,}6 + 576 = 8938{,}6$$



ESERCIZIO

SCALA 1 : 5

$$A_1 = 16\cdot 5 = 80\ cm^2 \qquad G(8;13{,}21)$$

$$A_2 = 4\cdot 15 = 60\ cm^2$$

$$S_x = 80\cdot 17{,}5 + 60\cdot 7{,}5 = 1400 + 450 = 1850$$

$$y_G = \frac{1850}{140} = 13{,}21\ cm$$

$$S_y = 80\cdot 8 + 60\cdot 8 = 640 + 480 = 1120\ cm^3$$

$$x_G = \frac{1120}{140} = 8\ cm$$

$$I_{xG} = \left(\frac{1}{12}\cdot 16\cdot 5^3\right) + 80(4{,}29)^2\Big] + \frac{1}{12}\cdot 4\cdot 15^3 + 60(7$$

$$= \left[(166{,}76) + 1472{,}33\right] + 1125 = 1639 + 3081{,}25 =$$

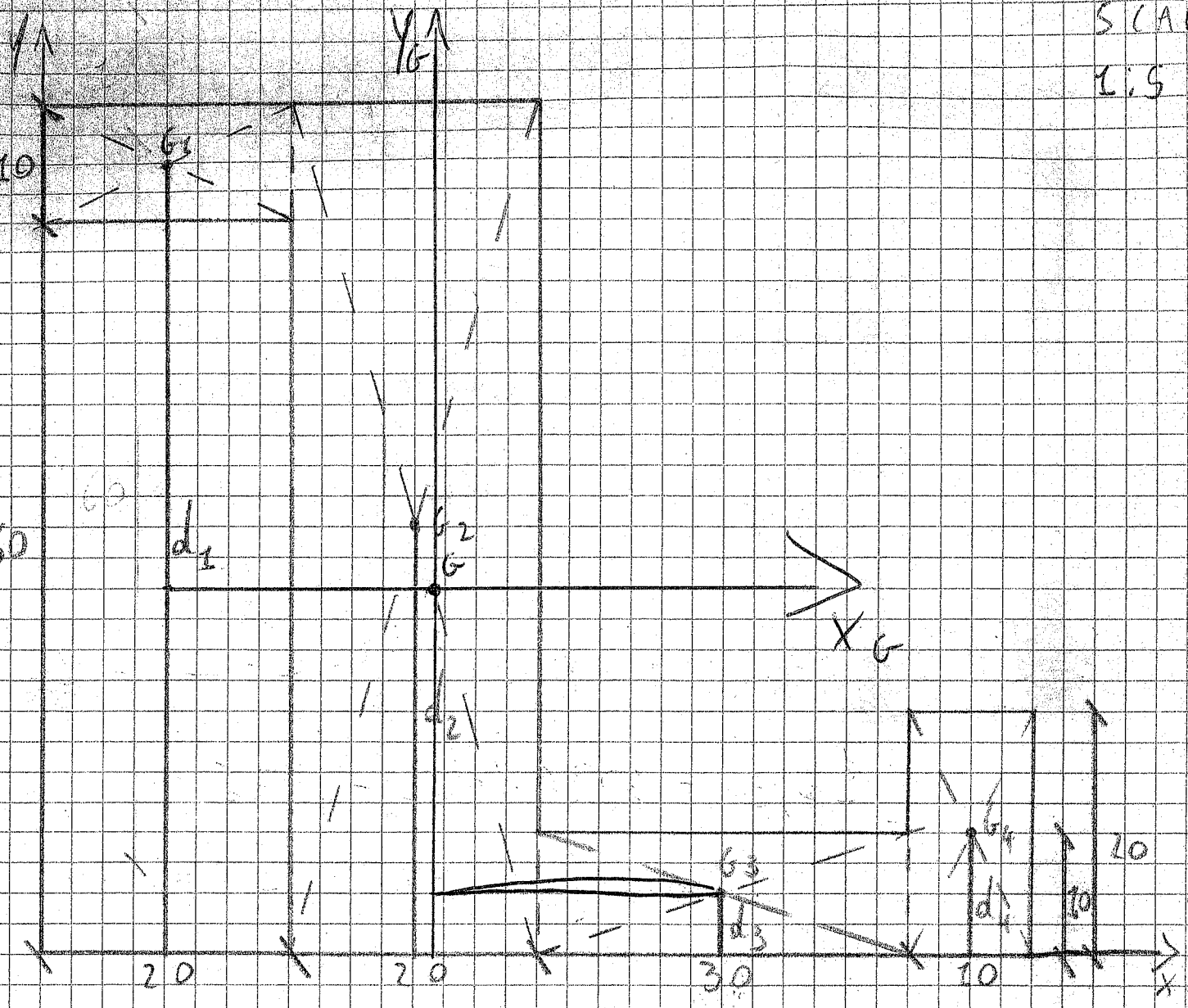
$$= 4720{,}25\ cm^4$$

$$I_{y} = \left[\left(\frac{1}{12}\cdot 5\cdot 16^3\right) + 80\right] + \left(\frac{1}{12}\cdot 15\cdot 4^3\right) + 60 =$$

35

SCA

1:5



$A_{tot} = A_1 + A_2 + A_3 + A_4 = 200 + 1400 + 300 + 200 = 2100\ cm^2$

$S_{x_{tot}} = S_x^{①} + S_x^{②} + S_x^{③} + S_x^{④} = A_1 \cdot d_1 + A_2 \cdot d_2 + A_3 \cdot d_3 + A_4 \cdot d_4 =$

$= 200 \cdot 65 + 1400 \cdot 35 + 300 \cdot 5 + 200 \cdot 10 = 75500\ cm^3$

$y_G = \frac{A_t}{S_{x_{tot}}} = \frac{75500}{2100} = 35,95\ cm$

$S_{y_{tot}} = 200 \cdot 10 + 1400 \cdot 30 + 300 \cdot 55 + 200 \cdot 75 = 82500\ cm^3$

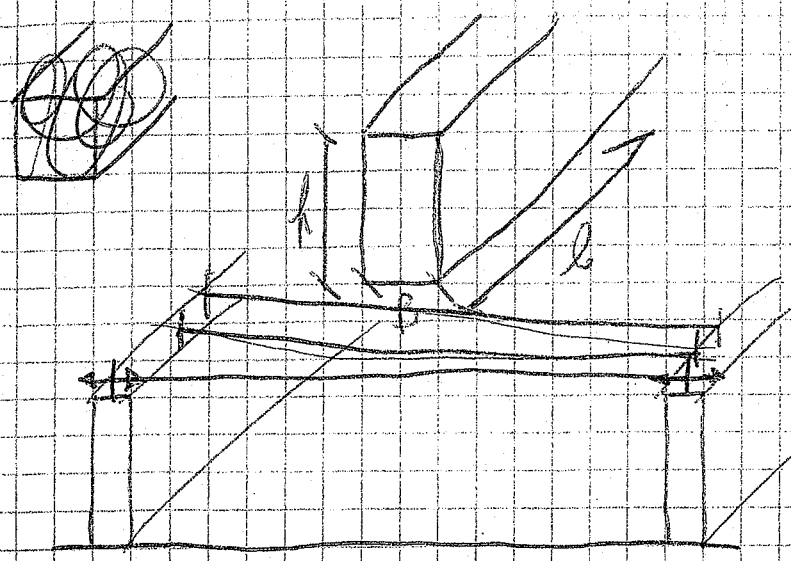
$x_G = \frac{75500}{2100} = 35,95\ cm$

$I_{x_{G\,tot}} = I_{x_G}^{①} + I_{x_G}^{②} + I_{x_G}^{③} + I_{x_G}^{④} = \left[\frac{1}{12} \cdot 20 \cdot 10^3 + 200 \cdot (33,9)^2\right] +$

$+ \left[\frac{1}{12} \cdot 20 \cdot 70^3 + 1400 \cdot (3,81)^2\right] + \left[\frac{1}{12} \cdot 30 \cdot 10^3 + 300 (26,19)^2\right] +$ *

36

La trave è un elemento lineare.



EQUAZIONI CARDINALI

$\sum F_x = 0$

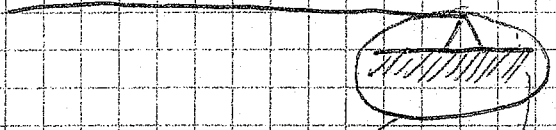
$\sum F_y = 0$

$\sum M = 0$

SCHEMA STATICO

CARRELLO (VINCOLO)

VINCOLI:
CARRELLO
CERNIERA



$* + \left[\frac{1}{12} \cdot 10 \cdot 20^3 + 200 \cdot (21,19)^2\right] =$

$= 1127017,8 \text{ cm}^4$

$I_{YG\,TOT} = \left[\frac{1}{12} \cdot \text{[illegible]} 10 \cdot 20^3 + 200 \cdot (25,95)^2\right] + \left[\frac{1}{12} \cdot 70 \cdot 20^3 + 2400 \cdot (5,95\right.$

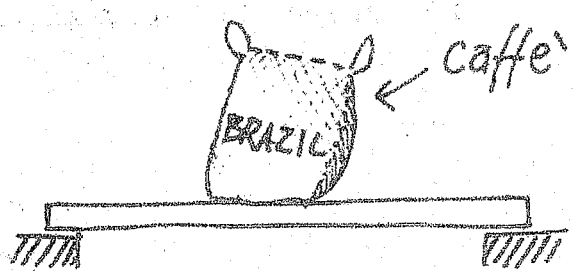
$+ \left[\frac{1}{12} \cdot 10 \cdot 30^3 + 300 \cdot (19,05)^2\right] + \left[\frac{1}{12} \cdot 20 \cdot 10^3 + 200 \cdot (39,05)^2\right] =$

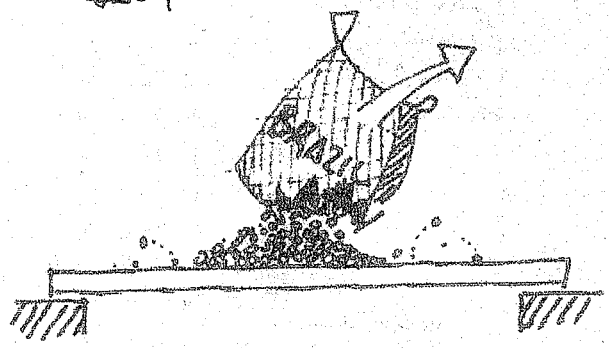
$= \left[6666,67 + 673,4025\right] + \left[46666,67 + 49563,5\right] + \left[22500 + 108870,75\right.$

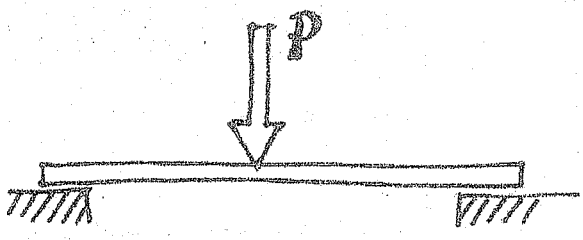
$+ \left[1666,67 + 304980,5\right] = 6740,0725 + 96230,17 + 131370,75 +$

$+ 306647,17 = 540988,16 \text{ cm}^4$

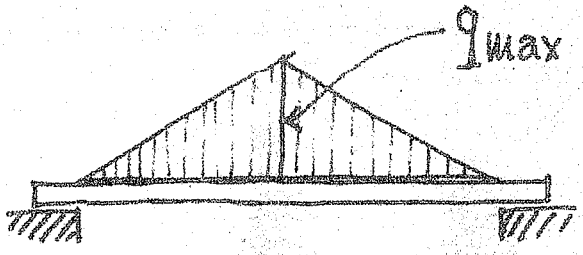
TIPI DI CARICHI TRAVI



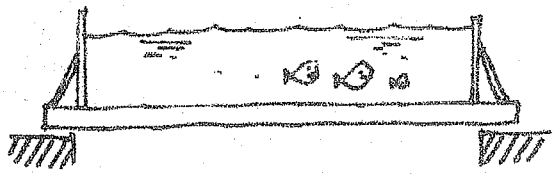


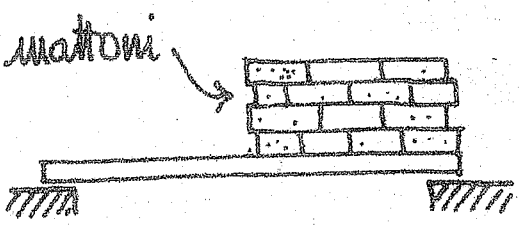


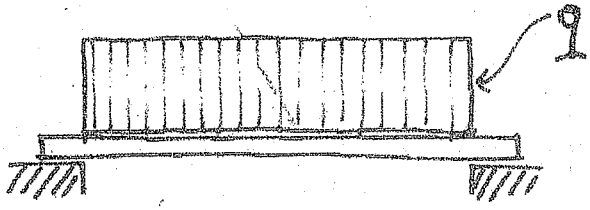
Carico concentrato



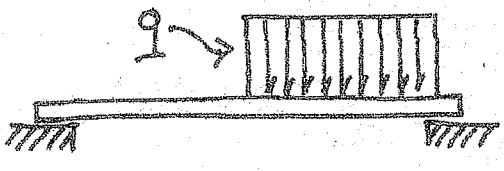
Carico distribuito
(CON LEGGE TRIANGOLARE)







Carico distribuito uniformemente
(CON LEGGE COSTANTE)



carico distribuito
(CON LEGGE COSTANTE)
su porzione di trave

LEZIONE 3°B

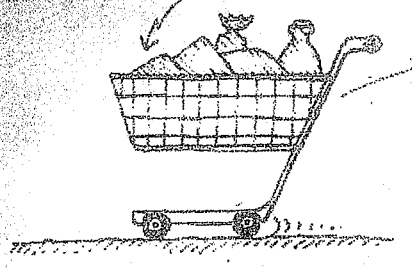
ITG NERVI / 2016

Gli esempi pratici

VINCOLI

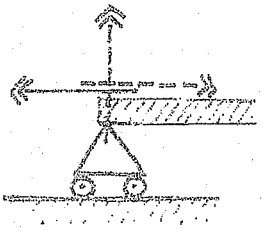
La schematizzazione statica

Spesa

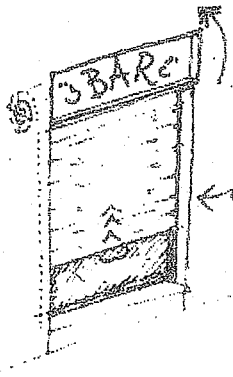


SUPER MERCATO

per ogni ruota questo è il vincolo!

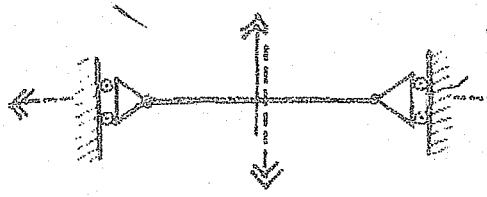


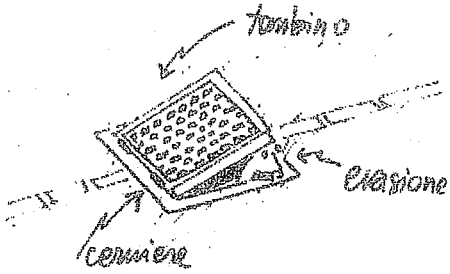
ABOLISCE N° 1 GRADO DI LIBERTÀ
BLOCCA

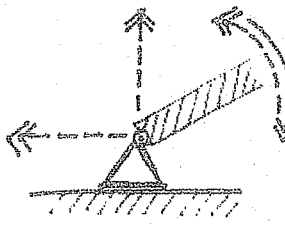


La serranda che si apre

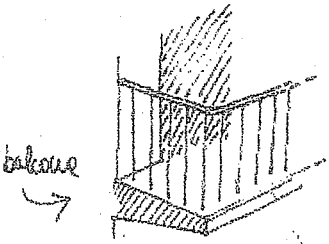
per ogni stecca della bandone questo è il vincolo!

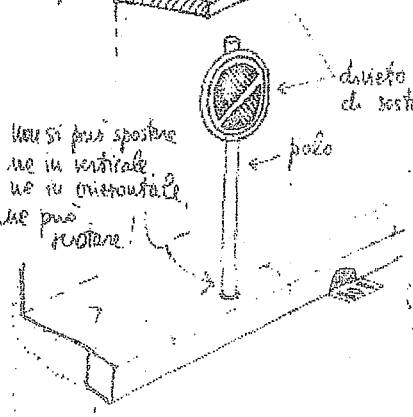


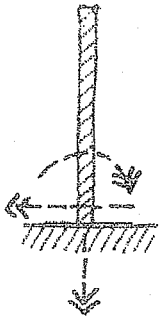




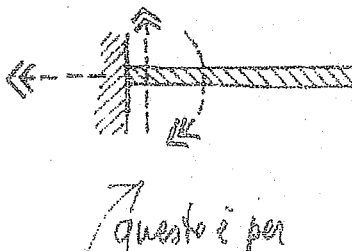
ABOLISCE N° 2 GRADI DI LIBERTÀ
BLOCCA





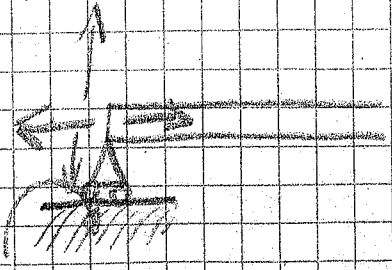


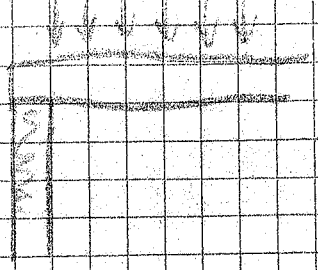
questo è per il palo!

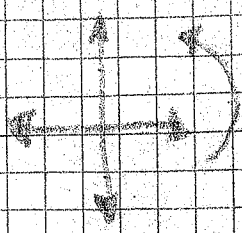


questo è per il balcone!

BLOCCA N° 3 GRADI DI LIBERTÀ

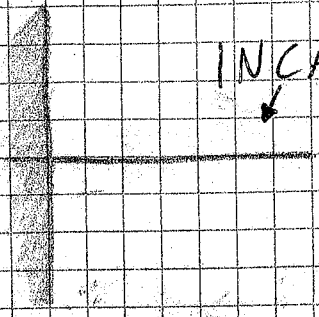






$\Delta l = l_0 \cdot \lambda \cdot \Delta T$

15 → 30°

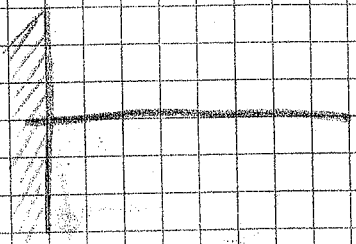


STRUTTURA:
LABILE
ISOSTATICA
IPERSTATICA

$3n \lessgtr \sum$ VINCOLI $(a + c + i)$

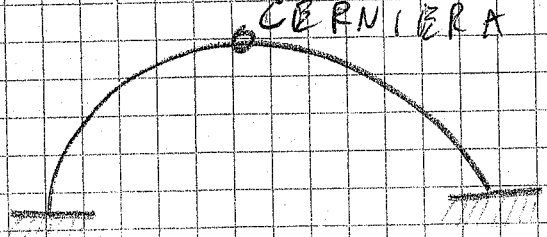
- (LABILE - NO) ~~(MINORE)~~ MAGGIORE
- (ISOSTATICA - IDEALE) (UGUALE)
- (IPERSTATICA - PIÙ DIFFICILE) ~~(MAGGIORE)~~ MINORE

n = numero travi
a = appoggio
c = cerniera
i = incastro



$3 \cdot 1 = a \cdot 0 + c \cdot 0 + 1 \cdot 3$

$3 = 3$ ISOSTATICA

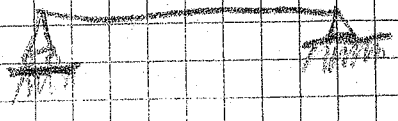


$3 \cdot 2 = a \cdot 0 + c \cdot 1 + 2 \cdot i$

$6 = 2 + 6 = 8$

$6 < 8$ IPERSTATICA

40



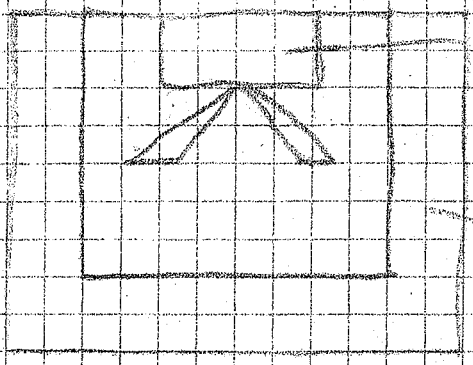
$3 \cdot 1 = 1 \cdot 2 + 1 \cdot 2 + 1 \cdot 0$

$3 \cdot 1 = 1 + 2$

$3 = 3$ ISOSTATICA

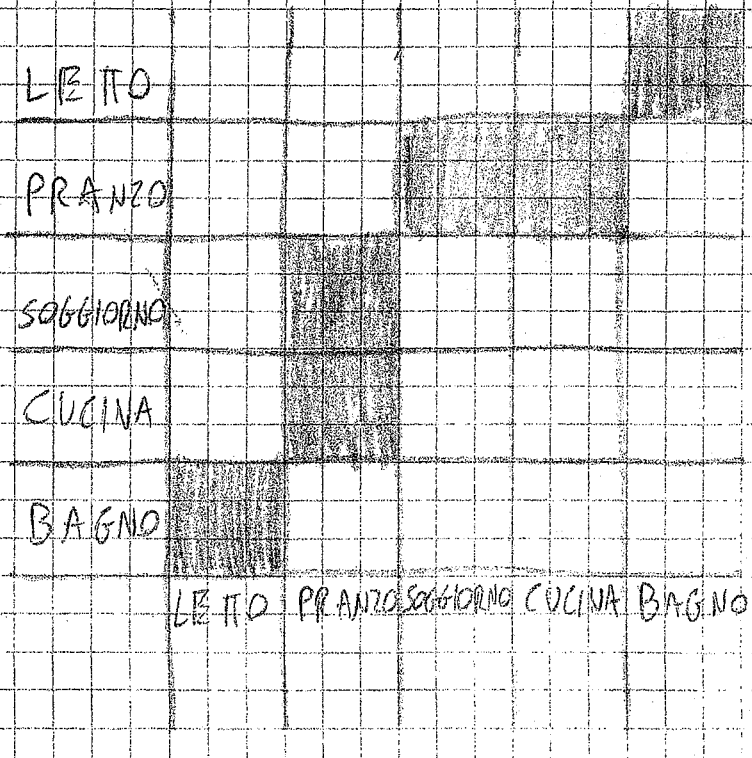
SPAZI E FUNZIONI

PROGETTARE
- ANALISI FUNZIONI
- ANALISI ATTIVITÀ
- DIMENSIONE SPAZI ELEMENTARI
- DIMENSIONE SPAZI FUNZIONALI



SPAZIO OCCUPATO DALL'OGGETTO
+
SPAZIO PER USARE L'OGGETTO

COLLEGAMENTI IMMEDIATI / COMPATIBILITÀ

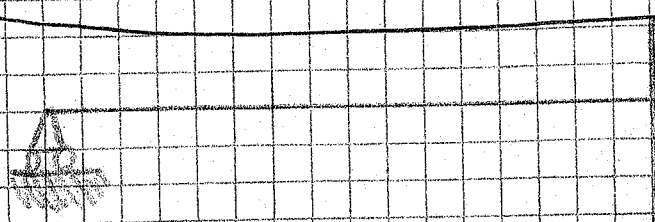


~~$\frac{1}{8}$~~ SUPERFICIE FINESTRATO

RAPPORTO AEREOILLUMINATO

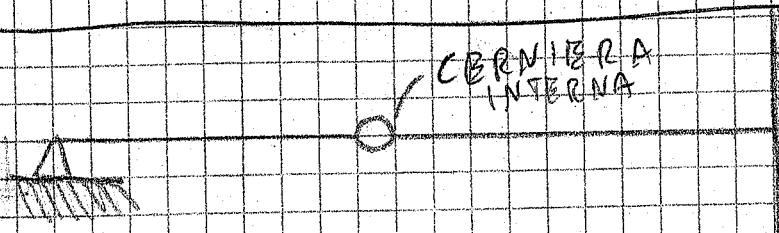
SUPERFICIE FINESTRATA $\geq \frac{1}{8}$ SUPERFICIE PAVIMENTATA

$\frac{4Q}{8} = 5\,m^2$



$3m \gtrless \Sigma\, a\cdot 1 + c\cdot 2 + 1\cdot 3 = 1+3 = 4$

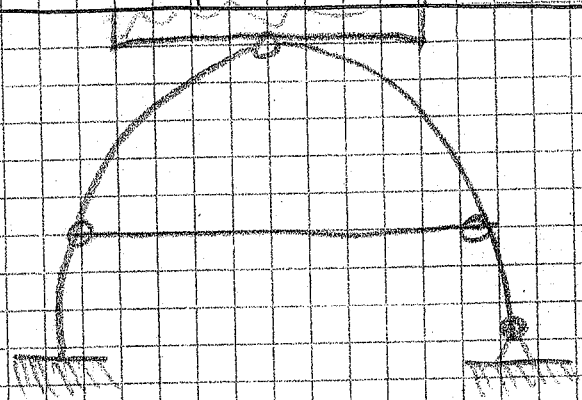
$3 < 4$ IPERSTATICA



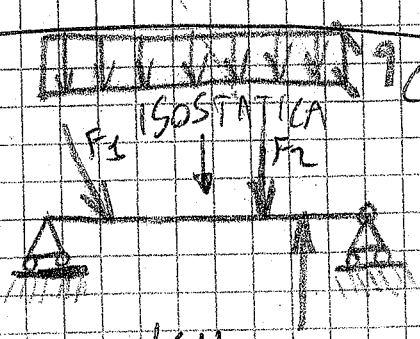
$3m \gtrless \Sigma\, a\cdot 1 + c\cdot 2 + i\cdot 3 =$

$= \cancel{4} + \cancel{2} + \cancel{0} + 1 + 3$

$6 < 7$



$3m \gtrless \Sigma\, a\cdot 1 + c\cdot 2 + i\cdot 3 = 3\cdot 3 \lessgtr \Sigma\, 0\cdot 1 + 4\cdot 2 + 1\cdot 3 = 8+3 = 11 = 9 < 11$ IPERSTATICA



CARICHI

DATI

$F_1 = 20\,KN$

$F_2 = 30\,KN$

$3\cdot m = \Sigma\, 1+2 = 3$

$3\cdot 1 = 3$

KN

$12\,KN = 12000\,N$

1200 Kg

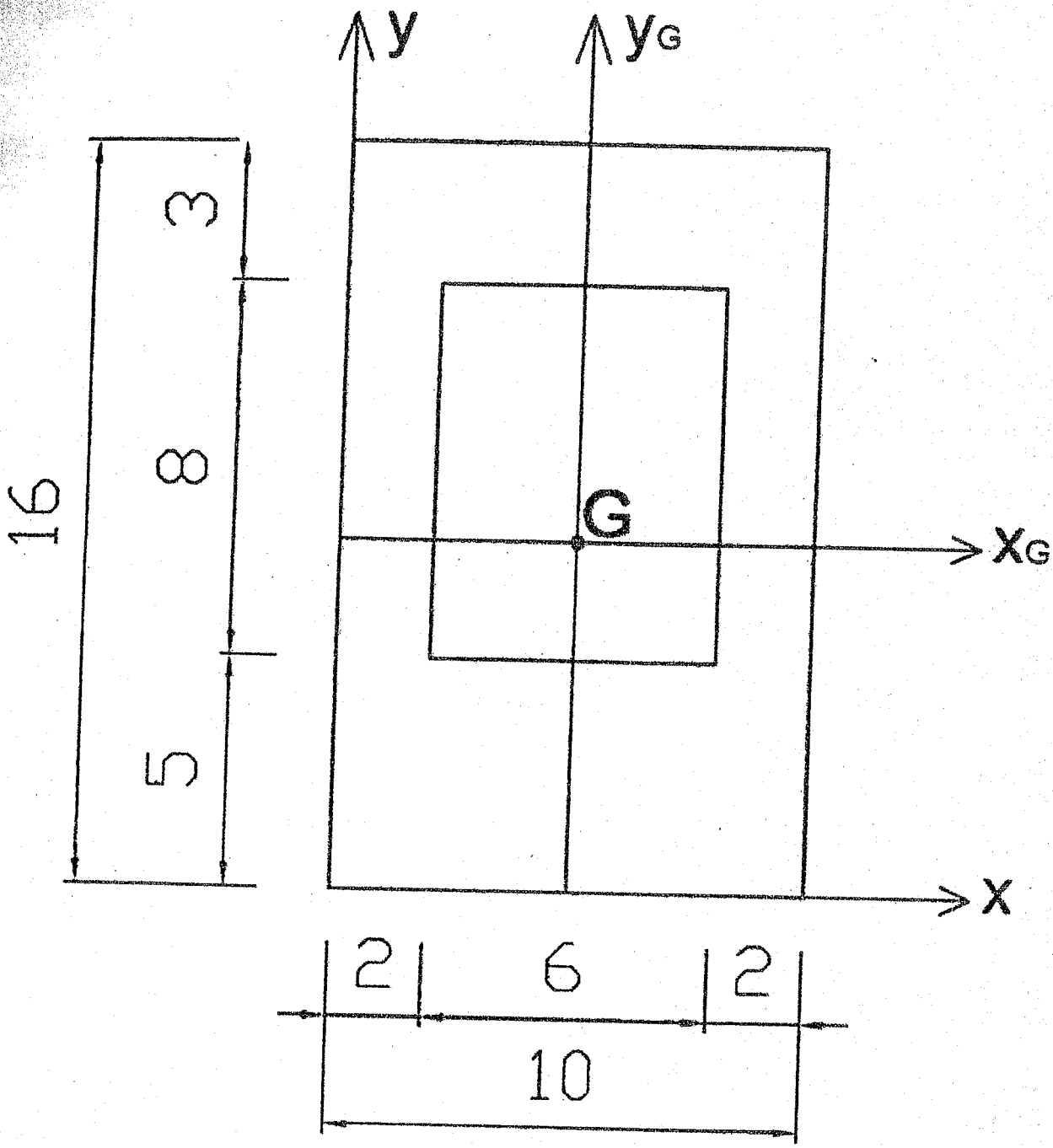
80000 Kg

8000000 N

800 KN

42

Determinare il nocciolo centrale di inerzia della seguente sezione



Dopo aver determinato le coordinate del baricentro :

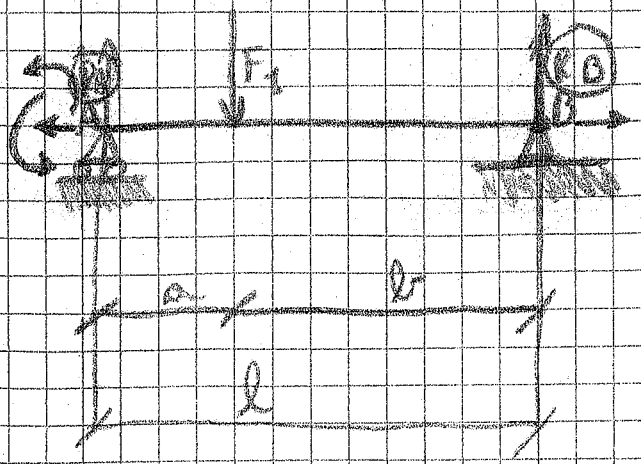
$X_G = 5{,}0$ cm
$Y_G = 7{,}57$ cm

Dopo aver determinato i momenti principali di inerzia :

$I_{XG} = 3091{,}18\ cm^4$
$I_{YG} = 1189{,}33\ cm^4$

43

EQUILIBRIO TRAVI



DATI

$F_1 = 80\,KN$

$l = 5\,m$

$a = 2\,m$

$b = 3\,m$

$R_A = ?$

$R_B = ?$

R = REAZIONE

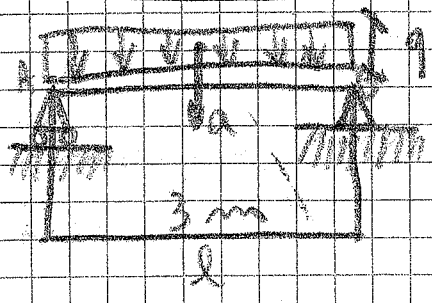
EQUAZIONE EQUILIBRIO STATICA

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \qquad \begin{cases} \sum F_y = F_1 + R_A + R_B = 0 \\ \sum M_A = R_A \cdot 0 + F_1 \cdot a + R_B \cdot l \end{cases} \qquad \begin{cases} \sum F_y = +80 - R_B - R_A = 0 \\ \sum M_A = R_A \cdot 0 + 80 \cdot 2 - R_B \cdot 5 \end{cases}$$

CONVENZIONE

$$\begin{cases} -R_B \cdot 5 = -80 \cdot 2 \\ R_B = \frac{160}{5} = 32\,KN \end{cases}$$

$$\begin{cases} +80 + 32 + 80 - 32 - R_A = 0 \\ R_A = 80 - 32 = 48\,KN \end{cases}$$



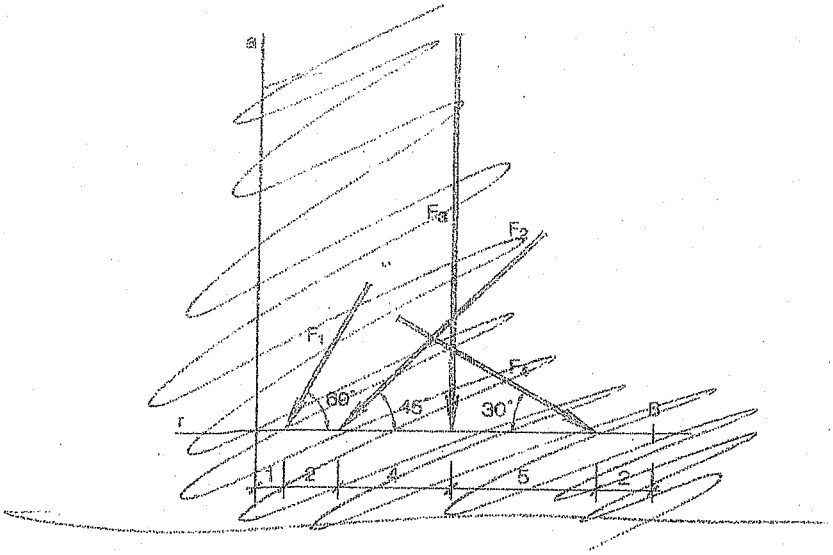
DATI

$l = 3\,m$

$q = 20\,KN/m$

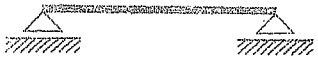
$Q = 60\,KN$

perpendicolare alla retta r, e una retta b passante per il punto B.

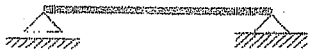


Effettuare il computo dei vincoli per le strutture qui di seguito riportate.

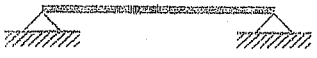
7



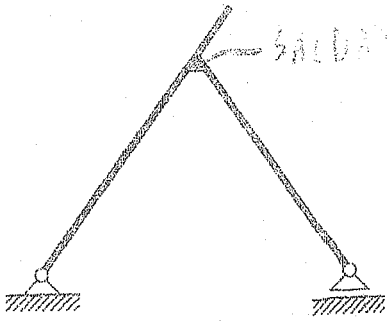
8



9

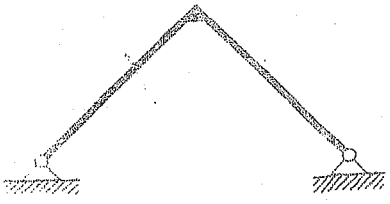


10



11

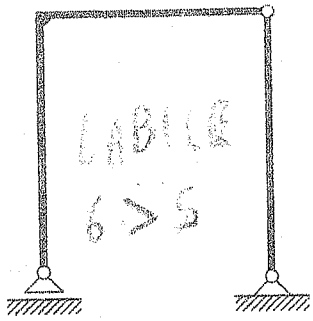
12



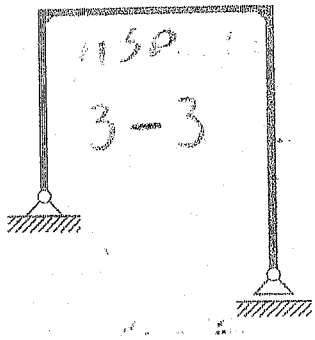
APPOGGIO

CERNIERA

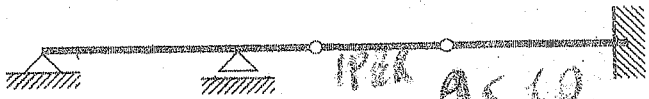
14



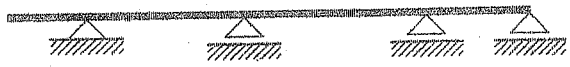
15



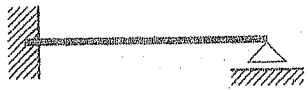
16



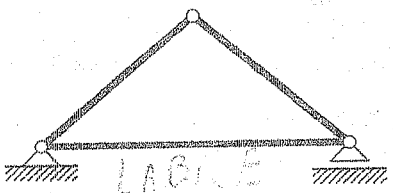
17



18

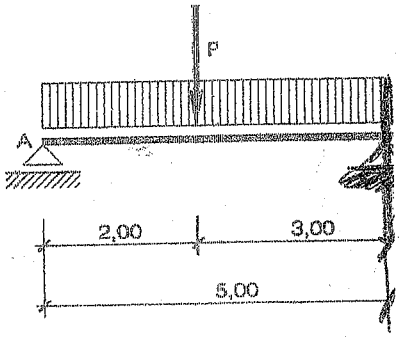


19

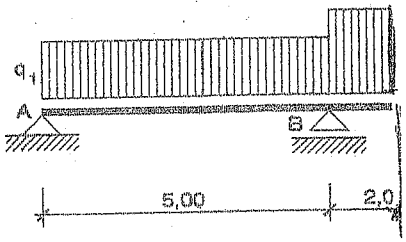


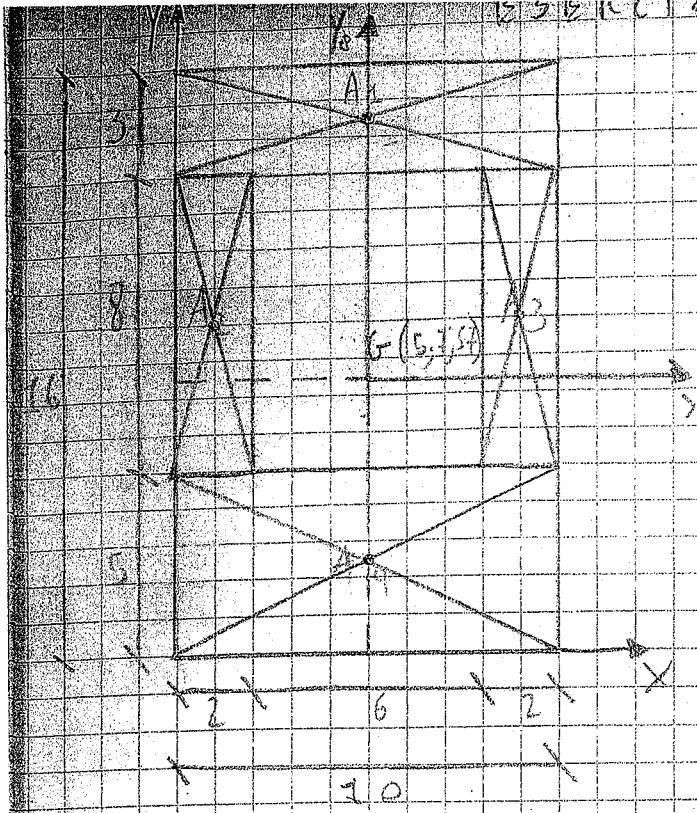
Calcolare le reazioni vincolari delle travi e dei portali qui di seguito riportati.

20 Procedimento analitico;
$P = 8$ kN, $q = 6$ kN/m.



21 Procedimento analitico;
$q_1 = 4$ kN/m, $q_2 = 6$ kN/m.





1 cm. = 1 □

$A_1 = 10 \cdot 3 = 30\ cm^2$

$A_2 = 2 \cdot 8 = 16\ cm^2$

$A_3 = 2 \cdot 8 = 16\ cm^2$

$A_4 = 10 \cdot 5 = 50\ cm^2$

$S_x = \sum A_i \cdot y_i$

$S_y = A_i \cdot x_i$

$S_x = 30 \cdot 14,5 + 16 \cdot 9 + 16 \cdot 9 + 50 \cdot 4,5$

$= 435 + 144 + 144 + 225 = 948\ cm^3$

$y_6 = \frac{948}{112} = 7,57\ cm$

$S_y = 30 \cdot 5 + 16 \cdot 1 + 16 \cdot 9 + 50 \cdot 5 =$

$= 150 + 16 + 144 + 250 = 560\ cm^3$

$x_6 = \frac{560}{112} = 5\ cm$

$I_{x6_{TOT}} = \left[\frac{1}{12} \cdot 10 \cdot 3^3 + 30 \cdot (6,93)^2\right] + \left[\frac{1}{12} \cdot 2 \cdot 8^3 + 16 \cdot (1,43)^2\right] + \left[\frac{1}{12} \cdot 2 \cdot 8^3 + 16 \cdot (1,43)^2\right] + \left[\frac{1}{12} \cdot 10 \cdot 5^3 + 50 \cdot (5,07)^2\right] = \left[22,5 + 1440,747\right] +$

$+ \left[85,33 + 32,7184\right] + \left[85,33 + 32,7184\right] + \left[104,17 + 1285,245\right] =$

$= 1463,247 + 118,0484 + 118,0484 + 1389,415 = 3088,76\ cm^4$

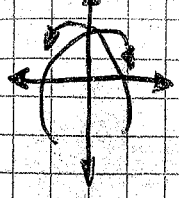
$I_{y6_{TOT}} = \left[\frac{1}{12} \cdot 3 \cdot 10^3 \text{ [illegible]}\right] + \left[\frac{1}{12} \cdot 8 \cdot 2^3 + 16 \cdot (4)^2\right] + \left[\frac{1}{12} \cdot 8 \cdot 2^3 + 16 \cdot (4)^2\right] +$

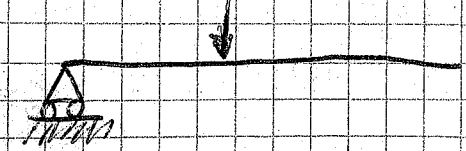
$+ \left[\frac{1}{12} \cdot 5 \cdot 10^3 \text{ [illegible]}\right] = \left[250 \text{ [illegible]}\right] + \left[5,3 + 256\right] + \left[5,3 + 256\right] +$

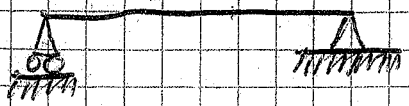
$+ \left[416,67 + \text{[illegible]}\right] = \sout{280 + 261,3 + 261,3 + 466,67 = 1269,27\ cm^4}$

$= 250 + 261,3 + 261,3 + 416,67 = 1189,27\ cm^4$

4 7







$$3 \cdot m = a \cdot 1 + 2 \cdot 2 + 3 \cdot 1$$

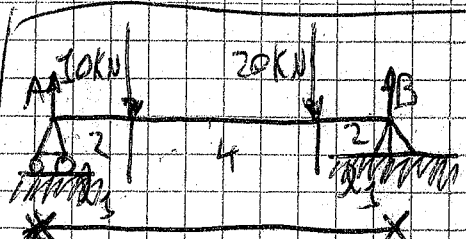
$$3 \cdot 1 = 1 \cdot 1 + 2 \cdot 1 + 3 \cdot 0$$

$$3 = 3$$

$3 < 4$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases}$$

DATI



CONVENZIONE

$F_1 = 10\,KN$

$F_2 = 20\,KN$

$l_1 = 2\,m$

$l_{TOT} = 8\,m$

$R_A = ?$

$R_B = ?$

DIAG. T.

DIAG. M.

$$\begin{cases} \Sigma F_y = F_1 + F_2 + R_A + R_B = 0 \\ \Sigma M_A = R_A \cdot 0 + F_1 \cdot l_1 + F_2 \cdot 6 + R_B \cdot 8 = 0 \end{cases}$$

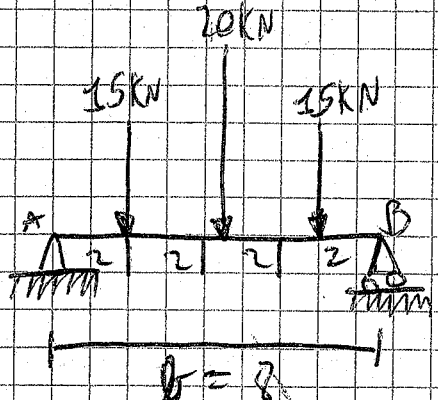
$10 + 20 - 17{,}5 = -12{,}5$

$0 + 20 + 120 - R_B \cdot 8 = 0$

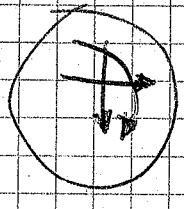
$140 - R_B \cdot 8 = 0$

$R_B = \frac{+140}{8} = -17{,}5\,KN$

Es.



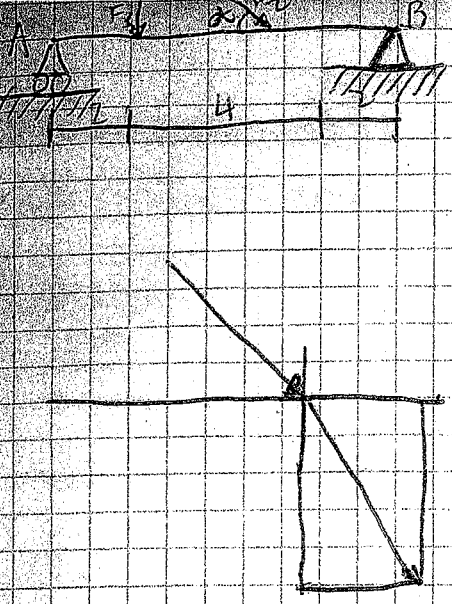
CONVENZIONE

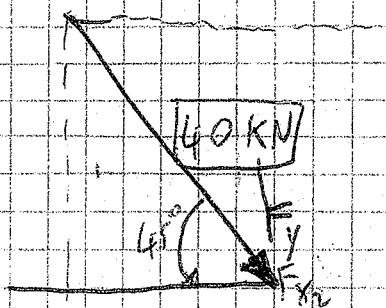


$$\begin{cases} \Sigma F_y = 0 \\ \Sigma F_x = 0 \\ \Sigma M = 0 \end{cases}$$

$$\begin{cases} \Sigma F_y = F_1 + F_2 + F_3 + R_A + R_B = 0 \Rightarrow 15 + 20 + 15 - R_A - R_B = 0 \Rightarrow \\ \Sigma M = R_A \cdot 0 + F_1 \cdot 2 + F_2 \cdot 4 + F_3 \cdot 6 - R_B \cdot 8 = 0 \Rightarrow R_B \Rightarrow \frac{0 + 30 + 80 + 90}{8} = 25\,KN \end{cases}$$

$R_A = 15 + 20 + 15 - 25 = +25\,KN$



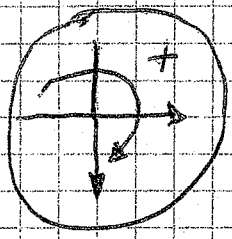


$$F_y = F_2 \cdot \cos 45^\circ$$
$$F_x = F_2 \cdot \operatorname{sen} 45^\circ$$

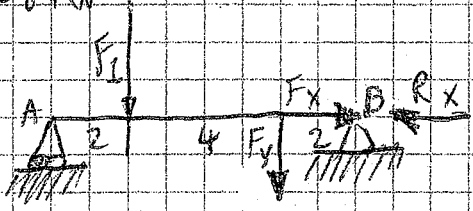
DATI

$F_1 = 30\,KN$

$F_2 = 40\,KN$



$$F_y = 40 \cdot \cos 45^\circ = 28{,}28\ KN$$
$$F_x = 40 \cdot \operatorname{sen} 45^\circ = 28{,}28\ KN$$

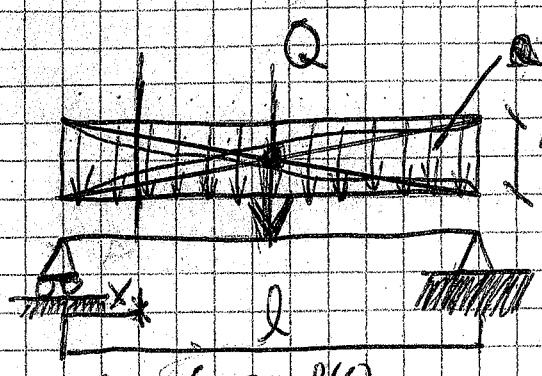


$$\begin{cases} \Sigma F_y = 0 \\ \Sigma F_x = 0 \\ \Sigma M = 0 \end{cases}$$

$$\begin{cases} \Sigma F_x = F_x + R_x = 0 \\ \Sigma F_y = R_A + F_1 + F_y + R_B = 0 \\ \Sigma M_A = -R_A \cdot 0 + F_1 \cdot 2 + F_y \cdot 6 + F_x \cdot 0 - R_B \cdot 8 - R_x \cdot 0 = 0 \Rightarrow 60 + 169{,}68 - R_B \cdot 8 = 0 \Rightarrow \end{cases}$$

$$\Rightarrow R_B = \frac{60 + 169{,}68}{8} = 28{,}71\ KN$$

$$R_A = F_1 + F_y - R_B \Rightarrow 30 + 28{,}28 - 28{,}71 = 29{,}57\ KN$$



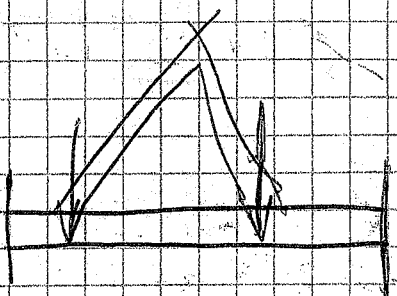
q = entità carico uniformemente distribuito

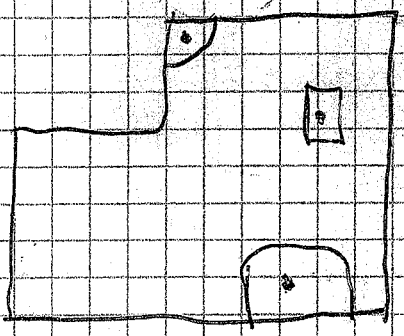
$q = 300\,N/ml$

$l = 6\,m$

Q = carico totale ⇒

$\Rightarrow q \cdot l = 300 \cdot 6 = 1800\,N$



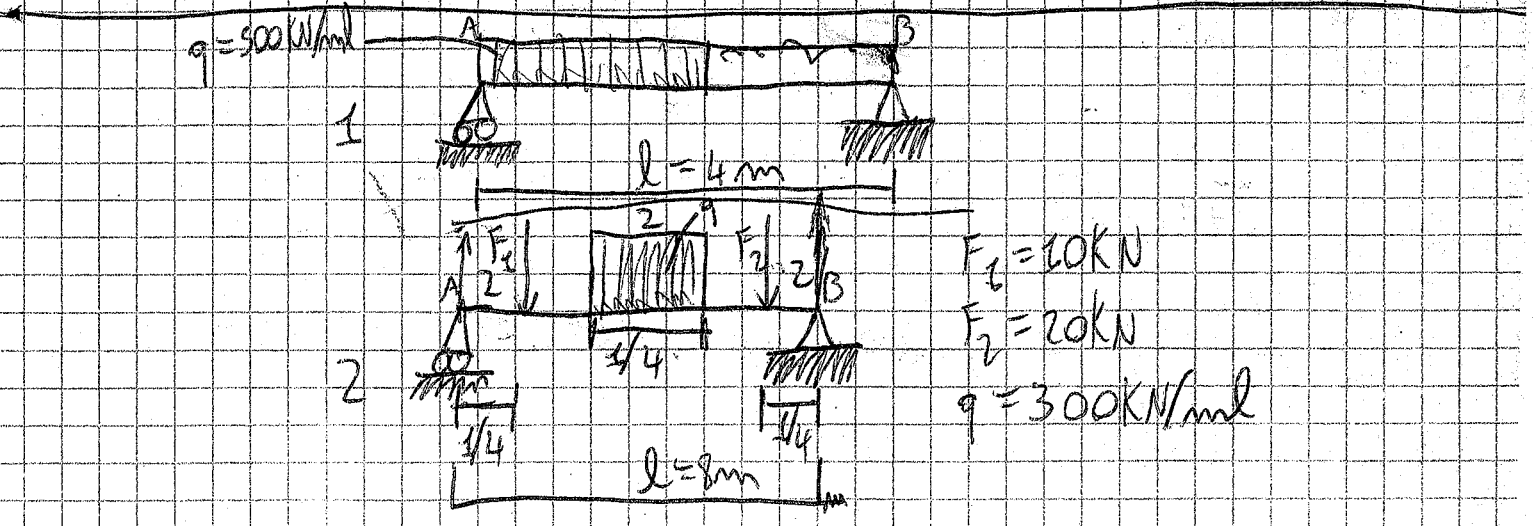


$\Sigma F_x = 0 \rightarrow = 0$

$\Sigma F_y = 0 \rightarrow R_A + Q\,(q \cdot l) + R_B = 0$

$\Sigma M_A = 0 \rightarrow R_A \cdot 0 + \left(q \cdot l \cdot \frac{l}{2}\right) - R_B \cdot l \Rightarrow \frac{1}{2} q \cdot l^2 - R_B \cdot l = 0 \Rightarrow$

$\Rightarrow R_B = \frac{1}{2} q \cdot l$



$F_1 = 20\,kN$

$F_2 = 20\,kN$

$q = 300\,kN/ml$

50

1)
$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \rightarrow R_A + (q \cdot l) + R_B = 0 \\ \Sigma M = 0 \rightarrow R_A \cdot 0 + (q \cdot l \cdot \frac{l}{2}) - R_B \cdot l \rightarrow \frac{1}{2} \cdot q \cdot l^2 - R_B \cdot l = 0 \Rightarrow \\ \Rightarrow R_B = \frac{1}{2} \cdot q \cdot l = 0{,}5 \cdot 500\,KN/ml \cdot 4 = 1000\,KN/ml \\ R_A = \end{cases}$$

$q = 500\,KN/$
$l = 4\,m$
$Q = q \cdot l =$
$= 500 \cdot 4 =$
$= 2000\,KN$

DATI
$l = 8\,m$
$F_1 = 10\,KN$
$F_2 = 20\,KN$
$q = 300\,KN/$
$R_A = ?$
$R_B = ?$
$Q = q \cdot \frac{l}{4}$

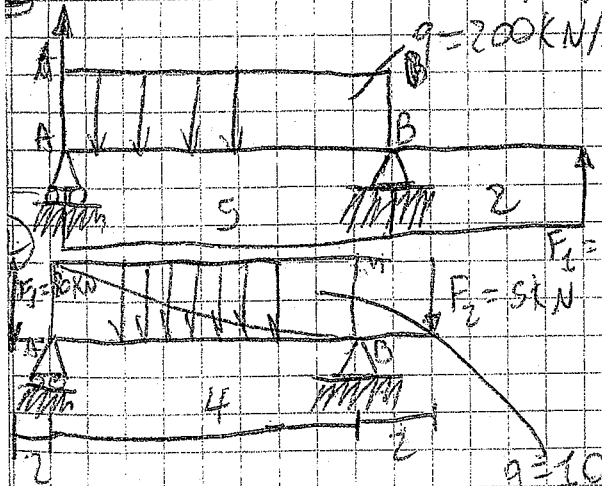
2)
$$\begin{cases} \Sigma F_x = 0 \rightarrow 0 \\ \Sigma F_y = 0 \rightarrow R_A + F_1 + q \cdot \frac{l}{4} + F_2 + R_B = 0 \\ \Sigma_A = R_A \cdot 0 + F_1 \cdot 2 + q \cdot \frac{l}{4} \cdot 4 + F_2 \cdot 6 - R_B \cdot 8 \Rightarrow \\ \Rightarrow -R_B \cdot 8 = 20 + 2400 + 120 = \frac{2540}{8} = 317{,}5\,KN = 317{,}5\,KN \\ \Rightarrow 630 \end{cases}$$

CONVENZIONE

R_A

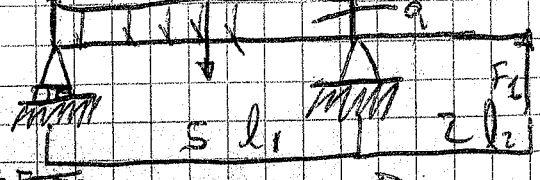
TRAVI A SBALZO

3)



$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \Rightarrow \\ \Sigma M = 0 \Rightarrow \end{cases}$$

1)



$Q = q \cdot l = 200 \cdot 5 = 1000\,KN$ $q = 200\,KN/m$

$F_1 = 10\,KN$

$l = ?$

$R_A = ?$

$R_B = ?$

$\sum F_x = 0$

$\sum F_y = 0 \Rightarrow R_A + q \cdot l_1 + R_B + F_1 = 0$

$\sum M_A = 0 \Rightarrow R_A \cdot 0 + Q \cdot \frac{l_1}{2} - R_B \cdot l_1 + F_1 \cdot l = 0$

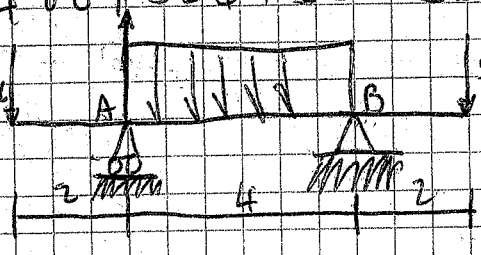
$-R_B \cdot l_1 = -2500 + 70 \Rightarrow$

$-R_B = \frac{-2430}{5} = -486\,KN$

$R_A = -1000 - 486 - 10 = 504\,KN$

$486 + 504 + 10 = 1000\,KN$

2)



$q = 100\,KN/m$

$Q = 400\,KN$

$\sum F_x = 0$

$\sum F_y = 0 \Rightarrow F_1 + R_A + q \cdot l_1 + R_B + F_2 = 0$

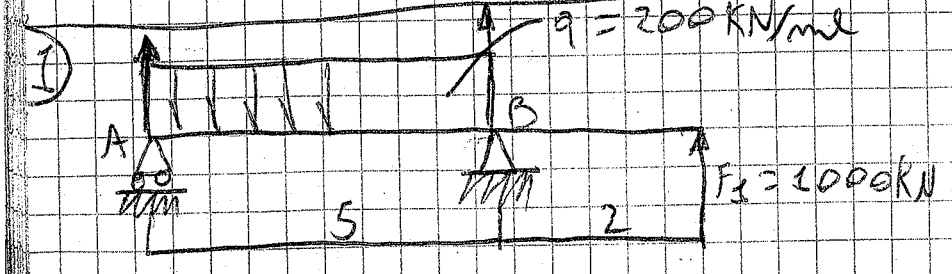
$\sum M_A = 0 \Rightarrow -F_1 \cdot 2 - R_A \cdot 0 + (q \cdot l \cdot \frac{l_1}{2}) - R_B \cdot 4 + F_2 \cdot 6 = 0$

$-60 + 800 - R_B \cdot 4 + 30 = 0$

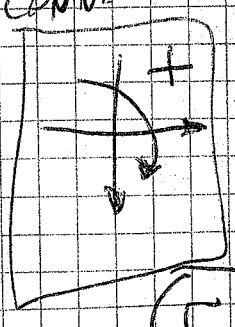
$-R_B = \frac{60 - 800 - 30}{4} \Rightarrow R_B = \frac{-770}{4} = -192,5\,KN$

$R_A = -437,5$

1)



CONV.



52

$\Sigma F_x = 0$

$\Sigma F_y = 0 \rightarrow R_A + Q + R_B + F_L = 0$

$\Sigma M_A = 0 \Rightarrow \cancel{R_A \cdot 0} + Q \cdot 2{,}5 - R_B \cdot 5 - F_L \cdot 7 = 0$

$\Rightarrow 1000KN \cdot 2{,}5 - R_B \cdot 5 - 7000KN = 0 \Rightarrow$

$\Rightarrow 2500KN - R_B \cdot 5 - 7000KN = 0 \Rightarrow$

$\Rightarrow -R_B \cdot 5 = -2500 + 7000 = \frac{4500\,KN}{5} = 900KN$

$R_A + 1000 - R_B - F_L = 0 \Rightarrow R_A = -900KN$

1) GLI ORGANISMI STRUTTURALI IN UNA COSTRUZIONE

Ogni costruzione si compone di elementi tra loro assemblati

elementi < di finitura / strutturali

2) LA PROGETTAZIONE DEGLI ELEMENTI STRUTTURALI

a) Individuazione dello schema statico
b) Analisi dei carichi
c) Risoluzione dello schema statico con i carichi (reazioni vincolari)
d) Determinazione delle caratteristiche di sollecitazione
e) Analisi tensionale con progetto e verifica degli elementi strutturali

3) UNITÀ DI MISURA

FORZE → N (Newton) $\quad 1\,Kg = 9{,}81\,N$

$1\,Kg = 1\,daN$

TENSIONI → $1\,Pa = \frac{1\,N}{m^2}$ (Pascal)

$$1\,\frac{Kg}{cm^2} = 10\,\frac{N}{cm^2} = 0{,}10\,\frac{N}{mm^2} = 0{,}10\,MPa$$

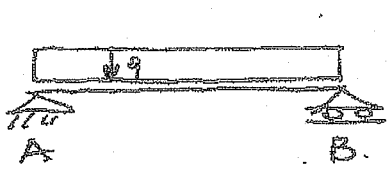
$$1\,MPa = \frac{10^6\,N}{m^2} = 1\,\frac{N}{mm^2} = 10\,\frac{Kg}{cm^2}$$

4) LE STRUTTURE ISOSTATICHE

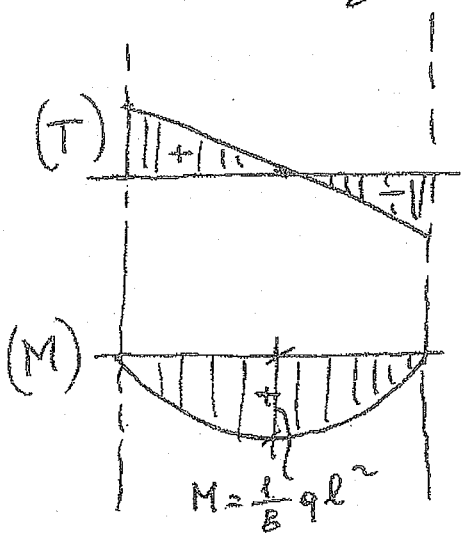
Le equazioni fondamentali della statica dei sistemi rigidi

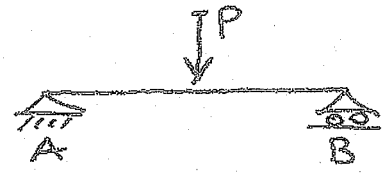
$$\begin{cases} \Sigma Fx = 0 \\ \Sigma Fy = 0 \\ \Sigma M = 0 \end{cases}$$

4) RIEPILOGO STRUTTURE ISOSTATICHE

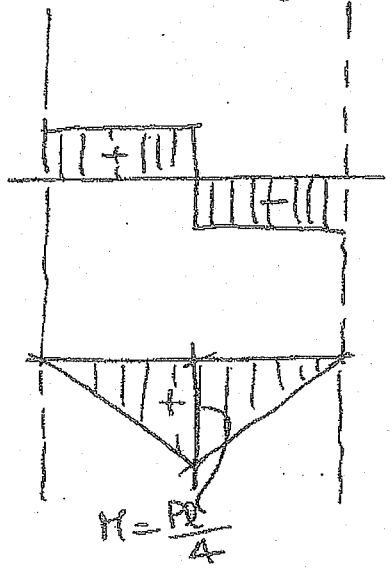


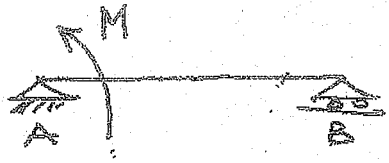
$R_A = R_B = \frac{ql}{2}$



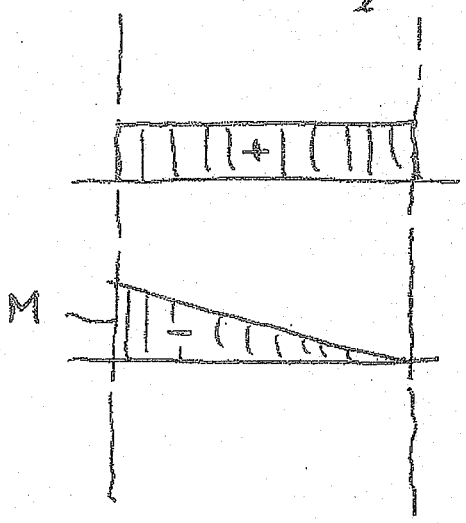


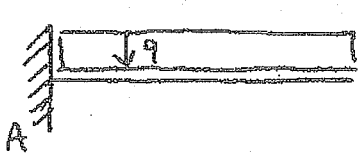
$R_A = R_B = \frac{P}{2}$



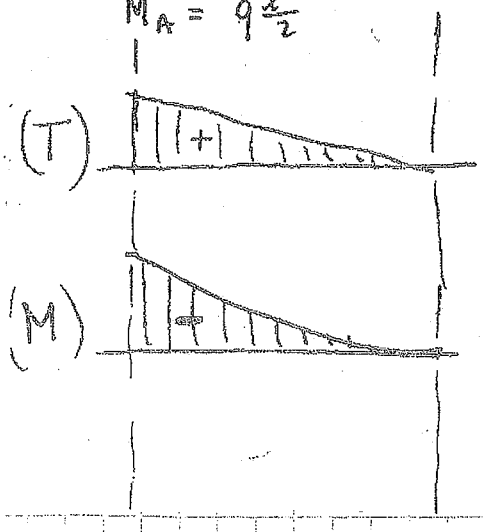


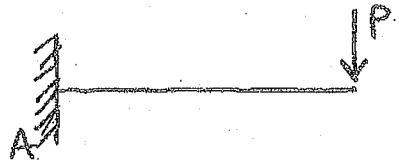
$R_A = -R_B = \frac{M}{l}$



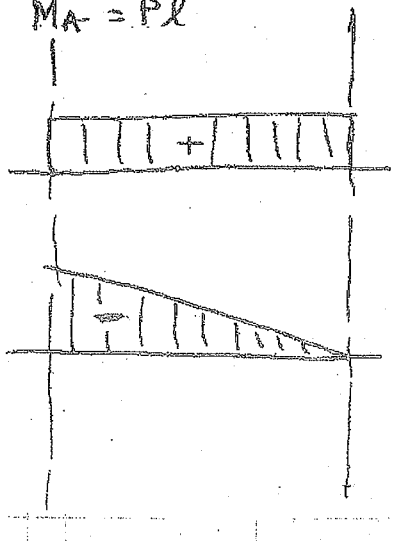


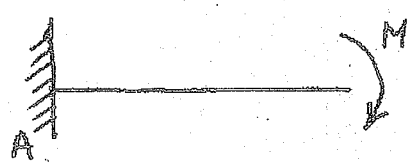
$R_A = ql$
$M_A = q\frac{l^2}{2}$



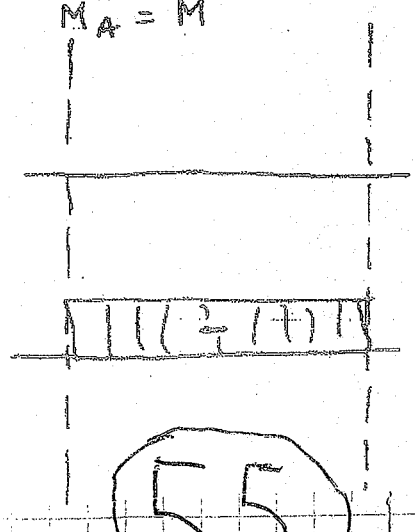


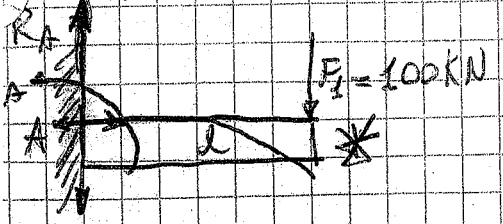
$R_A = P$
$M_A = Pl$

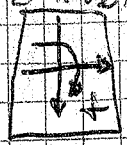




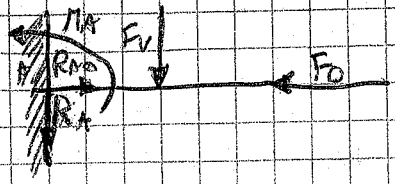
$R_A = 0$
$M_A = M$







$\sum M_A = F \cdot l - M_A = 0$



$-M_A = -F \cdot l$

1) ISOSTATICA ?

2) $3 \cdot m = 1 \cdot a + 2 \cdot c + 3 \cdot i$

2) CONVENZIONE

3) EQ STATICA $\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \rightarrow \begin{cases} \sum F_x = \text{non ci sono forze orizzontali quindi } F_x = 0 \end{cases}$

*

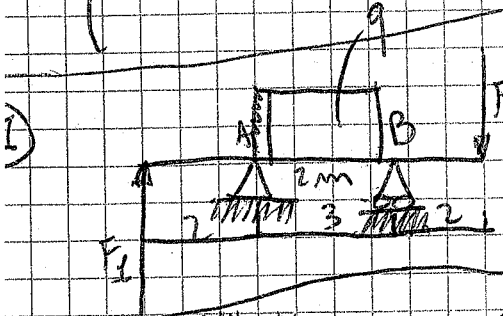
$$\begin{cases} R_A + F_1 = 0 \\ M_A + F_1 \cdot l = 0 \end{cases} \quad \begin{cases} R_A = -F_1 \\ M_A = -F_1 \cdot l \end{cases}$$

$F_1 = 100KN$

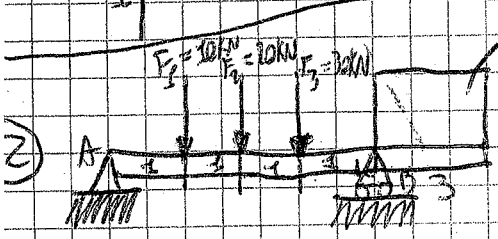
$F_2 = 20KN$

$q = 50KN/ml$

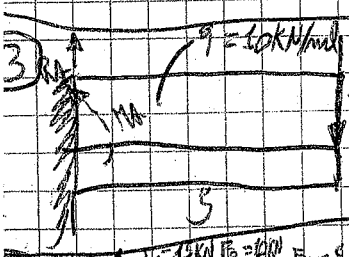
1)



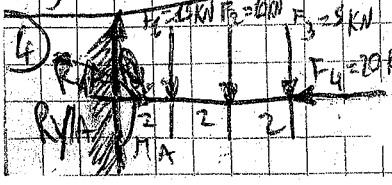
2)



3)



4)



56

1)
$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = F_1 + R_A + q\cdot l + R_B + F_2 = 0 \\ \Sigma M_A = F_1\cdot 2 + R_A\cdot 0 + (q\cdot l)\cdot 1{,}5 + R_B\cdot 3 + F_2\cdot 5 = 0 \end{cases}$$

$$-200 + 150 - R_B\cdot 3 + 250 = 0$$

$$-R_B\cdot 3 = -200 + 150 + 100$$

$$R_B = \frac{-200+150+100}{3} = 66{,}67\,KN$$

$$R_A = -F_1 - 100KN - R_B - F_2 = 0 \Rightarrow R_A = -286{,}67\,KN$$

2)
$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = R_A + F_1 + F_2 + F_3 + R_B + q\cdot l = 0 \\ \Sigma M_A = R_A\cdot 0 + F_1\cdot 1 + F_2\cdot 2 + F_3\cdot 3 + R_B\cdot 4 + (q\cdot l)\cdot 3{,}5 = 0 \end{cases}$$

$$10 + 40 + 90 + R_B\cdot 4 + 112{,}5 = 0$$

$$+R_B\cdot 4 = \frac{10+40+90+112{,}5}{4} = 138{,}125\,KN$$

$$R_A = -F_1 - F_2 - F_3 - R_B$$

3) $3\cdot m = 1\cdot a + 2\cdot c + 3\cdot i \Rightarrow 3 = 1\cdot 0 + 2\cdot 0 + 3\cdot 1 \Rightarrow 3 = 3$ ISOSTATIC

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \Rightarrow R_A + q\cdot l + F_1 = 0 \\ \Sigma M = 0 \Rightarrow R_A\cdot 0 + \left(q\cdot l\cdot \frac{l}{2}\right) + F_1\cdot 5 - M_A = 0 \end{cases}$$

~~$M_A = 125 + 120 = 245$~~

$$R_A = -74$$

$$M_A = 125 + 120 = 245$$

57

4) $\begin{cases} \Sigma F_x = 0 \Rightarrow R_{xA} - F_4 = 0 \; R_{xA} = F_4 \\ \Sigma F_y = 0 \Rightarrow R_A + F_1 + F_2 + F_3 = 0 \\ \Sigma M_A = 0 \Rightarrow \cancel{R_A \cdot 0} + F_1 \cdot l_1 + F_2 \cdot l_2 + F_3 \cdot l_3 - \cancel{F_4 \cdot 0} - M_A = 0 \\ -M_A = -(15 \cdot 2) + (10 \cdot 3) + (5 \cdot 6) = -100 \text{ KN} \\ -R_A 2 + \cancel{F_1 + F_2 + F_3} - 15 - 10 - 5 = -30 \text{ KN} \end{cases}$

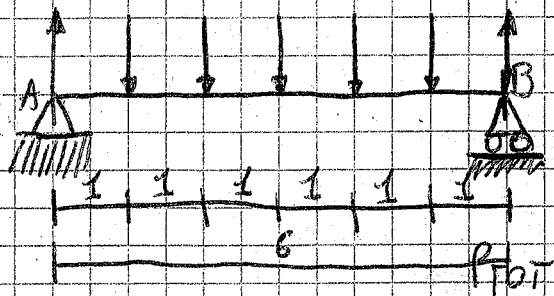
---

*

$l = 6 m$

$i = 1 m$

$\gamma = 850 \text{ Kg/m}^3$



$P_{TOT} = 325 \text{ Kg}$

$\begin{cases} P_1 = 3,25 \\ P_2 = // \\ P_3 = // \\ P_4 = // \\ P_5 = // \end{cases}$

$P_A = 1 \cdot 1 \cdot 0,06 \cdot 850 = 51 \text{ Kg}$

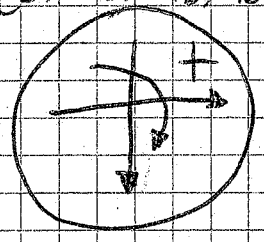
$P_{TS} = 0,14 \cdot 0,18 \cdot 1 \cdot 850 = 21,42 \text{ Kg}$

$P_{TP} = 0,22 \cdot 0,28 \cdot 1 \cdot 850 = \underline{52,36 \text{ Kg}}$

$\approx 125 \text{ Kg/ml}$

$P_{\text{DESIDERATO PERSONALE}} = 200 \text{ Kg/m}^2 = 200 \text{ Kg/ml}$

CONVENZIONE



$P_{TOT} = 125 + 200 = 325 \text{ Kg/ml}$

$P = 325 \cdot 10 = 3250 N \Rightarrow 3,25 KN$

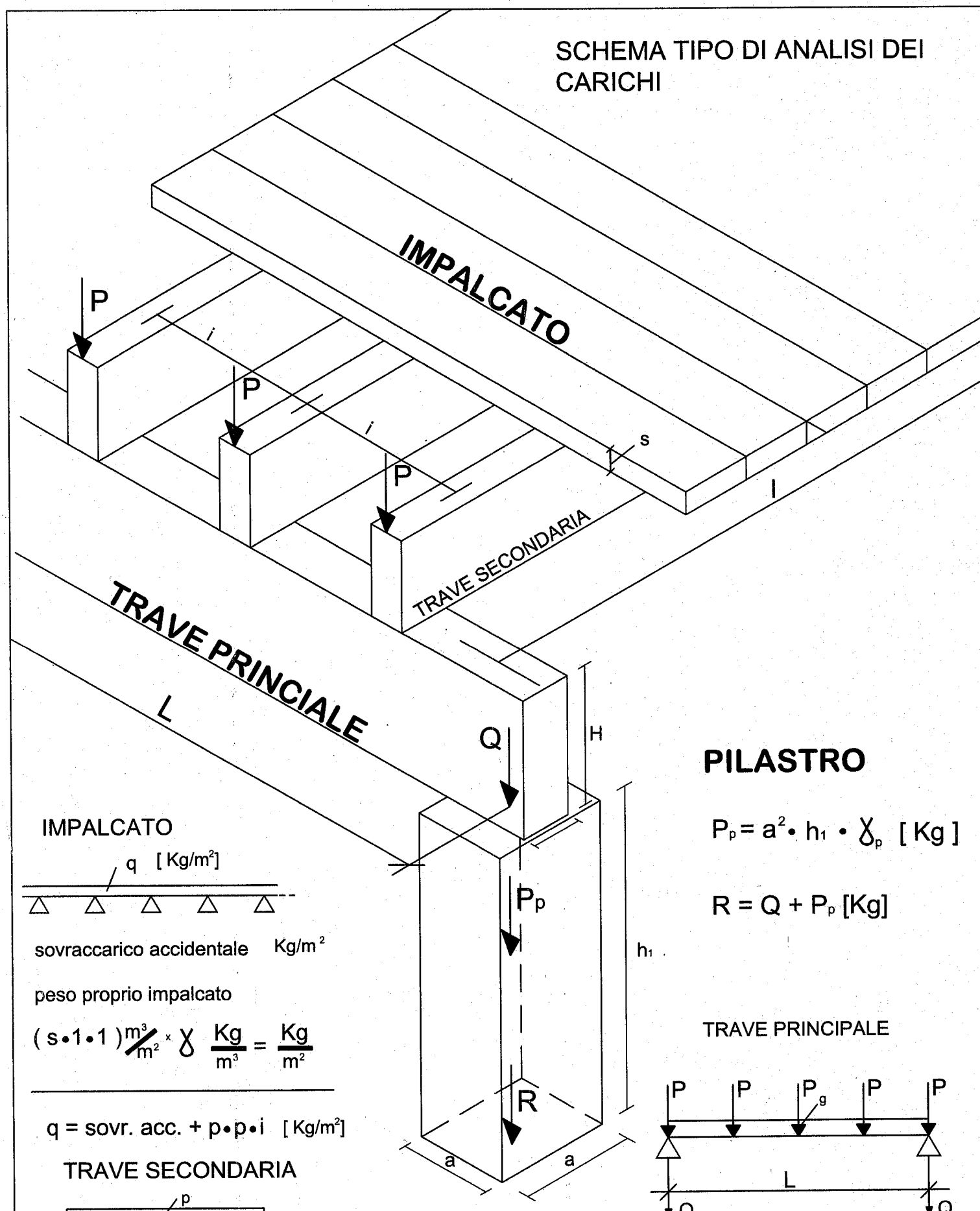
$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \Rightarrow R_A + P_1 + P_2 + P_3 + P_4 + P_5 + R_B = 0 \\ \Sigma M = 0 \Rightarrow \cancel{R_A \cdot 0} + P_1 \cdot 1 + P_2 \cdot 2 + P_3 \cdot 3 + P_4 \cdot 4 + P_5 \cdot 5 + R_B \cdot 6 = 0 \end{cases}$

$R_B \cdot 6 = 3,25 - 6,5 - 9,75 - 13 - 16,25$

$-R_B = 8,12 \text{ KN}$

$-R_A = +3,25 + 3,25 + 3,25 + 3,25 + 3,25 - 8,12 = 8,13 \text{ KN}$

SCHEMA TIPO DI ANALISI DEI CARICHI



PILASTRO

$P_p = a^2 \bullet h_1 \bullet \gamma_p$ [Kg]

$R = Q + P_p$ [Kg]

IMPALCATO

sovraccarico accidentale Kg/m²

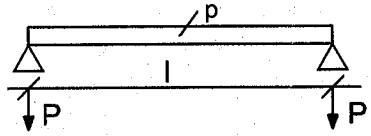
peso proprio impalcato

$(s \bullet 1 \bullet 1)\ m^3/m^2 \times \gamma\ \frac{Kg}{m^3} = \frac{Kg}{m^2}$

---

q = sovr. acc. + p•p•i [Kg/m²]

TRAVE SECONDARIA



$p = q \bullet i$ [Kg/m] + $b \bullet h \bullet 1 \bullet \gamma$ [Kg/m]

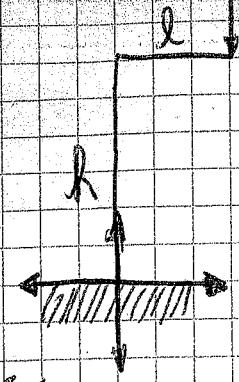
$P = \frac{p \bullet l}{2}$ [Kg]

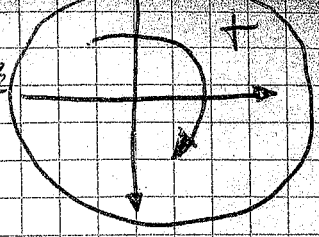
TRAVE PRINCIPALE

$P = \frac{p \bullet l}{2}$ [Kg]

$g = B \bullet H \bullet 1 \bullet \gamma$ [Kg/m]

$Q = \frac{\Sigma P}{2} + \frac{g \bullet L}{2}$ [Kg]

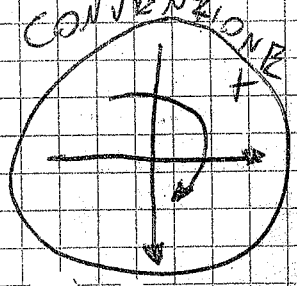


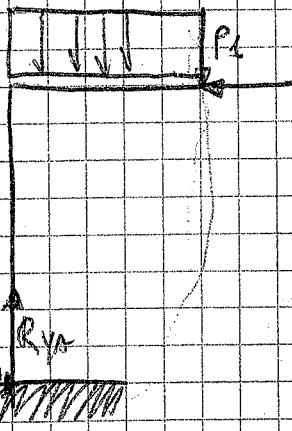


$P_1 = 10K$
$h = 3m$
$l = 2m$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \Rightarrow P_1 - R_A = 0 \\ \Sigma M_A = 0 \Rightarrow R_A \cdot 0 + P_1 \cdot 2 \end{cases}$$

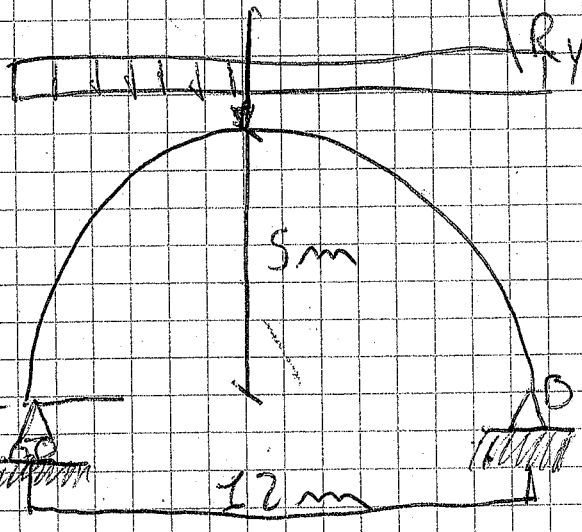
$R_A = -10$





$$\begin{cases} F_x = 0 \Rightarrow -F_1 + R_{xA} = 0 \\ F_y = 0 \Rightarrow \cancel{R_A \cdot 0} + q \cdot l + P_1 - R_{yA} = 0 \\ M_A = 0 \Rightarrow \cancel{R_A \cdot 0} + (q \cdot l) \cdot \frac{l}{2} + P_1 \cdot 2 - F_1 \cdot 3 - M_A \\ 10 + 20 - 9 - M_A = 0 \end{cases}$$

$M_A = -21\ KN/m$
$R_{xA} = +3\ KN/m$
$R_{yA} = +20\ KN/m$

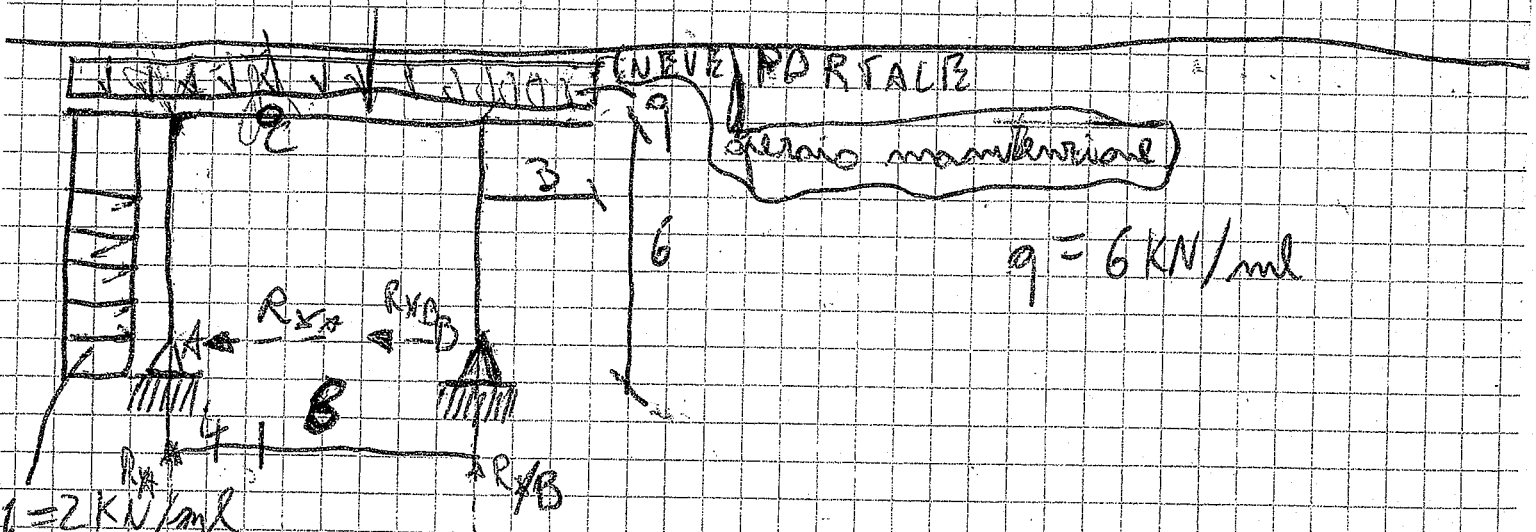


60

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \\ \Sigma M_c = 0 \end{cases}$$

$$\begin{cases} \Sigma F_x \Rightarrow R_{xA} + R_{xB} = 0 \\ \Sigma F_y \Rightarrow R_{yA} - R_{yB} + (q \cdot l) = 0 \\ \Sigma M_A \Rightarrow (q \cdot l \cdot \frac{l}{2}) + R_{yB} \cdot 12 = 0 \\ \Sigma M_c = -R_{yA} \cdot 6 - R_{xA} \cdot 5 + q \cdot 3 \cdot \frac{l}{2} = -86,4 \end{cases}$$

$R_{yA} = -48$



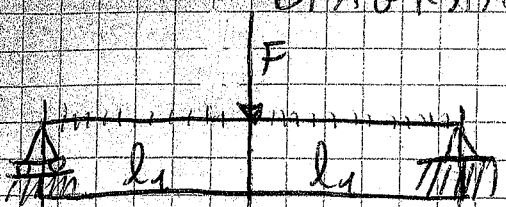
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_B = 0 \\ \sum M_C = 0 \end{cases}$$

$$\begin{cases} \sum F_x = -R_{xA} - R_{xB} + q \cdot l = 0 \\ \sum F_y = -R_{yA} - R_{yB} + q_i \, l = 0 \\ \sum M_B = q \cdot l \cdot \frac{l}{2} + q_i \, l \cdot \frac{l}{2} + \cancel{R_{xA} \cdot 0} + \cancel{R_{xB} \cdot 0} + \cancel{R_{yB} \cdot 0} + R_{yA} \cdot 12 = 0 \\ \sum M_C = \end{cases}$$

$$R_{yA} = \frac{972 + 36}{12} = 84 \text{ KN}$$

61

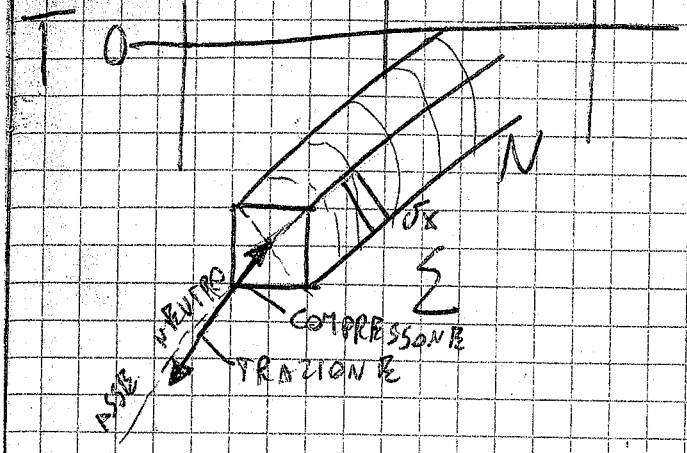
DIAGRAMMI DI SOLLECITAZIONE



$R_A = R_B = \frac{1}{2}F$

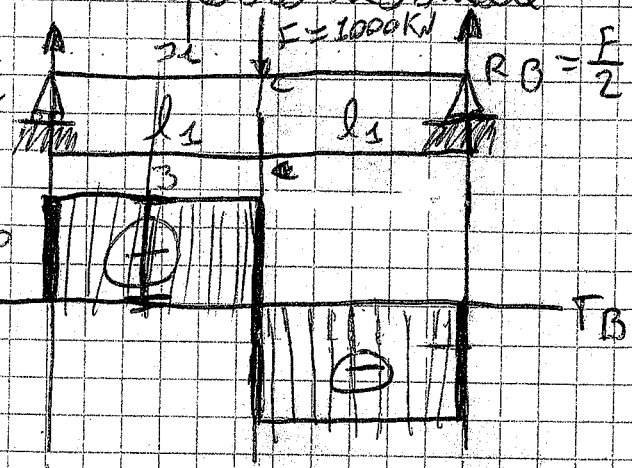
N = trazione e compressione

NON PRODUCE SFORZO NORMALE



sforzo normale

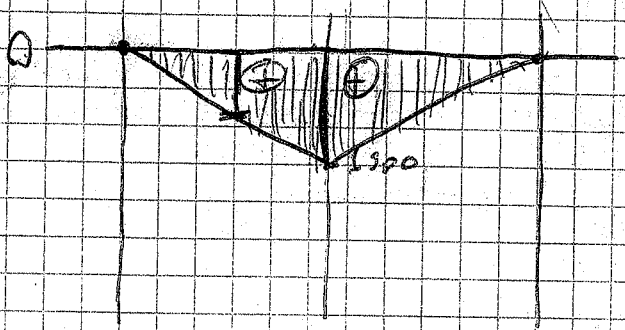
$R_A = \frac{F}{2}$



1) TIRARE DELLE LINEE VERTICALI SUGLI APPOGGI

2) TIRARE DELLE LINEE VERTICALI LUNGO LA DIREZIONE DEI CARICHI CONCENTRATI

3) AD UNA CERTA DISTANZA DALLE LINEE VERTICALI TIRARE UNA LINEA ORIZZONTALE (LINEA FONDAMENTALE)



$T_{SC} = 500KN$

$T_C = 1000KN$

$T_{DC} = -500KN$

$T_B = -500KN$

$T_A = 500KN$

$M_A = 0$

$M_{AD} = F \cdot l_1 - R_B \cdot 2l_1 = 0$

$F \cdot l_1 - F \cdot l_1 = 0$

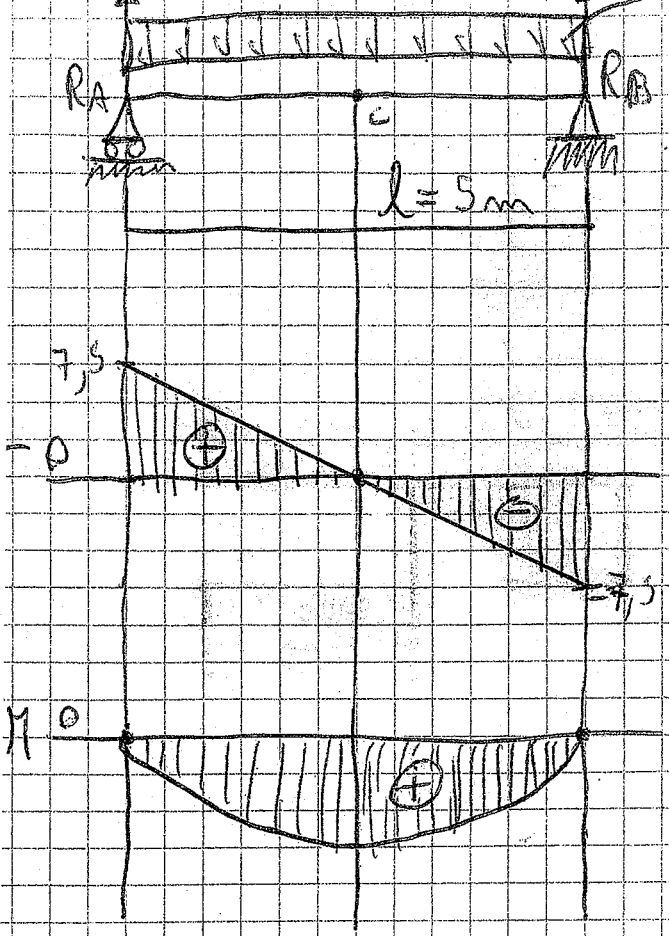
$M_C = R_A \cdot l_1 =$

$500 \cdot 3 = 1500KN \cdot m$

$M_B = -F \cdot l_1 + R_A \cdot 2l_1$

$-1000 \cdot 3 + 500 \cdot 6$

62



$$R_A = R_B = \frac{q \cdot l}{2}$$

$$R_A = R_B = \frac{5 \cdot 5}{2} = 7,5\,kN$$

$$T_A = R_A = 7,5\,kN$$

$$T_{SA} = 0$$

$$T_C = R_A + q \cdot \frac{l}{2} = -7,5 + 7,5 = 0$$

$$T_B = R_A - q \cdot l + R_B = 0$$

$$= R_A - q \cdot l = -7,5\,kN$$

$$M_A = 0$$

$$M_B = 0$$

$$M_C = R_A \cdot \frac{l}{2} - q \cdot \frac{l}{2} \cdot \frac{l}{4} =$$

$$= (7,5 \cdot 2,5) - 5 \cdot 2,5 \cdot 1,25 =$$

$$M_C = \frac{1}{8} \cdot q \cdot l^2$$

$$M_C = \frac{q \cdot l}{2} \cdot \frac{l}{2} - q \cdot \frac{l^2}{8}$$

$$M_C = \frac{q \cdot l^2}{4} - \frac{q \cdot l^2}{8} = \frac{1}{8} \cdot q \cdot l^2$$

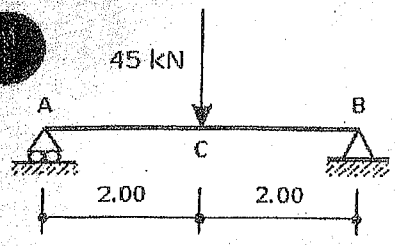
63

Corso di COSTRUZIONI

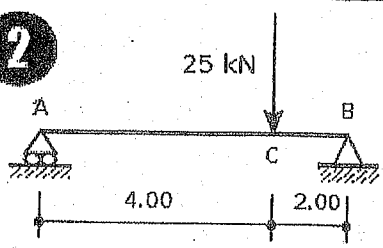
Esercizi su travi isostatiche

Calcolare le reazioni vincolari e le caratteristiche di sollecitazione delle seguenti strutture isostatiche:

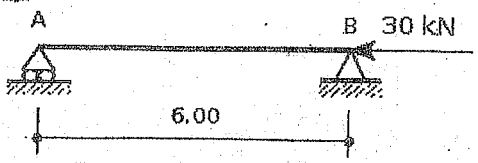
1



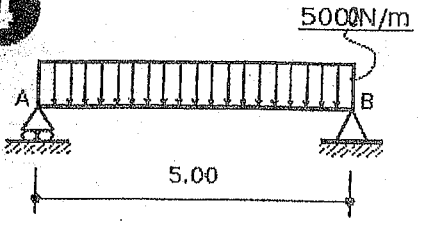
2



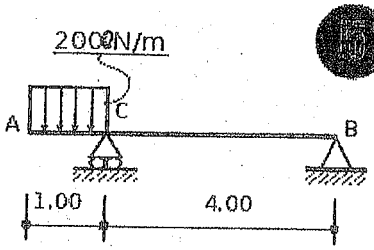
3



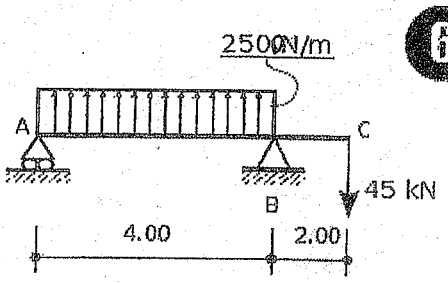
4



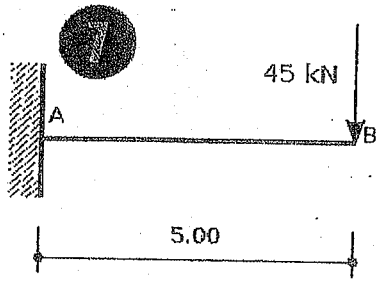
5



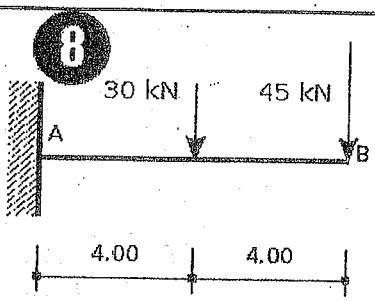
6



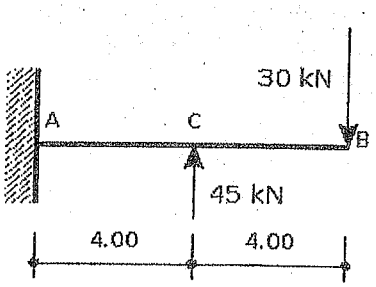
7



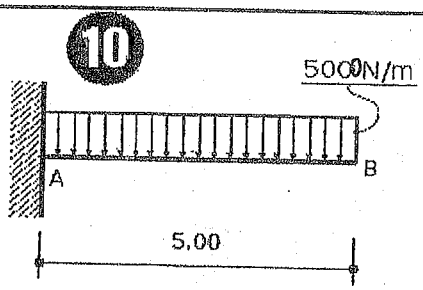
8



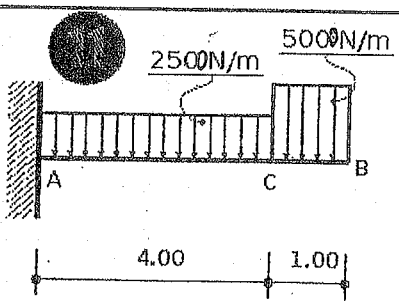
9



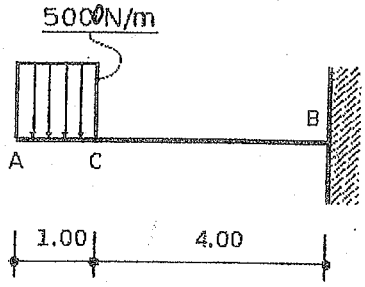
10



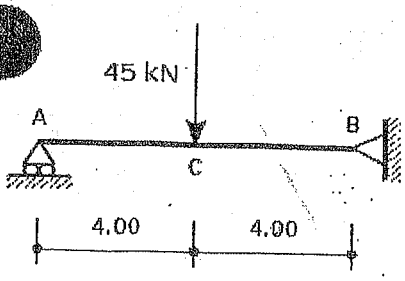
11



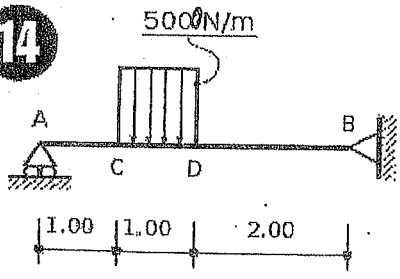
12



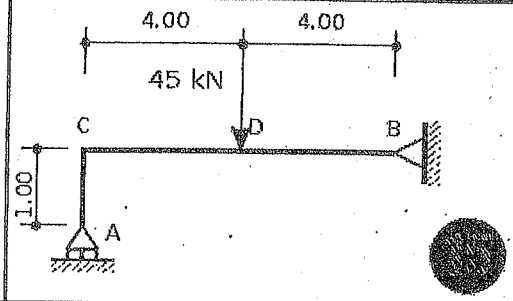
13



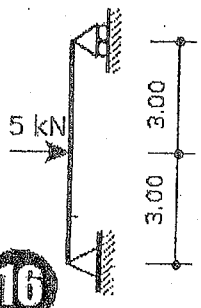
14



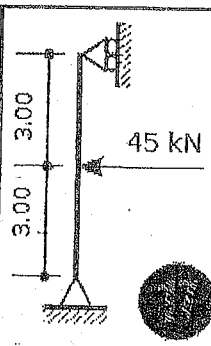
15



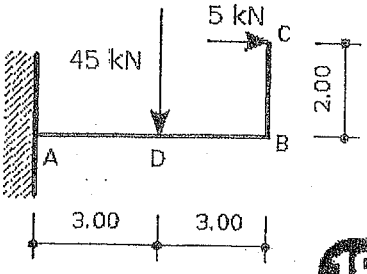
16



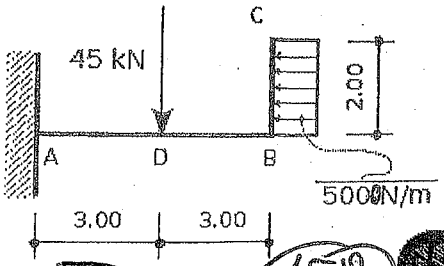
17



18



19

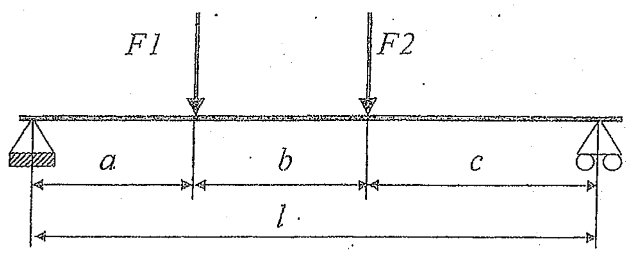


64

159

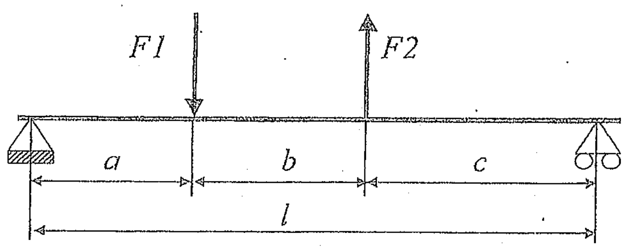
Esercizio n° 1

Data la struttura schematizzata in figura calcolare le reazioni vincolari.
Dimensioni: a= 2 m, b= 2 m, c= 3 m; Forze F1= 10 KN, F2 = 15 KN
Risolvere prima in modo letterale e poi passare ai calcoli.
STAI ATTENTO ! Equazioni della statica; convenzioni verso, VERIFICA CHE LA TRAVE SIA ISOSTATICA , ALTRIMENTI TI METTO 2 !!! CAPITO



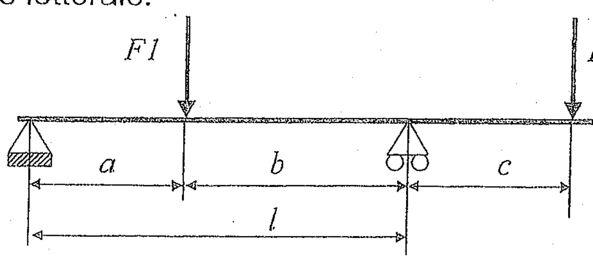
Esercizio n 2

Data la struttura schematizzata in figura calcolare le reazioni vincolari.
Dimensioni: a= 2 m, b= 2,50 m, c= 3 m; Forze F1= 100 KN, F2 = 150 KN
Risolvere prima in modo letterale.



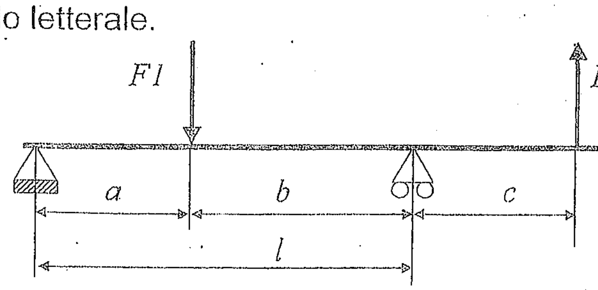
Esercizio n° 3

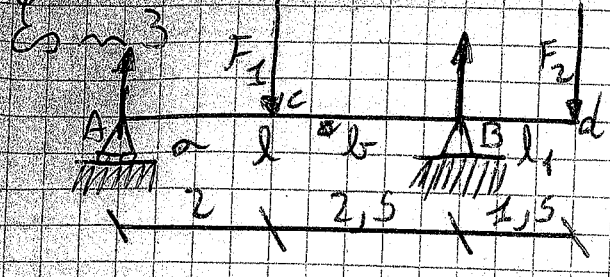
Data la struttura schematizzata in figura calcolare le reazioni vincolari.
Dimensioni: a= 2 m, b= 2,50 m, c= 1,50 m; Forze F1= 1000 N, F2 = 1500 N
Risolvere prima in modo letterale.



Esercizio n° 4

Data la struttura schematizzata in figura calcolare le reazioni vincolari.
Dimensioni: a= 2,50 m, b= 3,00 m, c= 1,50 m; Forze F1= 3500 N, F2 = 1500 N
Risolvere prima in modo letterale.



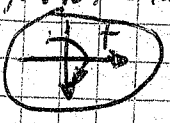


$$\begin{cases}\Sigma F_x = 0 \\ \Sigma F_y = 0 \Rightarrow -R_A + F_1 - R_B + F_2 = 0 \\ \Sigma M_A = 0 \Rightarrow R_A \cdot 0 + F_1 \cdot a - R_B \cdot l + F_2 \cdot l_1 = 0\end{cases}$$

$F_1 = 10\,kN$

$F_2 = 15\,kN$

CONVENZIONE

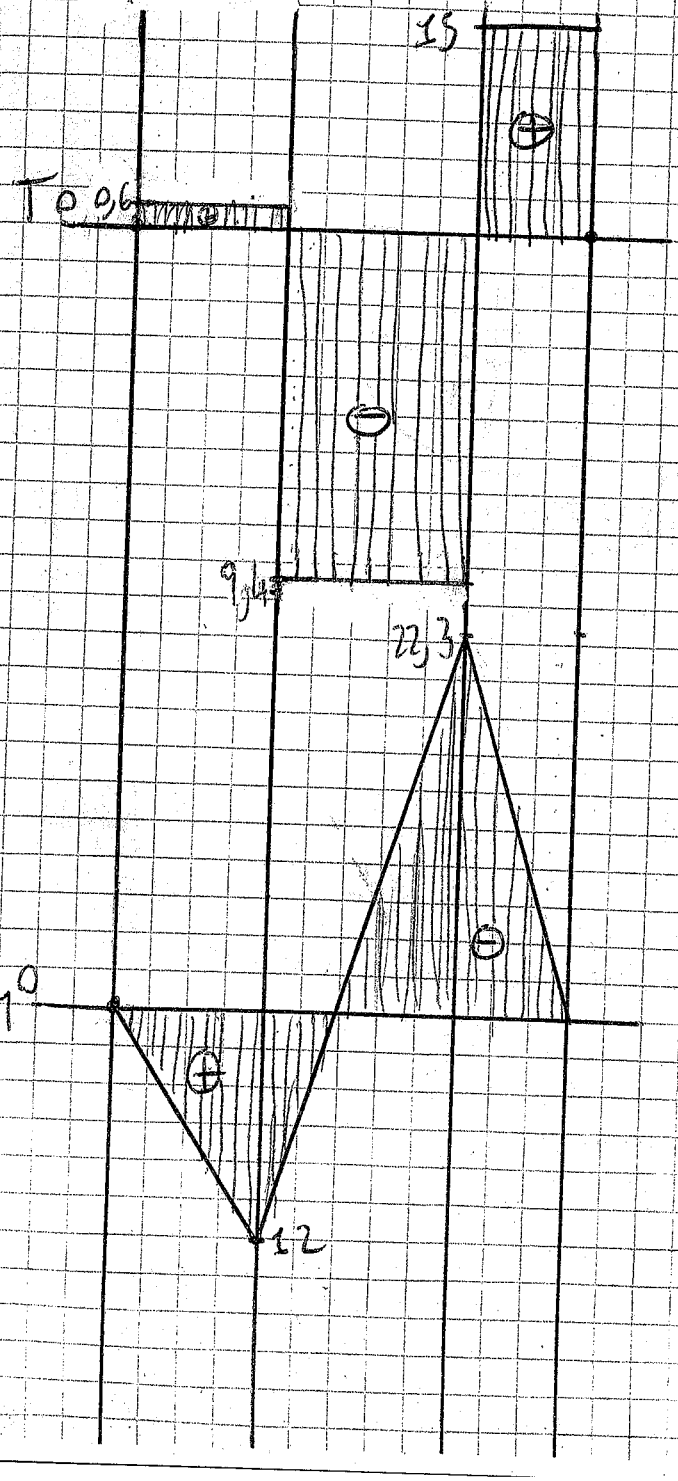


$$\begin{cases}\Sigma F_x = 0 \\ \Sigma F_y = 0 \Rightarrow R_A + 10 - 24{,}40 + 15 = 0 \Rightarrow -R_A = -10 + 24{,}40 - 15 \Rightarrow -R_A = -0{,}6\,kN \\ \Sigma M_A = 0 \Rightarrow 10 \cdot 2 - R_B \cdot 4{,}5 + 15 \cdot 6 \Rightarrow -R_B = \dfrac{-20 - 22{,}5}{4{,}5} = -90\end{cases}$$

$$= \frac{-20-90}{4{,}5} = -24{,}40\,kN$$

SCALA/TAGLI

1 = 1 □



$T_{AS} = 0$

$T_A = R_A = 0{,}6\,kN$

$T_{F_1 S} = R_A = 0{,}6\,kN$

$T_{F_1} = F_1 - R_A = 9{,}4\,kN$

$T_{R_B} = F_1 - R_A = 9{,}4\,kN$

$T_{R_B} = F_1 - R_A - R_B =$

$-25 + 10 = -15\,kN$

$F_2 = F_1 + F_2 - R_A - R_B = 0$

SCALA/MOMENTI

1 □ = 2

$M_{AS} = 0$

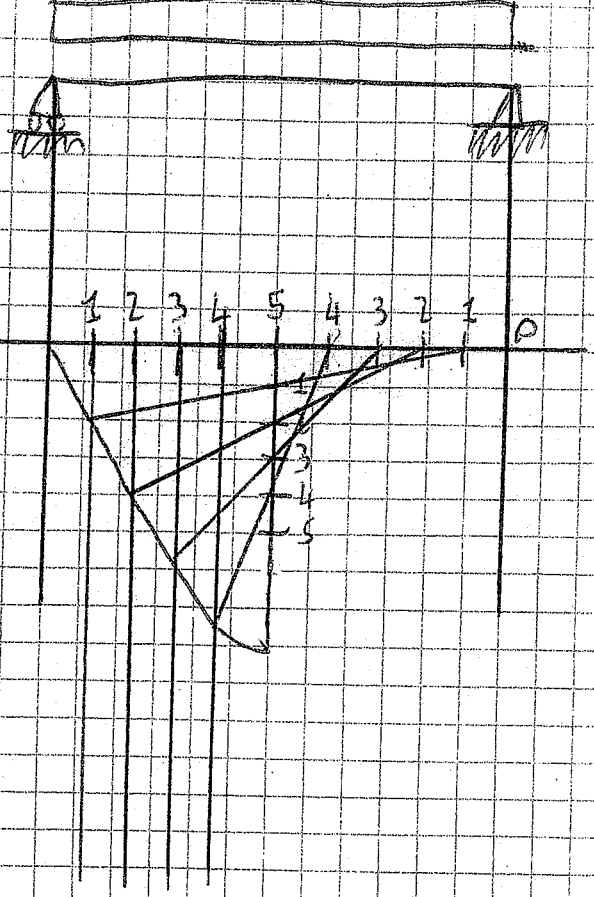
$M_C = R_A \cdot ac = 0{,}6 \cdot 2 = 12\,kN \cdot m$

$M_B = R_A \cdot a + b - F_1 \cdot b + c =$

$= 0{,}6 \cdot 4{,}5 - 10 - 2{,}5 = -22{,}3\,kN \cdot m$

$M_D = 0$

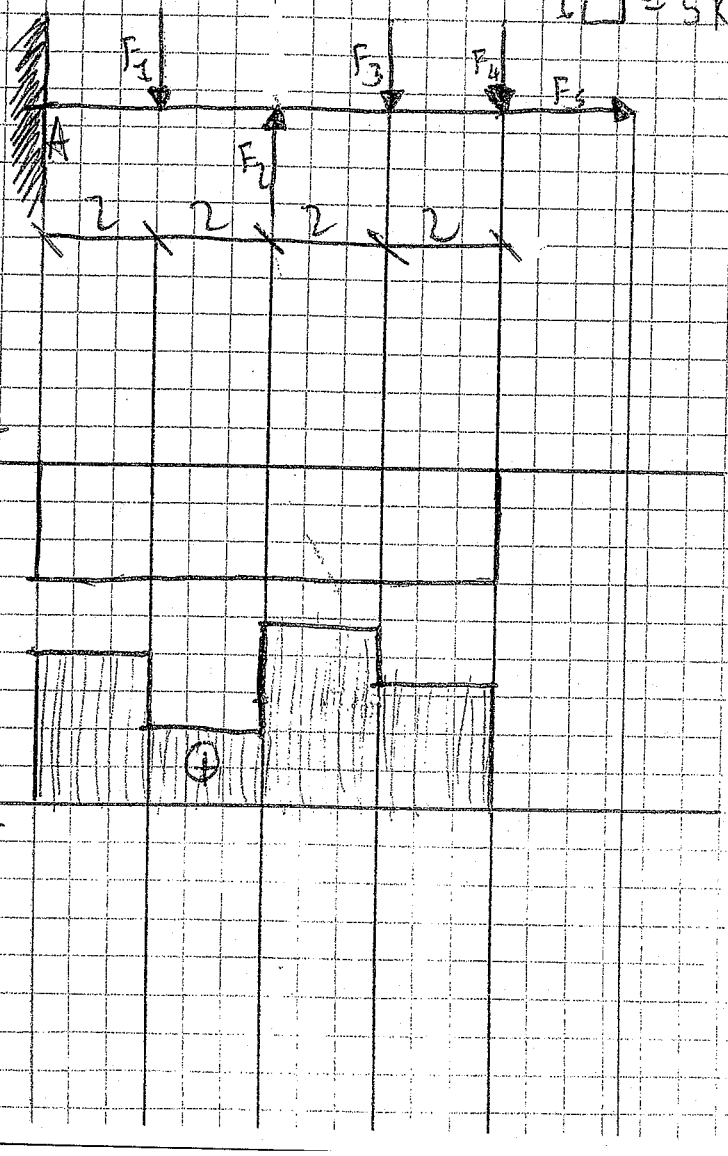
66

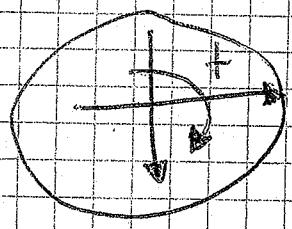


$$M = \frac{1}{8} q \cdot l^2$$

SCALA T

1 □ = 5 KN





$F_1 = 10\,KN$

$F_2 = 4\,KN$

$F_3 = 6\,KN$

$F_4 = 8\,KN$

$F_5 = 12\,KN$

$T_{AS} = 0$

$T_A = R_A = -20\,KN$

$T_{F_1} = F_1 - R_A = -20 - 10 R_A = -20\,KN$

$= 10\,KN$ $\quad R_0 = -12\,KN$

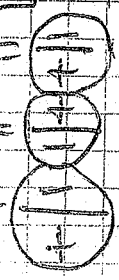
$T_{F_2} = -20 + 10 - 4 = -14$ $\quad M_A = -10 \cdot 4\,KN$

$T_{F_3} = -20 + 10 - 4 + 6 = -8\,KN$

$T_{F_4} = -20 + 10 - 4 + 6 + 8 = 0\,KN$

SCALA N

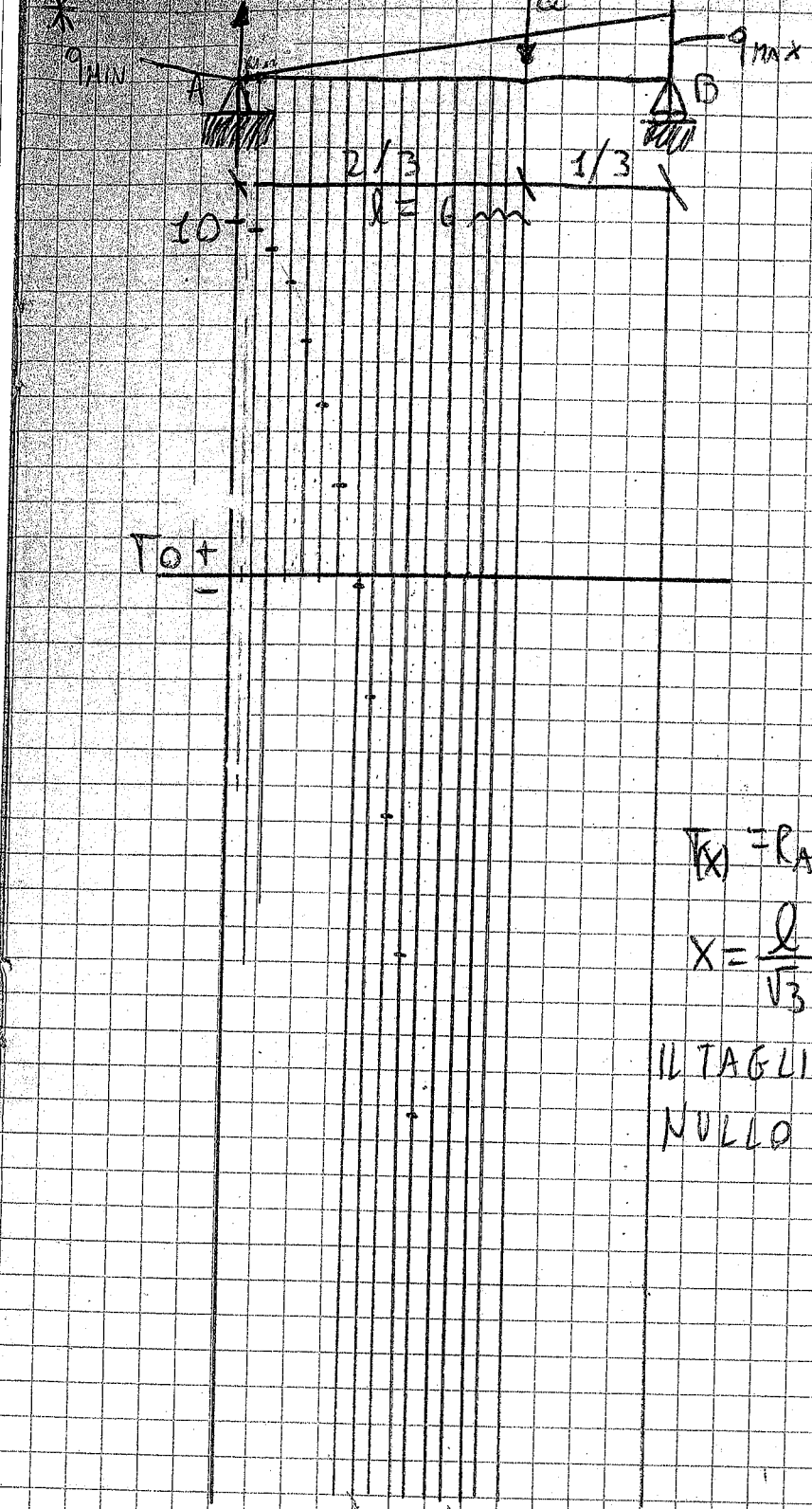
1 □ = 4 KN



67

N = trazione verso l'alto
compressione verso il basso

SCALA VOLUTA	MISURA IN m.	MISURA IN cm.	MISURA IN mm.
1 :1	1000 :1	10 :1	1 :1
1 :2	500 :1	5 :1	0.5 :1
1 :5	200 :1	2 :1	0.2 :1
1 :10	100 :1	1 :1	0.1 :1
1 :20	50 :1	0.5 :1	0.05 :1
1 :25	40 :1	0.4 :1	0.04 :1
1 :50	20 :1	0.2 :1	0.02 :1
1 :100	10 :1	0.1 :1	0.01 :1
1 :200	5 :1	0.05 :1	0.005 :1
1 :250	4 :1	0.04 :1	0.004 :1
1 :500	2 :1	0.02 :1	0.002 :1
1 :1.000	1 :1	0.01 :1	0.001 :1
1 :2.000	0.5 :1	0.005 :1	0.0005 :1
1 :2,500	0.4 :1	0.004 :1	0.0004 :1
1 :4.000	0.25 :1	0.0025 :1	0.00025 :1
1 :5.000	0.2 :1	0.002 :1	0.0002 :1
1 :10.000	0.1 :1	0.001 :1	0.0001 :1
1 :20.000	0.05 :1	0.0005 :1	0.00005 :1
1 :25.000	0.04 :1	0.0004 :1	0.00004 :1
1 :50.000	0.02 :1	0.0002 :1	0.00002 :1
1 :100.000	0.01 :1	0.0001 :1	0.00001 :1
1 :200.000	0.005 :1	0.00005 :1	0.000005 :1
1 :500.000	0.002 :1	0.00002 :1	0.000002 :1



1□ = 1KN

$q_{MAX} = 10\,KN/m$

$l = 6\,m$

$$R_A = \frac{1}{6} \cdot q_{MAX} \cdot l = \frac{1}{6} \cdot 10 \cdot 6 = 10\,KN$$

$$R_B = \frac{1}{3} \cdot q_{MAX} \cdot l = \frac{1}{3} \cdot 10 \cdot 6 = 20\,KN$$

$$T(x) = R_A - \left(\frac{q_x}{2}\right) = 0$$

$$X = \frac{l}{\sqrt{3}} = 3,46\,KN$$

IL TAGLIO È NULLO

$q_{x_1} = \frac{q_{MAX} \cdot 0,50}{6} = 0,83$

$q_{x_2} = \frac{10 \cdot 1}{6} = 1,67\,KN$

$q_{x_3} = \frac{10 \cdot 1,5}{6} = 2,5\,KN$

$q_{x_4} = \frac{10 \cdot 2}{6} = 3,33\,KN$

$q_{x_5} = \frac{10 \cdot 2,5}{6} = 4,17\,KN$

$q_{x_6} = \frac{10 \cdot 3}{6} = 5\,KN$

$q_{x_7} = \frac{10 \cdot 3,5}{6} = 5,83\,KN$

$q_{x_8} = \frac{10 \cdot 4}{6} = 6,67\,KN$

$q_{x_9} = \frac{10 \cdot 4,5}{6} = 7,5\,KN$

$q_{x_{10}} = \frac{10 \cdot 5}{6} = 8,33\,KN$

$q_{x_{11}} = \frac{10 \cdot 5,5}{6} = 9,17\,KN$

$q_{x_{12}} = \frac{10 \cdot 6}{6} = 10\,KN$

$T(0,50) = R_A - \left(\frac{0,83 \cdot 0,50}{2}\right) = 9,8\,KN$

$T(1,00) = R_A - \left(\frac{1,67 \cdot 1}{2}\right) = 9,1\,KN$

$T(1,50) = R_A - \left(\frac{2,50 \cdot 1,50}{2}\right) = 8,1\,KN$

$T(2,00) = R_A - \left(\frac{3,33 \cdot 2}{2}\right) = 6,67\,KN$

$T(2,50) = R_A - \left(\frac{4,17 \cdot 2,50}{2}\right) = 4,79\,KN$

$T(3,00) = R_A - \left(\frac{5 \cdot 3}{2}\right) = 2,5\,KN$

$T(3,50) = R_A - \left(\frac{5,83 \cdot 3,50}{2}\right) = -0,20\,KN$

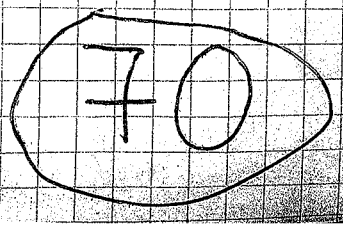
$T(4,00) = R_A - \left(\frac{6,67 \cdot 4}{2}\right) = -3,31\,KN$

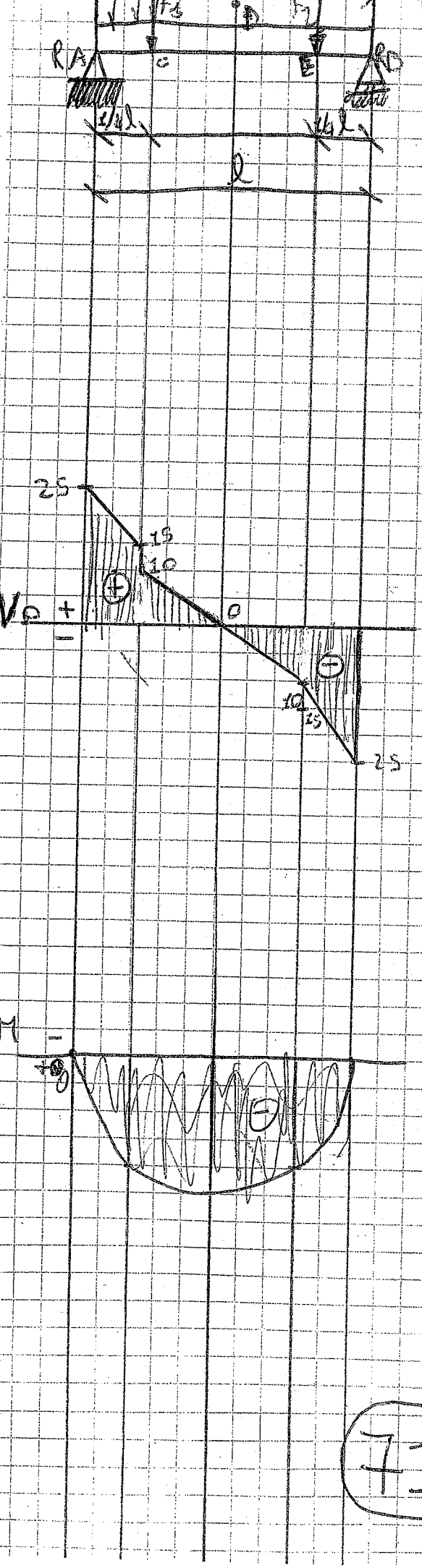
$T(4,50) = R_A - \left(\frac{7,5 \cdot 4,50}{2}\right) = -6,88\,K$

$T(5,00) = R_A - \left(\frac{8,33 \cdot 5}{2}\right) = -10,82\,K$

$T(5,50) = R_A - \left(\frac{9,17 \cdot 5,50}{2}\right) = -13,21$

$T(6,00) = R_A - \left(\frac{10 \cdot 6}{2}\right) = 0\,K$ 20K





$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases}$ $\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = R_A + F_1 + F_2 + q \cdot l + R_B = 0 \\ \Sigma M_A = F_1 \cdot 1 + q \cdot l \cdot 2 + F_2 \cdot 3 + R_B \cdot 4 \end{cases}$

$q = 10\,KN/m$

$F_1 = F_2 = 5\,KN$

$l = 4\,m$

$R_A = ?$

$R_B = ?$

$T = ?$

$M = ?$

$R_B = \frac{-20 - 15 - 3}{4} = -25\,KN$

$R_A + 5 + 15 + 40 - 25 = 0$

$R_A = -5 - 5 - 40 + 25 = -25$

SCALA TAGLIO

$1\square = 5\,KN$

TAGLIO = V

$T_{SA} = 0$

$T_A = R_A = -25\,KN$

$T_{SE} = R_A - q \cdot \frac{1}{4} l = 25 - 10 = 15\,KN$

$T_C = R_A - q \cdot \frac{1}{4} l + F_1 = 25 - 10 - 5 = 10$

$T_{SB} = R_A - q \cdot \frac{l}{2} - F_1 = 25 - 20 - 5 = 0$

$T_B = 0$

$T_{SD} = F_2 - q \cdot \frac{1}{4} l - R_B = 5 - 10 + 25 = -10\,KN$

$T_D = R_B - q \cdot l = 25 - 10 = 15$

$M_A = 0$

$M_C = -R_A \cdot 1 + q \cdot \frac{1}{4} l \cdot \frac{1}{8} l$

$= -25 + 5 = -20\,KN$

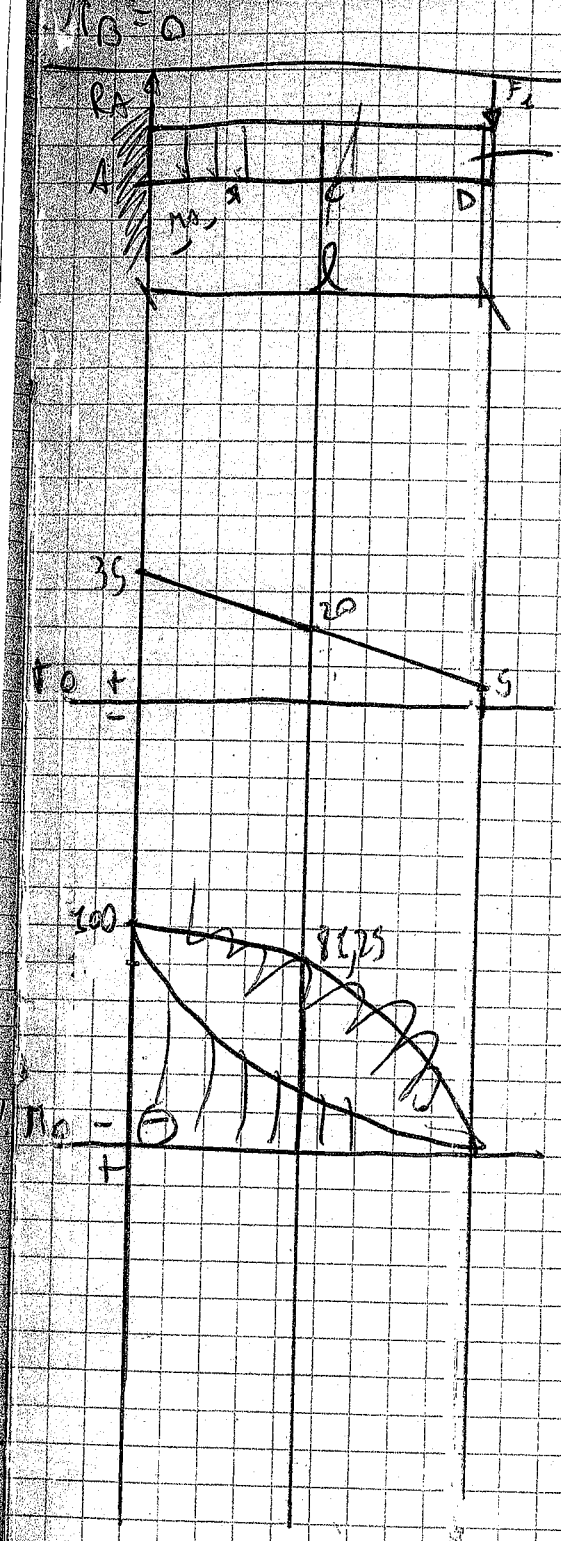
$M_D = -R_A \cdot \frac{l}{2} + q \cdot \frac{l}{2} \cdot \frac{l}{4} + F_1 \cdot \frac{l}{4} = -50 + 20 + 5 =$

$= -25\,Kn/m$

$M_E = -R_A \cdot \frac{l}{3} + q \cdot \frac{l}{3} \cdot \frac{l}{6} + F_1 \cdot \frac{l}{1} + F_2 \cdot 0 =$

POS		T[KN]	M[K
A	0		
B	0,50		
C	1,00		
D	1,50		
F	2,00		

71



$M_B = 0$

$F_1 = 5$

$q = 6KN$

$l = 5m$

$R_A = ?$

$R_B = ?$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases} \quad \begin{cases} \Sigma F_y = q \cdot l + F_1 = 30 + 5 = 35KN \\ \Sigma M_A = q \cdot l \cdot \frac{l}{2} + F_1 \cdot 5 = \end{cases}$$

SCALA T

$1\square = 10KN$

DIAGRAMMI

$= -75 - 25 = -100KN$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \Rightarrow q \cdot l + F_1 - R_A = 0 \\ \Sigma M_B = 0 \Rightarrow -q \cdot l \cdot \frac{l}{2} - F_1 \cdot l - R_A \cdot 0 + M_A = 0 \end{cases}$$

$-R_A = -30 - 5$

$-R_A = -35KN$

$M_A = -75 - 25 = -100KN \cdot m$

$T_{(C_S)} = +R_A - q \cdot \frac{l}{2} = 35 - 15 = 20KN$

$T_{(D_S)} = R_A - q \cdot l = 35 - 30 = 5$

SCALA M

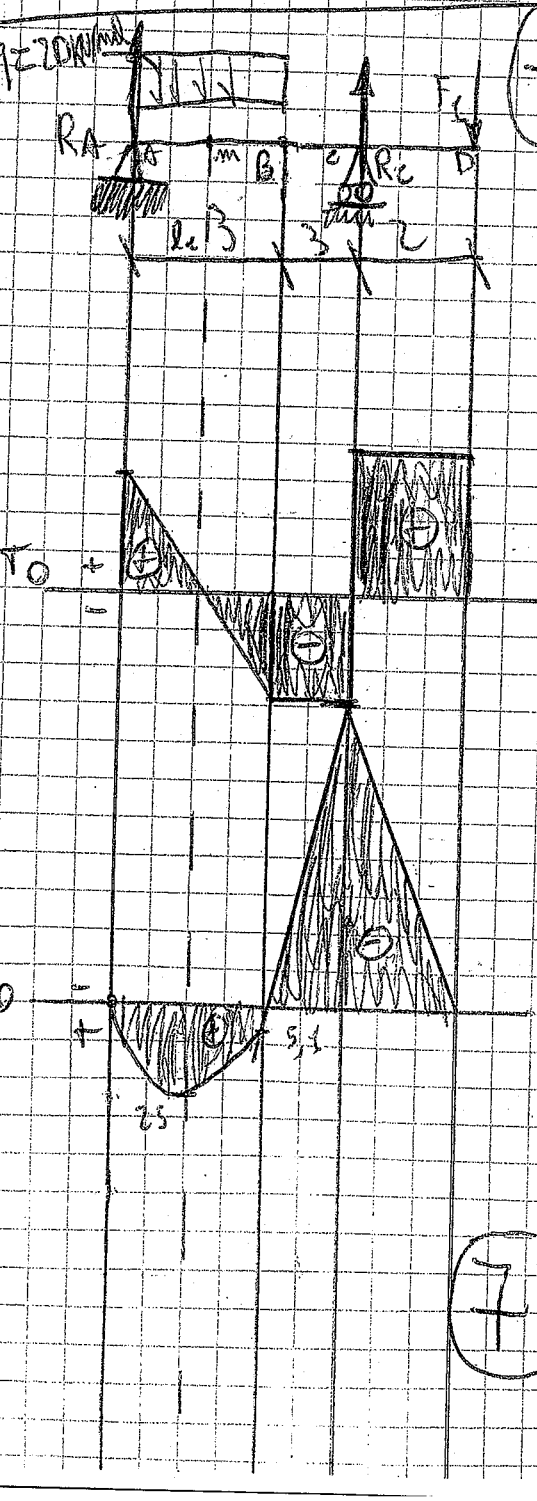
$1\square = 20KN$

$M_{C_S} = 100 - q \cdot \frac{l}{2} \cdot \frac{l}{4} = 100 - 18,75 =$

$= 81,25KN \cdot m$

72

DOVE IL TAGLIO È 0
IL MOMENTO È MASSIMO



$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \Rightarrow \\ \Sigma M_1 = 0 \Rightarrow \end{cases}$$

$q = 20\,KN/ml$

$F_1 = 40\,KN$

$$\begin{cases} \Sigma F_y = 0 \Rightarrow R_A + q \cdot l + R_C + F_1 \\ \Sigma M_A = 0 \Rightarrow \cancel{R_A \cdot 0} + q \cdot l \cdot \frac{l}{2} - R_C \cdot 6 + F_1 \cdot 8 \end{cases}$$

$$-R_C = \frac{-90 - 320}{6} = -68{,}3\,KN$$

$$R_A = -60 + 68{,}3 - 40 = 31{,}7\,KN$$

TAGLIO = SCALA 1 □ = 10 KN

$T_{R_A} = R_A = 31{,}7\,KN$

$(T_{R_D}) = R_A - q \cdot \frac{l_1}{2} = 31{,}7 - 30 = 1{,}7\,KN$

$T = 0 = R_A - q \cdot l_m = 0$

$l_m = \frac{R_A}{q} = \frac{31{,}7}{20} = 1{,}585\,m$

MOMENTO = SCALA 1 □ = 10 KN·m

$M_A = 0$

$M_m = R_A \cdot 1{,}5 - (q \cdot \frac{l_1}{2} \cdot 0{,}75) = 25\,KN \cdot m$

$M_B = R_A \cdot 3 - (q \cdot l_1 \cdot 1{,}5) = 5{,}1\,KN \cdot m$

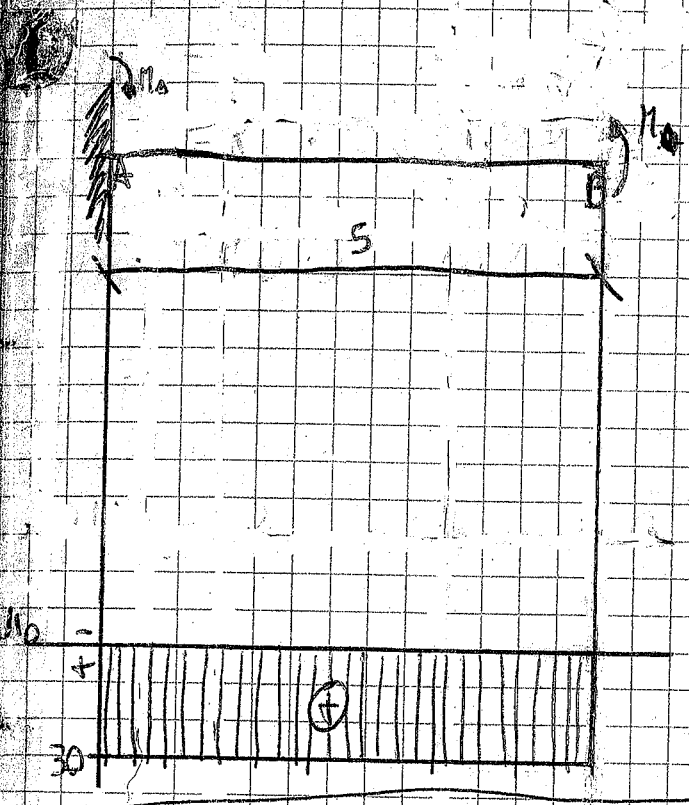
$M_C = \cancel{R_A} \cdot F_1 \cdot 2 = 80\,KN \cdot m$

73

$T_{SC} = R_A - q \cdot l = -28,3\,KN$

$T_C = R_A - q \cdot l + R_C = 40\,KN$

$T_D = R_A - q \cdot l + R_C - F_1 = 3[,\cancel{77}] - 60 + 68,3 - 40 = 0$



$\begin{cases} \Sigma F_x = 0 \Rightarrow \text{NON CI SONO FORZE ORIZZONTALI} \\ \Sigma F_y = 0 \Rightarrow \text{NON CI SONO FORZE VERTICALI} \\ \Sigma M_A = 0 \Rightarrow M_A = -30 \end{cases}$ $R_A = 0\,KN$ $M = 30\,KN \cdot m$

DATI

$A.B = 5\,m$

$R_A = ?$

$M_A = ?$

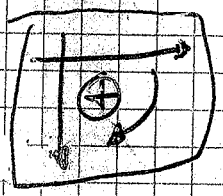
DIAGRAMMI = ?

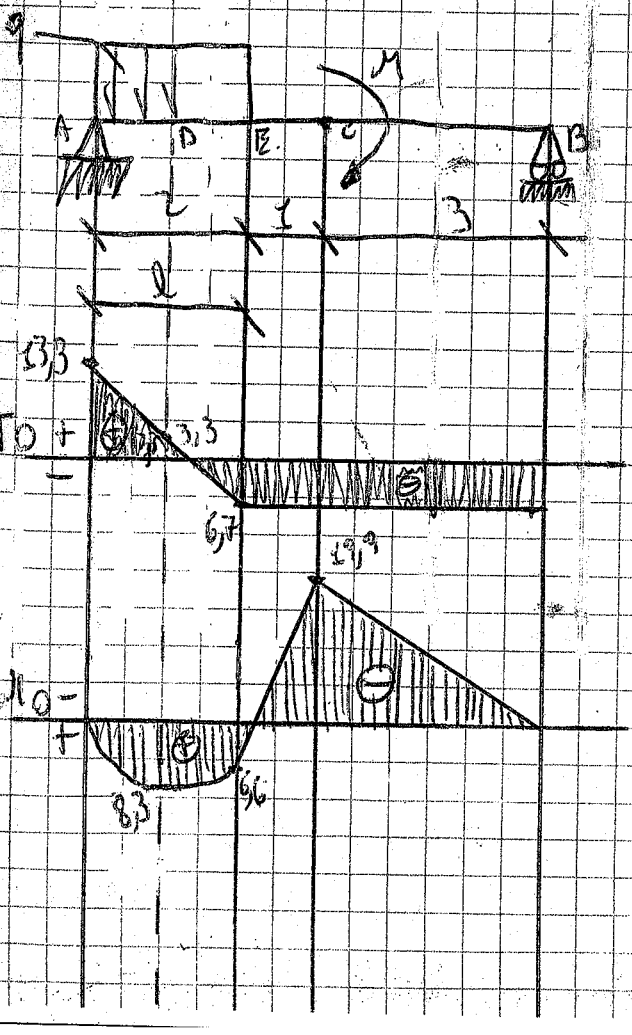
$\Sigma M_A = M_A - M = 0$

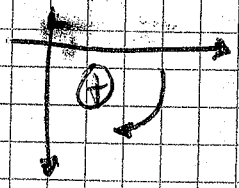
$M_A = M$

SCALA M

1 □ = 10 KN · m







$q = 10\,KN$

$M_C = 20\,KN$

$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases}$

$\begin{cases} \Sigma F_y = 0 - R_A + q \cdot l - R_B = 0 \\ \Sigma M_A \Rightarrow q \cdot l + M_C - R_B \cdot 6 \end{cases}$

$\begin{cases} R_A = q \cdot l - R_B \\ 20 + 20 - R_B \cdot 6 \end{cases}$ $\begin{cases} R_A = q \cdot l - R_B \\ R_B = \frac{40}{6} = 6,7\,KN \end{cases}$

$\begin{cases} R_A = 20 - 6,7 = 13,3\,KN \\ R_B = 6,7\,KN \end{cases}$

SCALA T □ = 5 KN

$T_A = R_A = 13,3\,KN$

$T_D = R_A - q \cdot \frac{l}{2} = 13,3 - 10 = 3,3\,KN$

$T_E = R_A - q \cdot l = 13,3 - 20 = -6,7\,KN$

$M_D = q \cdot \frac{l}{2} \cdot \frac{l}{4} + R_A = 13,3\,KN \cdot m$

$M_E = R_A^2 - q \cdot l = -6,6\,KN$

$M_C = R_A \cdot 3 + q \cdot l \cdot 2 + M_B = 19,9$

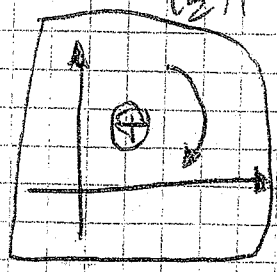
$M_{B(\Phi)} =$

74

A C D

$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases}$ $\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \Rightarrow R_A + R_B \\ \Sigma M_A = 0 \Rightarrow M \end{cases}$ $\begin{cases} AB = 5\ m \\ M = 20\ KN \cdot m \end{cases}$

DATE



$M \cdot 2 - R_B \cdot 5$

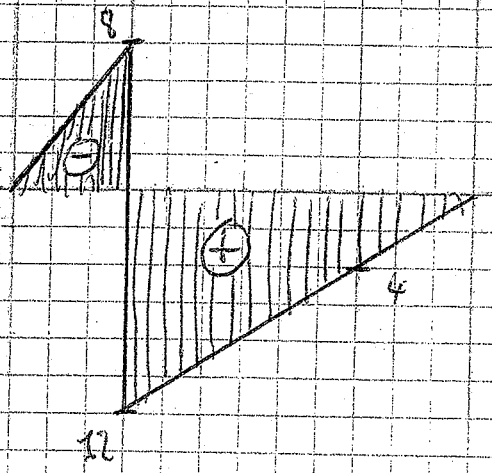
$+R_B = \frac{+20}{5} = +4\ KN$

$R_A = -R_B = -4\ KN$

SCALA 1 □ = 2 KN

$M_A = 0$

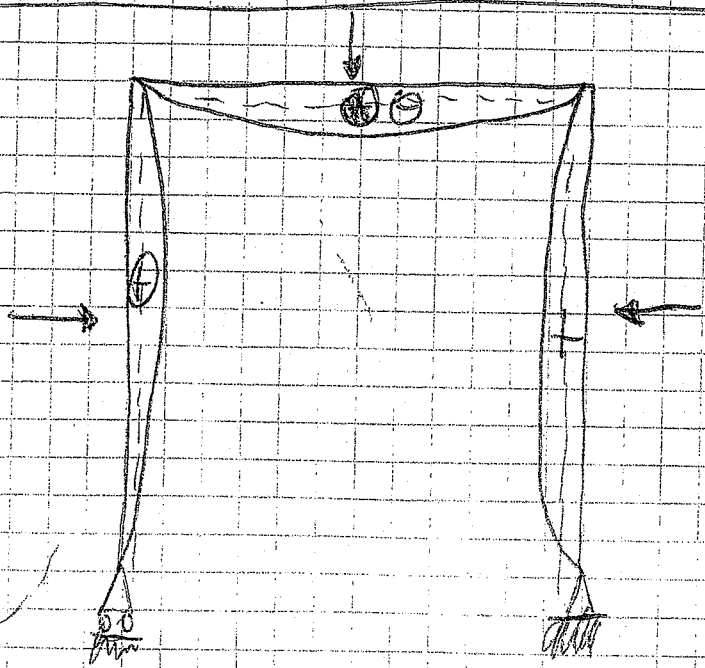
$M_{C(S)} = R_A \cdot 2 = -8\ KN \cdot m$



$M_{C(D)} = R_B \cdot 3 = +12\ KN \cdot m$

$M_{ZD} = R_B \cdot 1$

$M_{RD} = 0$



75

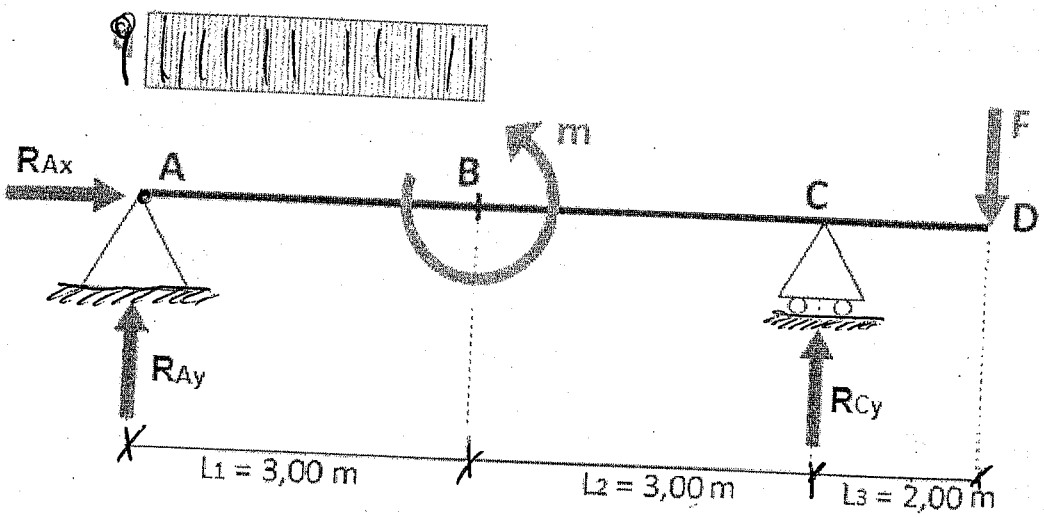
ESERCITAZIONE 3°B COSTRUZIONI MARZO 2017

Determinare i diagrammi delle caratteristiche della sollecitazioni N,T,M della trave riportata in figura e riportare tutto in opportuna scala (delle forze e delle lunghezze).

q	20 kN/m
M	30 kN m
F	40 kN

$Q = q L_1 = 60$ kN

M = MOMENTO APPLICATO



Il calcolo delle REAZIONI VINCOLARI avviene impostando 3 equazioni di equilibrio, le EQUAZIONI CARDINALI DELLA STATICA:

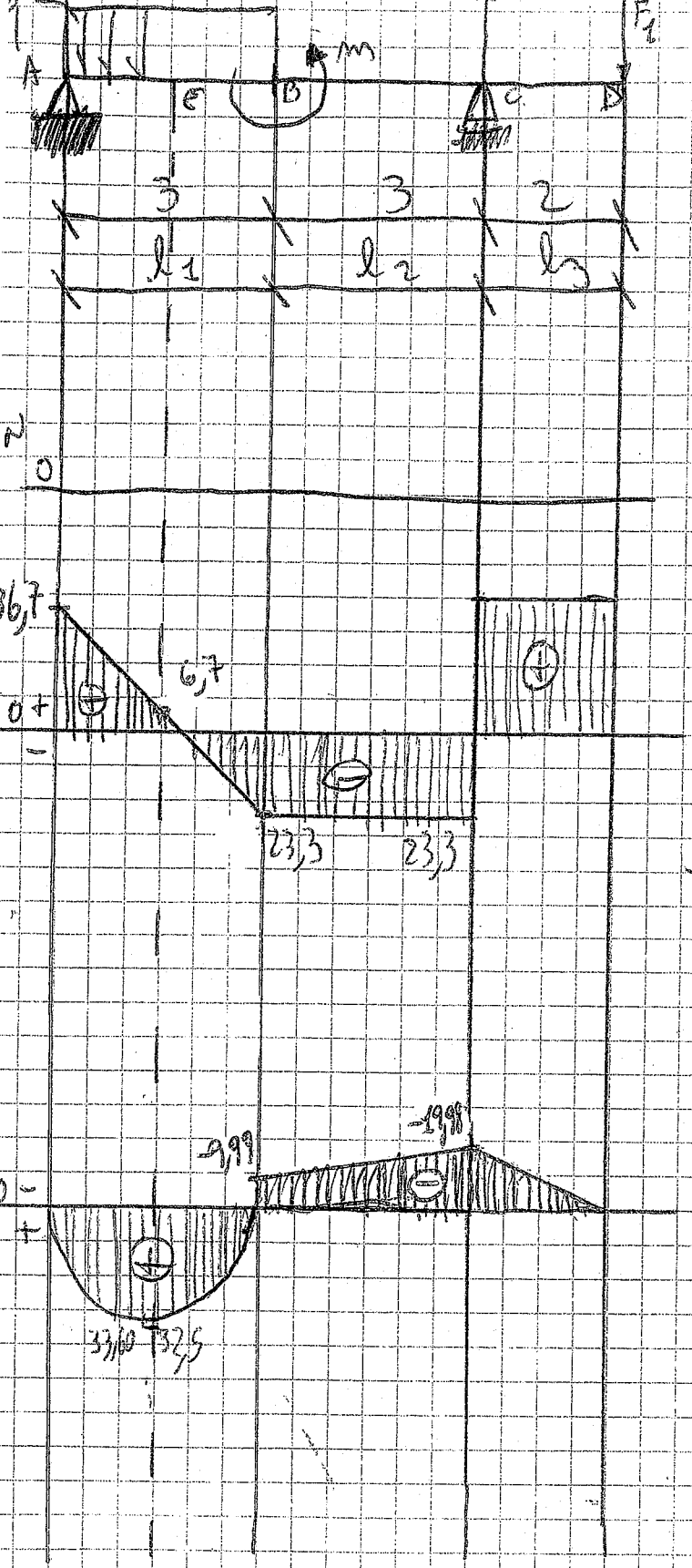
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_E = 0 \end{cases}$$

Nel caso in esame, tenendo conto della seguente convenzione dei segni si ha:

$$\begin{cases} R_{A,x} = 0 \\ R_{A,y} - Q + R_{C,y} - F = 0 \\ Q \cdot \frac{L_1}{2} - m - R_{C,y} \cdot (L_1 + L_2) + F \cdot (L_1 + L_2 + L_3) = 0 \end{cases}$$

L'equazione di equilibrio alla rotazione è stata effettuata rispetto al punto A

76



$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases}$$

$q = 20\ KN/m$

$m = 30\ KN \cdot m$

$F_1 = 40\ KN$

$$\begin{cases} R_{A1x} = 0 \\ R_{A1y} - q + R_{C1y} - F = 0 \\ q \cdot l \cdot \frac{l}{2} - m - R_{C1y}(l_1 + l_2) + F(l_1 + l_2 + l_3) = 0 \end{cases}$$

$$R_{C1y} = \frac{-90 + 30 - 320}{6} = -63{,}3\ KN$$

$$R_{A1y} = q \cdot l - R_{C1y} + F = 36{,}7\ KN$$

SCALA TAGLIO

1 □ = 10 KN

$N = 0$

$T = 0 = R_A - q \cdot l = \frac{R_A}{q} =$

$= 1{,}83\ m$

$T_A = R_A = 36{,}7\ KN$

$T_E = R_A - q \cdot \frac{l_1}{2} = 6{,}7\ KN$

$T_B = R_A - q \cdot l = -23{,}3\ KN$

$T_C = R_A - q \cdot l + 63{,}3 = 40\ KN$

$T_{F_1} = 0\ KN$

$M_A = 0$

$M_E = R_A \cdot 1{,}5 - q \cdot \frac{1{,}5}{2} \cdot 0{,}75 = 32{,}5\ KN$

$M_{B(S)} = R_A \cdot 3 - q \cdot l \cdot \frac{l}{2} + 0{,}9 = -9{,}99\ KN$

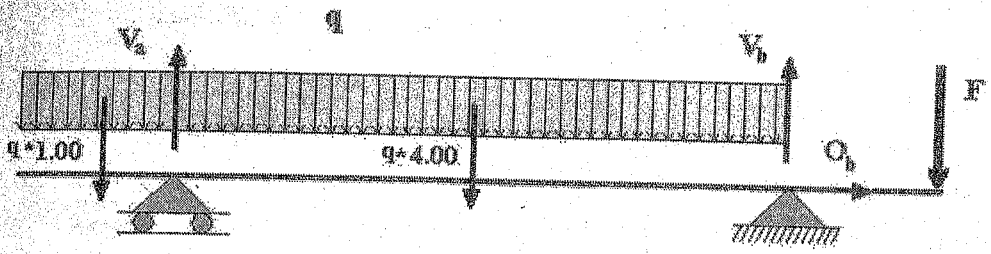
$M_B = R_A \cdot 3 - q \cdot l \cdot \frac{l}{2} - m = -19{,}98\ KN$

$M_C = R_A \cdot 6 - q \cdot l \cdot 4{,}5 - m =$

$M_{D(S)} = R_A \cdot 8 - (20 \cdot 6{,}5) - 30 + 63{,}3 = 0$

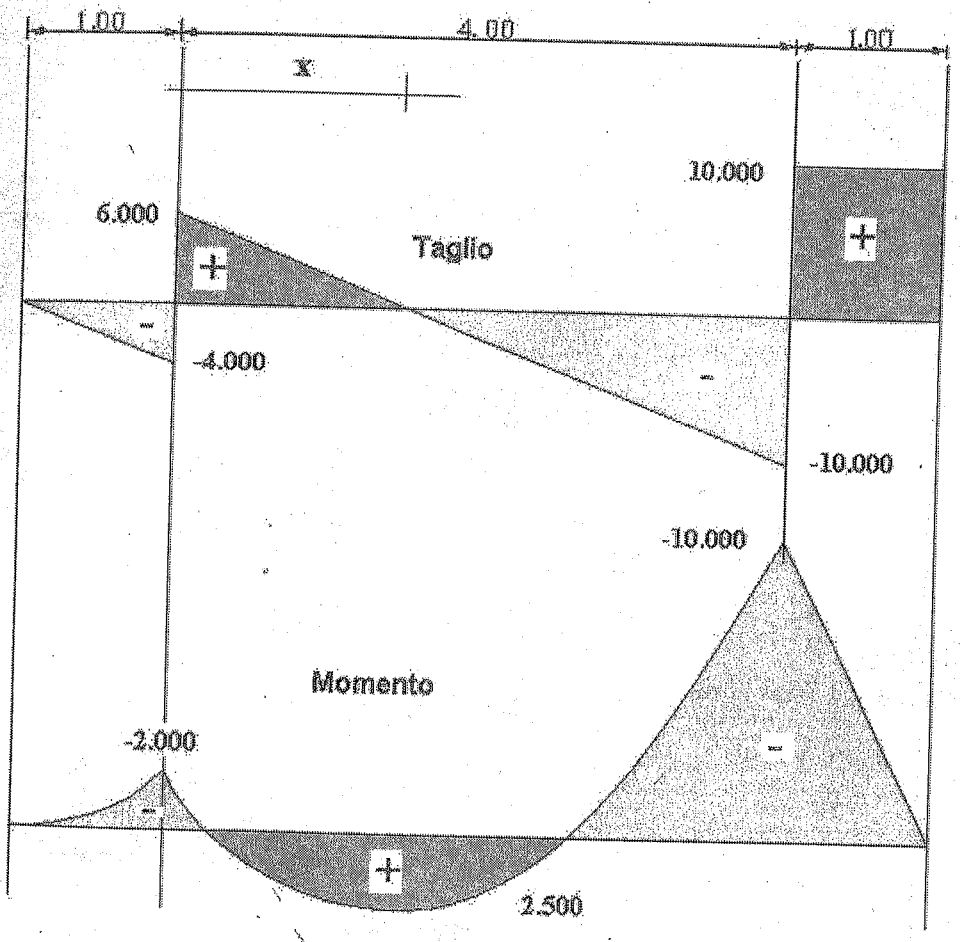
$M_{max} = 1{,}83 \cdot a \cdot \frac{1{,}83}{2} = 33{,}$

Esercizio N. 7 - trave appoggiata con sbalzi



q= 4.000 dN/m; 40 KN/m
F=10.000 dN; 100 KN

Convenzione dei segni



$$\begin{cases} V_a = 10.000 \ Kg \ 100 \ KN \\ V_b = 20.000 \ Kg \ 200 \ KN \\ O_b = 0 \ Kg \end{cases}$$

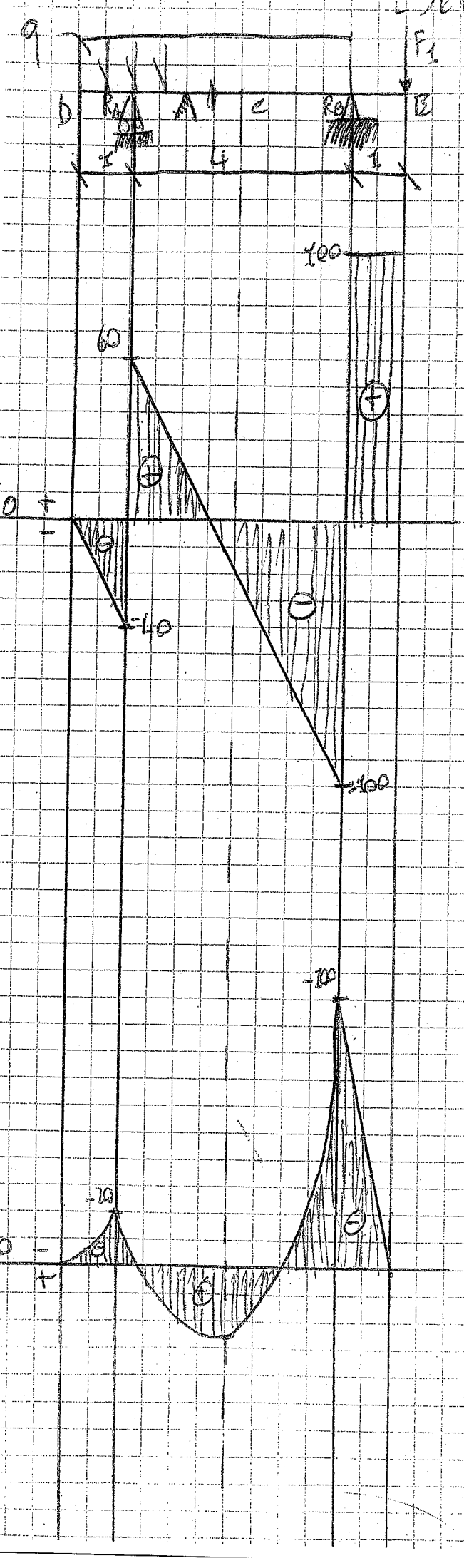
Calcoliamo i valori del taglio:

$$T_a^- = -q \cdot 1{,}00 = -4.000 \cdot 1{,}00 = -4.000 \ \ Kg$$

$$T_a^+ = -q \cdot 1{,}00 + V_a = -4.000 \cdot 1{,}00 + 10.000 = 6.000 \ \ Kg$$

$$T_b^- = -q \cdot 1{,}00 + V_a - q \cdot 4{,}00 = -4.000 \cdot 1{,}00 + 10.000 - 4.000 \cdot 4{,}00 = -10.000 \ \ Kg$$

$$T_b^+ = +F = +10.000 \ \ Kg \qquad T_F^- = +F = +10.000 \ \ Kg$$



$q = 40 KN/m$

$F_1 = 100 KN$

$R_A = 100 KN$

$R_B = 200 KN$

TAGLIO

SCALA 1 □ = 10KN

$T_A = -q \cdot 1 = -40 \cdot 1 = -40 KN$

$T_A^+ = -q \cdot 1 + V_A = -40 \cdot 1 + 100 = 60 KN$

$T_B = -q \cdot 1 + V_A - q \cdot 4 = -40 \cdot 1 + 100 - 40 \cdot 4 = -100 KN$

$T_B^+ = +F = 100 KN$

$T_F^- = +F = 100 KN$

MOMENTO

SCALA 1 □ = 10KN

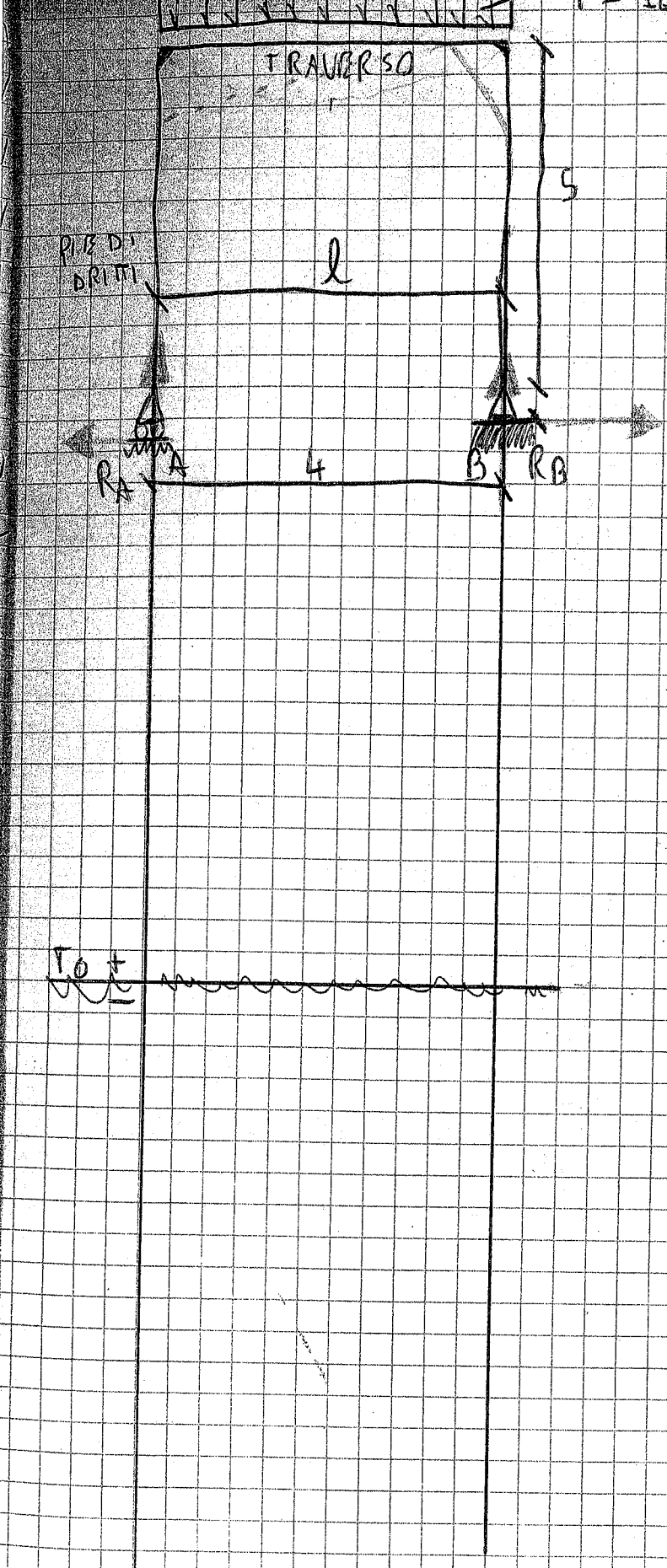
$\Pi_A = 0$

$\Pi_{RA} = -q \cdot l \cdot \frac{l}{2} = -20 KN$

$\Pi_C = -q \cdot l \cdot \frac{l}{2} + R_A \cdot 2,5 = 25 KN$

$\Pi_{RB} = -q \cdot l \cdot \frac{l}{2} + R_A \cdot 4 = -100 KN$

$\Pi_B = 0$

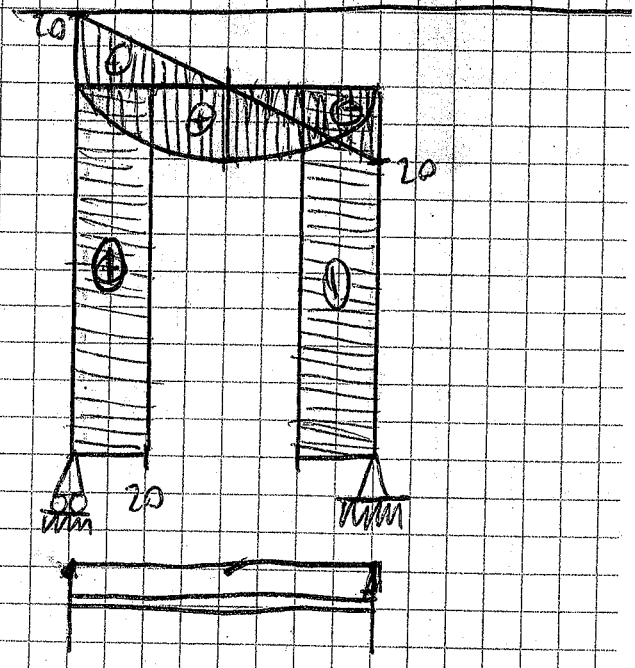


$$\begin{cases} \Sigma F_x = 0 \Rightarrow \nexists \\ \Sigma F_y = 0 \Rightarrow \\ \Sigma M = 0 \end{cases}$$

$$\begin{cases} \Sigma F_y = +R_A - q \cdot l + R_B = 0 \\ \Sigma M_B = R_B \cdot 0 + q \cdot l \left(\frac{l}{2}\right) - R_A \cdot 4 = 0 \end{cases}$$

$R_A = 20$

$R_B = 20$



$T_A = R_A = 20\,kN$

$T_C = R_A - q \cdot \frac{l}{2} = 20 - 20 = 0$

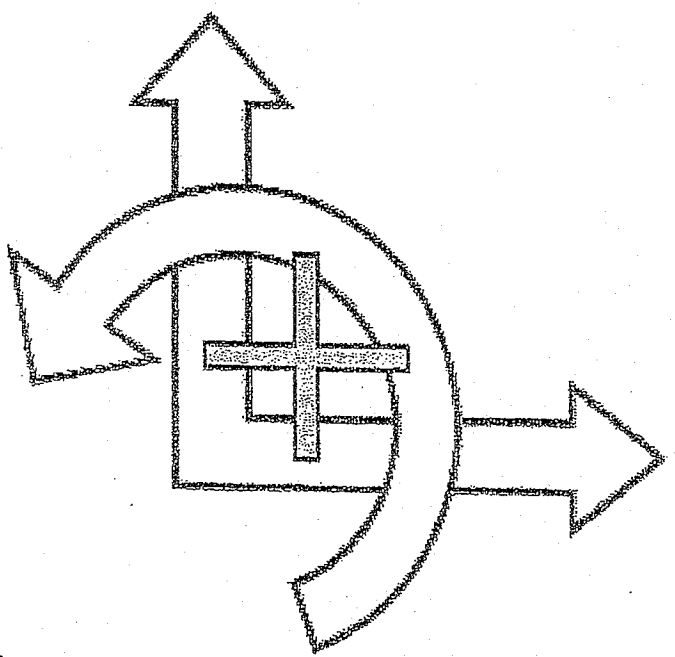
$M_A = 0$

$M_C = -R_A \cdot 2 + q \cdot \frac{l}{2} \cdot \frac{l}{4} = -20$

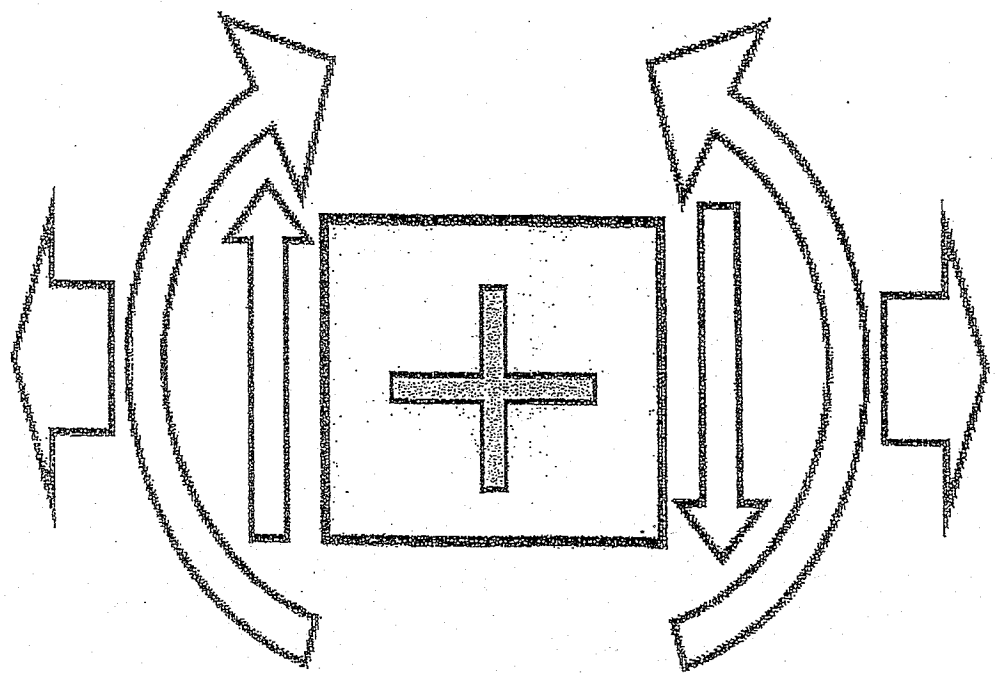
Studio delle Travature Isostatiche

4 - 1
4 - 2
4 - 3

Convenzioni sui segni:

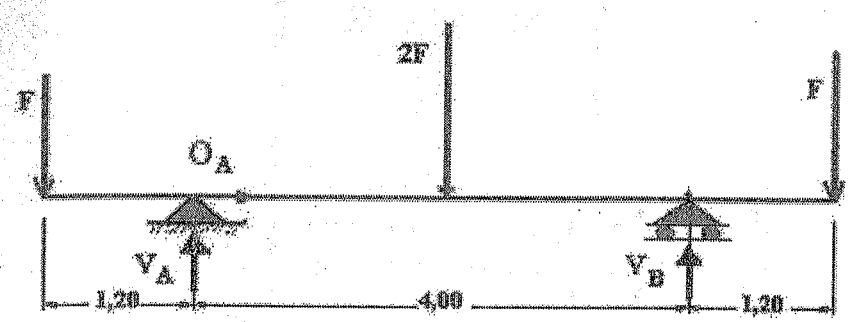


Forze e Momenti

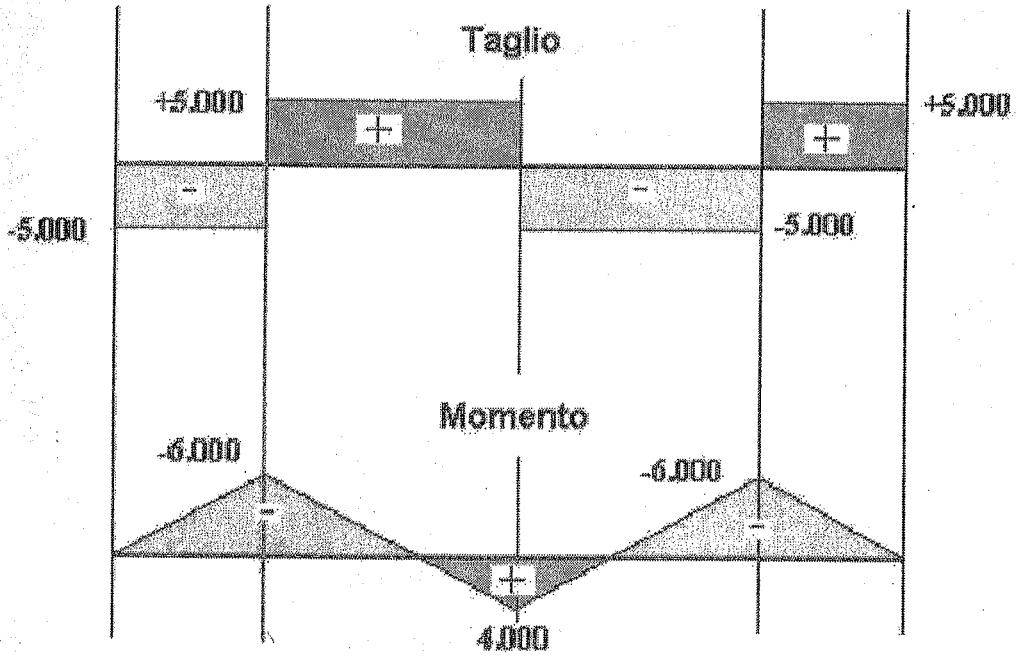


Trazione, Taglio e Momento flettente

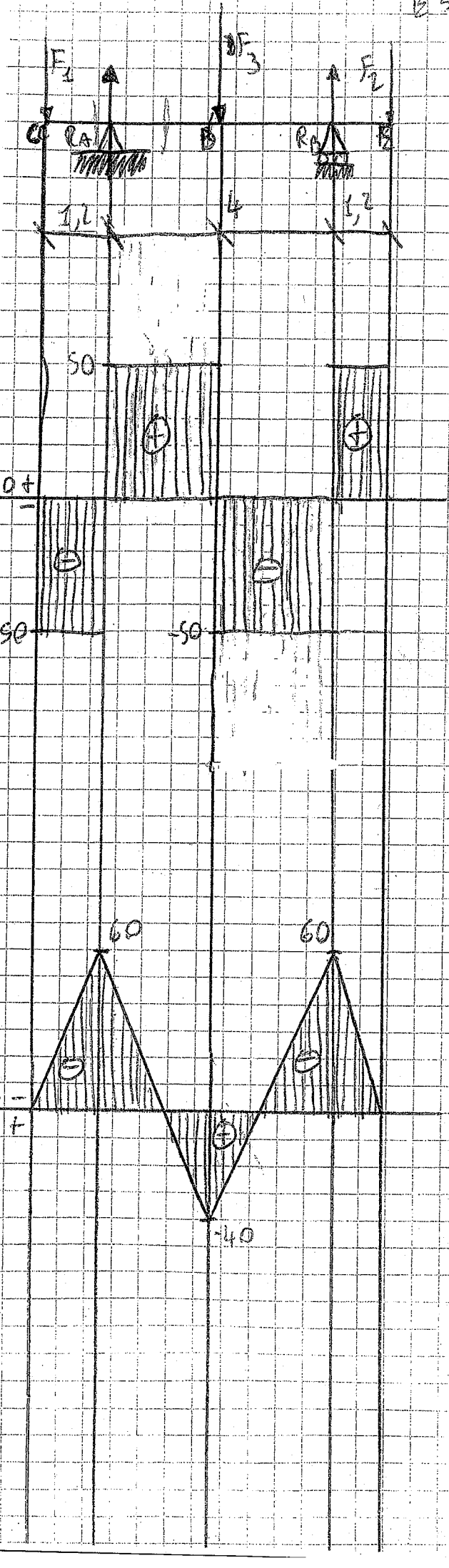
Esercizio N. 8 - Trave appoggiata con due sbalzi



Convenzione dei segni



F = 5.000 dN;
O_a=0;
V_a=10.000 dN ;
V_b=10.000 dN



SCALA

TAGLIO 1 □ = 10 KN

$T_C = F_1 = 50\,KN$

$T_A = F_1 - R_A = -50\,KN$

$T_D = F_1 - R_A + F_3 = 50\,KN$

$T_B = F_1 - R_A + F_3 = -50\,KN$

$T_E = F_1 - R_A + F_3 + R_B = 50$

$R_A = 100\,KN$

$R_B = 100\,KN$

$F_1 = 50\,KN$

$F_2 = 50\,KN$

$F_3 = 100\,KN$

MOMENTO SCALA 1 □ = 10 KN

$M_C = 0$

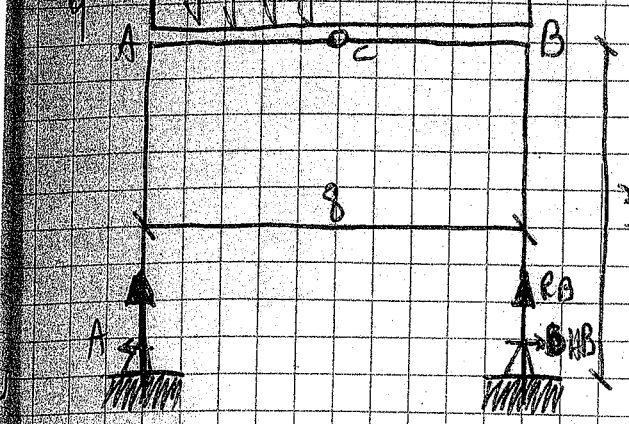
$M_A = F_1 \cdot 1,20 = 60\,KN \cdot m$

$M_D = F_1 \cdot 3,2 - R_A \cdot 2 = 160 - 200 = -40\,KN \cdot m$

$M_B = F_1 \cdot 5,2 - R_A \cdot 4 + F_3 \cdot 2 = 60\,KN \cdot m$

$M_E = 0$

83



$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \\ \Sigma M_C = 0 \end{cases}$ PORTALE

$q = 30\,KN/m$

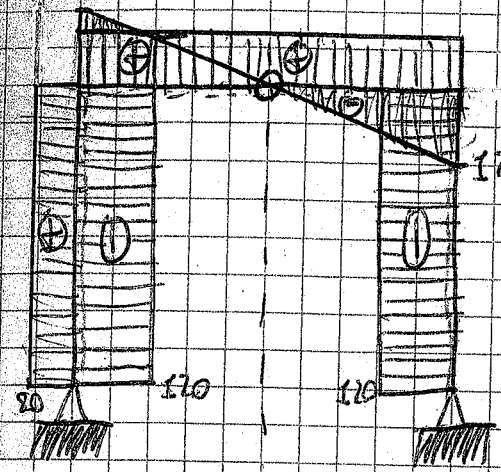
$R_A = 120KN$

$R_B = 120KN$

SCALA $1\square = 60KN$

$\Sigma M_C = (q \cdot 4 \cdot 2) + R_B \cdot 4 + H_B \cdot 3 =$

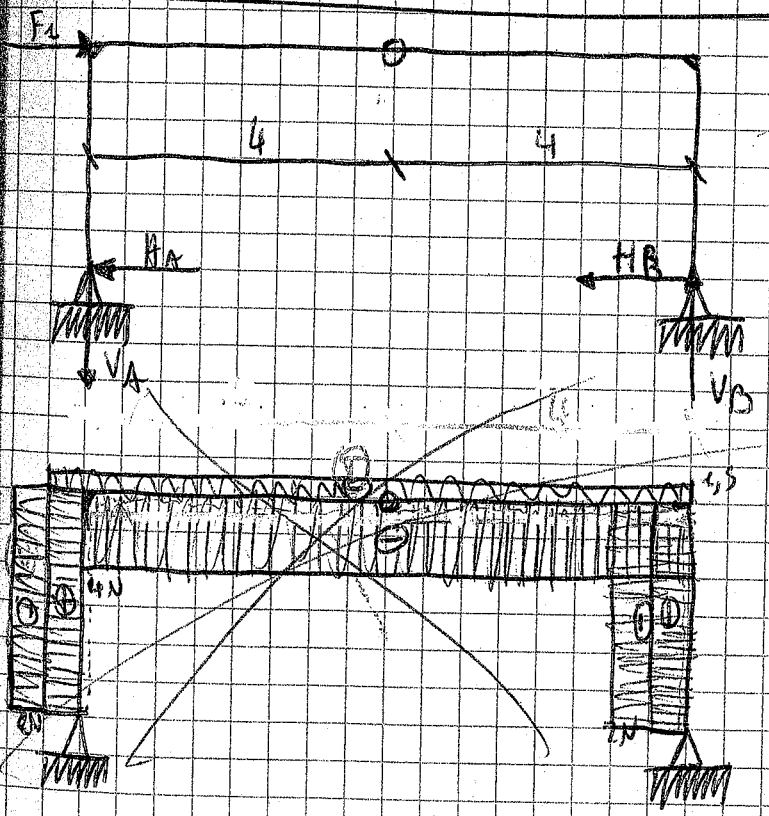
$H_B = \frac{-240+480}{3} = 80KN$



$T_A = 120KN$

$T_C = 120 - (q \cdot 4) = 0$

$T_B = -120KN$



$F_1 = 4KN$

$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M_A = 0 \end{cases} \begin{cases} 4N - H_A - H_B = 0 \\ -V_A + V_B = 0 \\ \Sigma M_A = -F_1 \cdot 3 + V_B \cdot 8 = 0 \\ \Sigma M_C = -H_A \cdot 3 + V_A \cdot 4 = \end{cases}$

$V_B = \frac{12}{8} = 1,5N$

$V_A = -1,5N$

$H_A = \frac{-6}{3} = 2N$

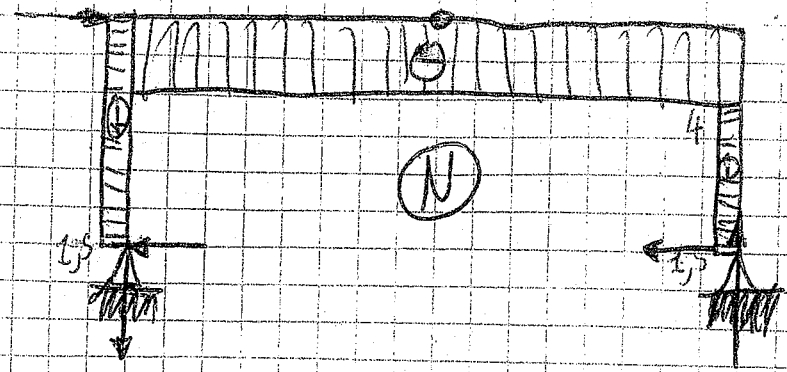
$H_B = -4+2 = 2N$

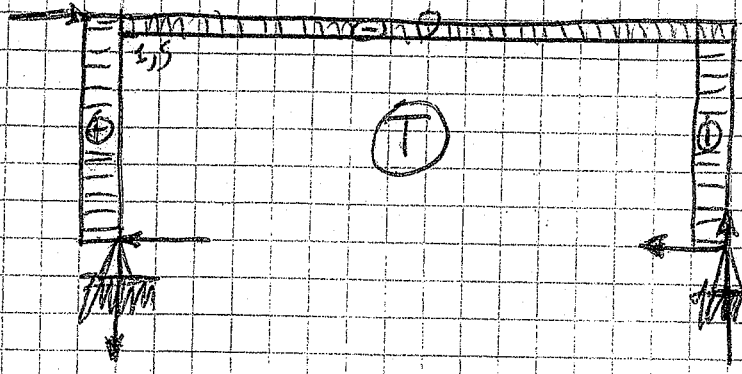
SCALA $1\square = 2N$

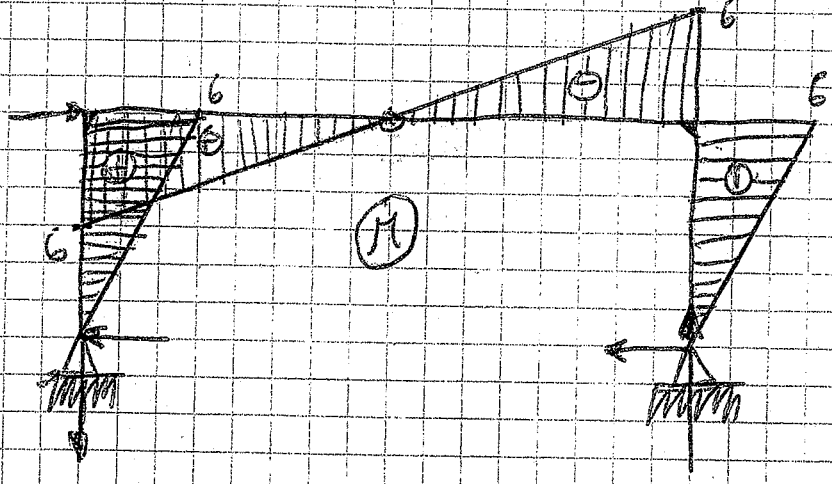
$\Pi_A = \frac{-2}{2} \cdot 3 + 2 \cdot 3 + 1,5 = 0$

84

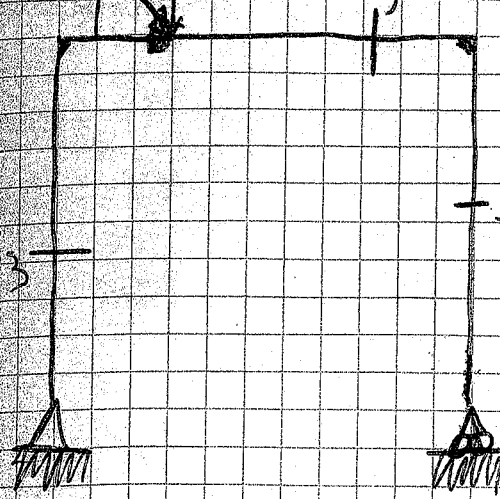
SCALA 1 □ = 2 N

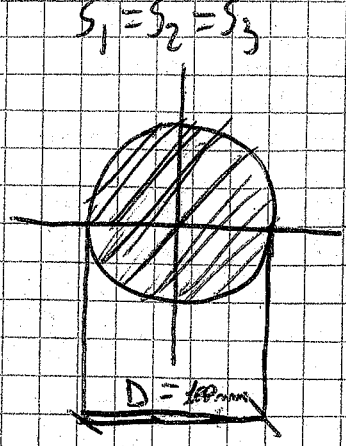






85





$A = \frac{\pi}{4} \cdot D^2$

$N_t = +200\,KN$

$N_{p_1} = -50\,KN$

$N_{p_2} = -180\,KN$

σ = tensione unitaria nel materiale di trazione o compressione

$\sigma = \frac{N}{A}$

$F_x = F \cdot \cos 60°$

$F_y = F \cdot \sin 60°$

$\pi r^2 = \pi \cdot \left(\frac{D}{2}\right)^2 =$

$= \pi \cdot \frac{D^2}{4} =$

$\sigma = \frac{N}{A} = \frac{200KN}{\frac{\pi}{4} \cdot 100^2} = 0{,}025\,KN/mm^2 \Rightarrow 25\,N/mm^2$

S235 = ferro meno costoso N/mm^2

S275

S355

S450

ACCIAIO (TRAZIONE, COMPRESS

LEGNO (TRAZIONE, COMPRESSI

DIPENDE DAL LEGNO

cls (TRAZIONE)

C.a. (TRAZIONE, COMPRESSIONE)

19/04/20

SOLLECITAZIONI

SFORZO NORMALE (COMPRESSIONE, TRAZIONE)

~~COMPRESSIONE~~

~~TRAZIONE~~ TAGLIO

FLESSIONE (MOMENTO FLETTENTE)

TORSIONE

TENSIONE

Sforzi che ogni particella della struttura tende a resistere

$\sigma_{ammissibile} = \frac{\sigma_{lim}}{gs}$ = sigma rottura / grado di sicurezza (1,05 ÷ 5-10)

$\sigma < \sigma_{limite}$

ES

CIAIO = S235 ⇒ $\frac{\sigma_s}{\gamma_s} = \frac{235}{1,05}$

S275

S355

S450

347 28 60 903

ACCIAIO	S235 $\sigma_{AM} = 223,8\ N/mm^2$ S275 $\sigma = 261,9\ N/mm^2$ S355 $\sigma_{AMMISSIBILE}$
LEGNO =	
LEGNO ⊥	
MURATURA COMPRESSIONE	
σ_b	

→ ⊥ FIBRE

COMPRESSIONE

ACCIAIO	γ_s	$\sigma_{AMMISSIBILE}$
S235	1,05	235/1,05 = 223,8
S275	1,05	275/1,05 = 261,9
355	1,05	355/1,05 = 338,09
LEGNO		
σ_{comp} ⊥		8 N/mm²
σ_{comp} //		2 N/mm²
FLESSIONE		10 N/mm²
TRAZIONE		9 N/mm²
MURATURE		
PROTON	1,5	1,6 N/mm²
UFO	0,8	0,8 N/mm²
σ_b		5,6 N/mm²

87

ESERCIZIO: Si calcoli la struttura di un pilastro in: acciaio, legno, muratura in tufo che deve resistere a un carico di 2500 KN alla compressione, si consideri inizialmente le seguenti dimensioni: acciaio sezione rettangolare

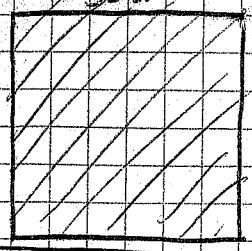
ACCIAIO

$\sigma_{AMMISSIBILE} = \frac{235}{1,05} = 223,8\ N/mm^2$

$A = 200 \cdot 100 = 20000\ mm^2$

$\sigma = \frac{N}{A} = \frac{2500 \cdot 2000\ N}{20000\ mm^2} = 125\ N/mm^2$

LEGNO



~~TRAZIONE~~ $\sigma_{AMMISSIBILE} = 8\ N/mm^2$

$A = 300 \cdot 300 = 90000\ mm^2$

$\sigma = \frac{N}{A} = \frac{2500 \cdot 10^3}{90 \cdot 10^3} = 27,7\ N/mm^2$ *

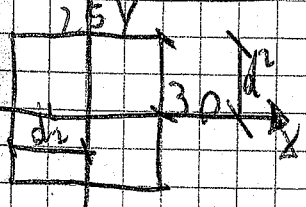
PILASTRO TUFO

$\sigma_{AMMISSIBILE} = 0,8\ N/mm^2$

$A = 500 \cdot 450 = 225000\ mm^2$

$\sigma = \frac{N}{A} = \frac{2500 \cdot 10^3}{225000} = 11,1\ N/mm^2$

* $\sigma = \frac{N}{A} = \frac{2500 \cdot 10^3}{600 \cdot 600} = 6,9\ KN/mm^2$



CARICO DI PUNTA

$H = 3,00\ m \quad P = 50\ KN$

$\sigma = \frac{P}{A} \leq \sigma_{AMMISSIBILE}$

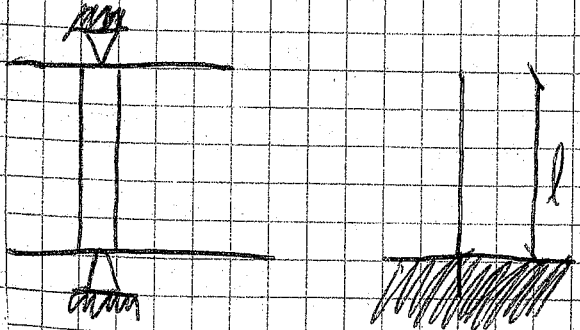
PILASTRI < 25 cm NO λ = snellezza / lambda $= \frac{l_0}{\rho_{(ro)}}$ (lunghezza libera di inflessione)

STRUTTURALI = CARICHI

$\rho = \sqrt{\frac{I}{A}}$

$I_x = A \cdot d^2$

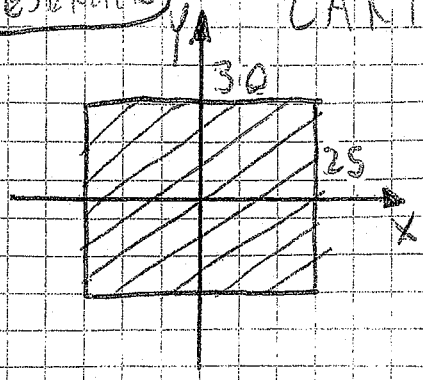
$I_y = A \cdot d^2$



88

1 ESERCIZIO

CARICO DI PUNTA (INSTABILITÀ DEL PILASTRO)



CALCESTRUZZO

I2 LEGNO

I4 d_5

$H_{MAX} < I4$

$N = 50\,KN$

$l = 3\,m$ $\quad \sigma_{AMMISSIBILE} = 5,2\,N/mm^2$

$\lambda = \frac{l_0}{\rho_{(min)}} = l_0 = l$

$\rho = \sqrt{\frac{I_{MINIMO}}{A}} = \sqrt{\frac{39062,5}{750}} = 7,22\,cm$

$I_x = \frac{1}{12} \cdot 25 \cdot 30^3$

$I_y = \frac{1}{12} \cdot 30 \cdot 25^3 = cm^4$

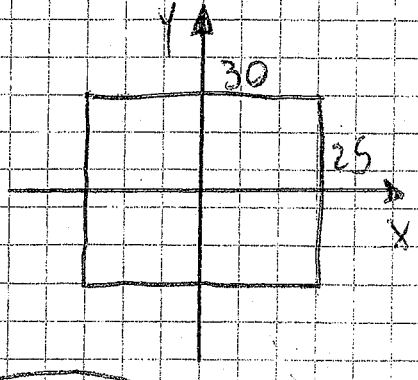
$\lambda = \frac{l_0}{\rho} = \frac{300}{7,22} = 41,55$ ADIMENSIONALE

$\omega = 0$

$\sigma = \frac{P}{A} = \frac{50000}{75000} = 0,7\,N/mm^2$

2 ESERCIZIO

CALCESTRUZZO



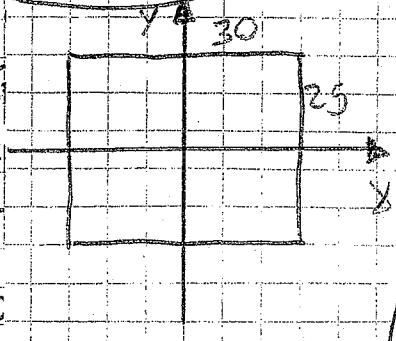
$l = 4\,m$

$\lambda = \frac{400}{7,22} = 55,40 \approx 56 \Rightarrow \omega = 1,02$

$\sigma = 0,7\,N/mm^2$

$\sigma = \frac{P}{A} \cdot \omega = 0,714$

3 ESERCIZIO



$\sigma_{AMMISSIBILE} = 8\,N/mm^2$

$l = 4\,m$

$x = \frac{(\omega_{MAX} - \omega_{MIN}) \cdot (\lambda_{CALC.} - \lambda_{MIN})}{(\lambda_{MAX} - \lambda_{MIN})} =$

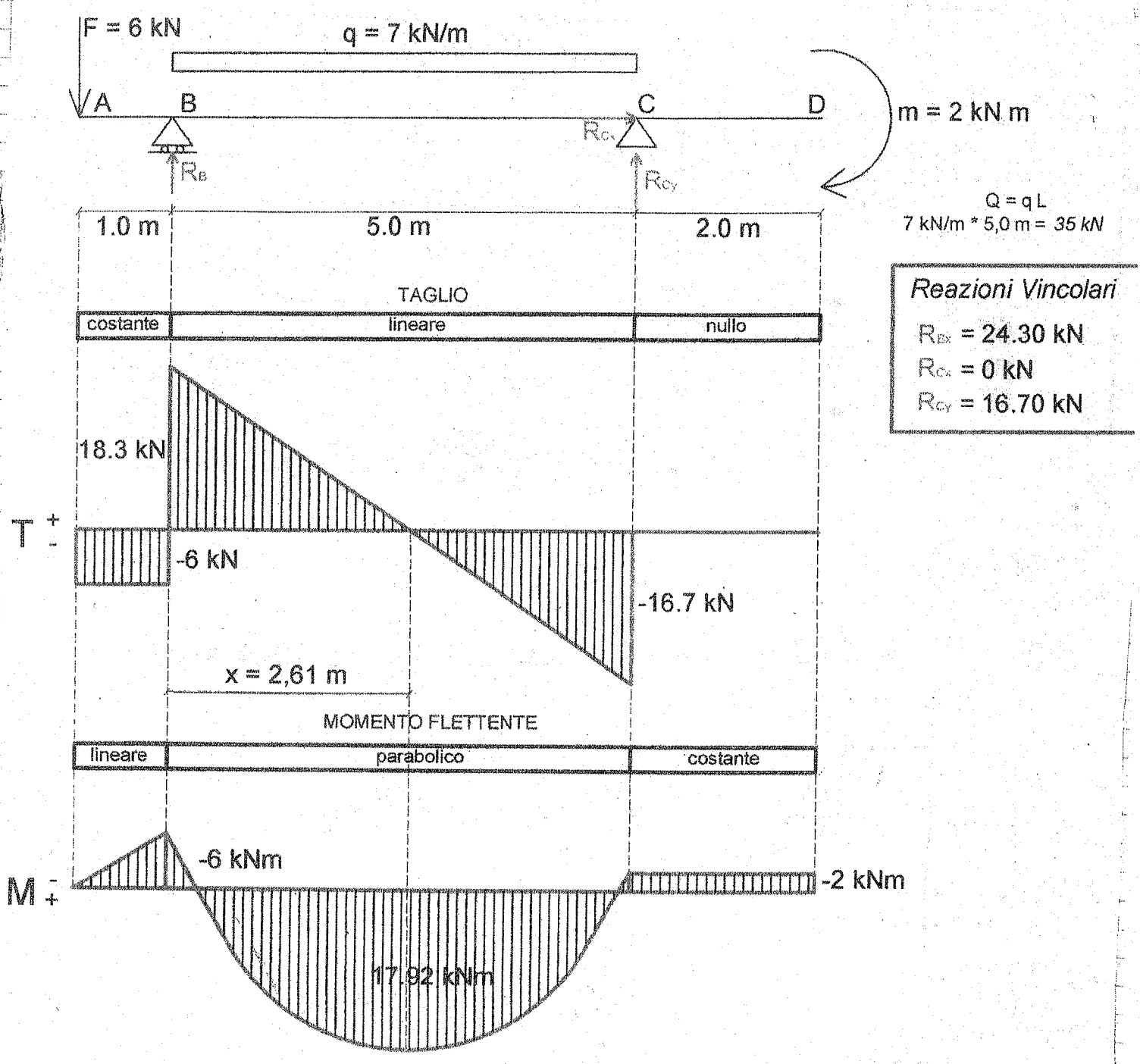
$= \frac{(1,62 - 1,42) \cdot (56 - 50)}{(60 - 50)} = \frac{0,26}{10} = 0,12 +$

$+ 1,42 = 1,54 \Rightarrow \omega = \text{omega}$

$\omega = 1,54 \quad \sigma = \frac{P \cdot \omega}{} = 1,02 < \sigma_{AMMISSIBILE}$

LEGNO MASSICCIO

TRAVE



Q = q L
7 kN/m * 5,0 m = *35 kN*

Reazioni Vincolari

R_{Bx} = 24.30 kN
R_{Cx} = 0 kN
R_{Cy} = 16.70 kN