A.S. 2016/2017 — ALTAMURA

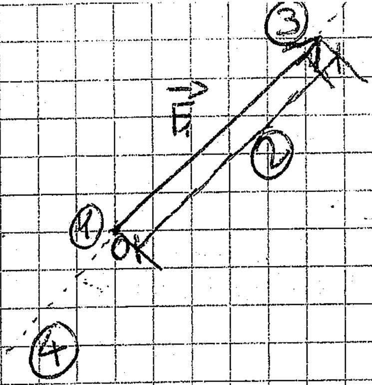
ALFABETO GRECO

APPUNTI DI COSTRUZIONI 3°B ITG

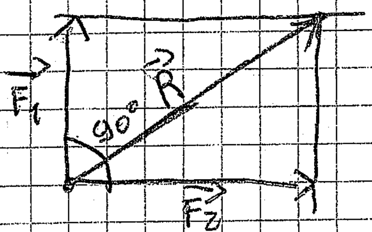
Maiuscole	minuscole	pronuncia	traslitterazione
A	α	alfa	a
B	β	beta	b
Γ	γ	gamma	g *(dura)*
Δ	δ	delta	d
E	ϵ	epsilon	é *(e chiusa)*
Z	ζ	zeta	z
H	η	eta	è *(e aperta)*
Θ	θ	theta	th
I	ι	iota	i
K	κ	cappa	c *(dura)*
Λ	λ	lambda	l
M	μ	mi *o* mu	m
N	ν	ni *o* nu	n
Ξ	ξ	csi	x
O	o	òmicron	ò *(o aperta)*
Π	π	pi	p
P	ρ	ro	r *(aspirata)*
Σ	σ	sigma	s
T	τ	tau	t
Y	υ	üpsilon	y $\rightarrow$ ü *(ue tedesca)*
Φ	ϕ	phi	ph $\rightarrow$ f
X	χ	chi	ch *(aspirata)*
Ψ	ψ	psi	ps
Ω	ω	oméga	ó *(o chiusa)*

1

VETTORE - GRANDEZZE VETTORIALI



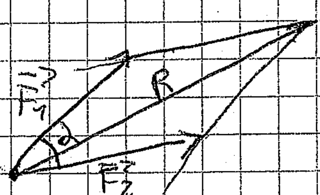
① PUNTO DI APPLICAZIONE
② MODULO O INTENSITÀ
③ DIREZIONE O VERSO
④ LINEA DI AZIONE O TRAETTORIA



$\vec{R}$ = Risultante

$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + \cancel{\vec{F_1}\cdot\vec{F_2}\cdot\underbrace{\cos 90°}_{0}}} =$$

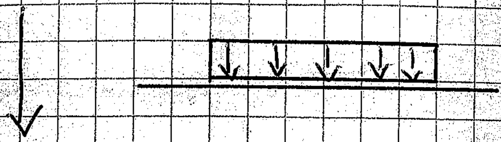
$$= \sqrt{\vec{F_1}^2 + \vec{F_2}^2} \longleftarrow \text{PITAGORA}$$



CARNOT

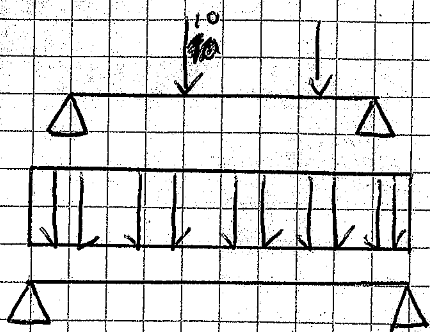
$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2\vec{F_1}\cdot\vec{F_2}\cos\alpha}$$

DISTRIBUITO



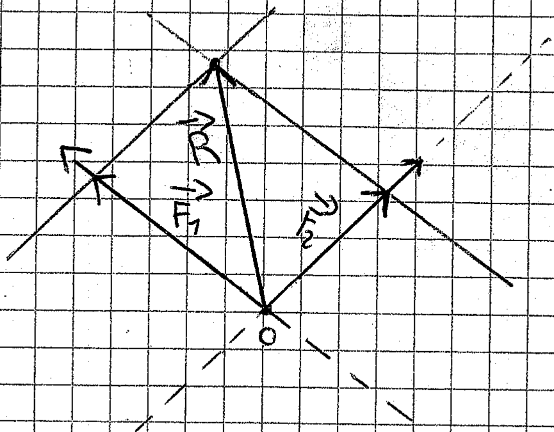
$P = m \cdot a$

$g = 9{,}81\ m/s^2$



CARICO PUNTUALE

CARICO UNIFORMEMENTE DISTRIBUITO



$\vec{F_1} + \vec{F_2} = ?$

$\vec{R} = \vec{F_1} + \vec{F_2}$

$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2\, F_1 \cdot F_2 \cdot \cos \alpha}$$

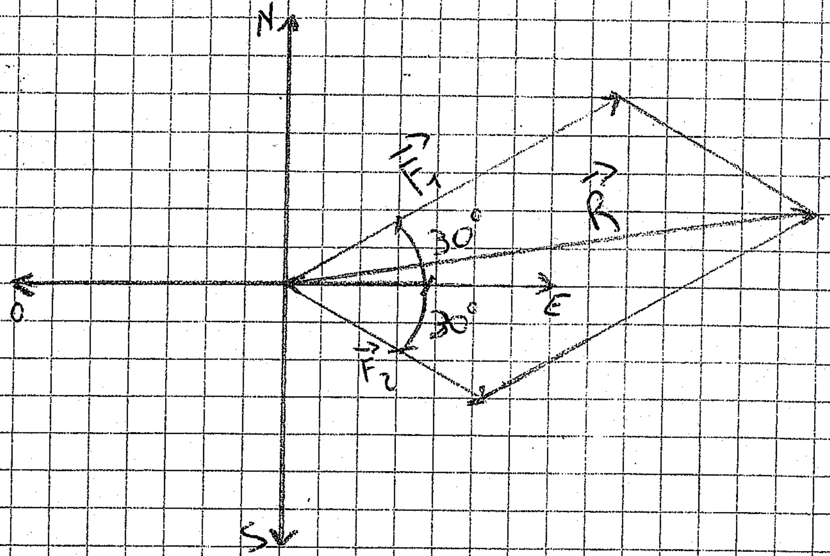
CAR NOT QUANDO

L'ANGOLO NON È 90°

ESERCITAZIONE IN CLASSE

$\vec{F_1} = 10\,N$ (N.E.) $\vec{F_2} = 6\,N$

SCALA → 1cm = 2N



$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2\vec{F_1}\cdot\vec{F_2}\cdot\cos\alpha}$$

$$= \sqrt{100 + 36 + 2(10\cdot 6)\,0{,}5}$$

$$= \sqrt{100 + 36 + \cancel{60}\,120\,(0{,}5)}$$

$$= \sqrt{256 \cdot 0{,}5}$$

$$= \sqrt{128} = \boxed{14\,N}$$

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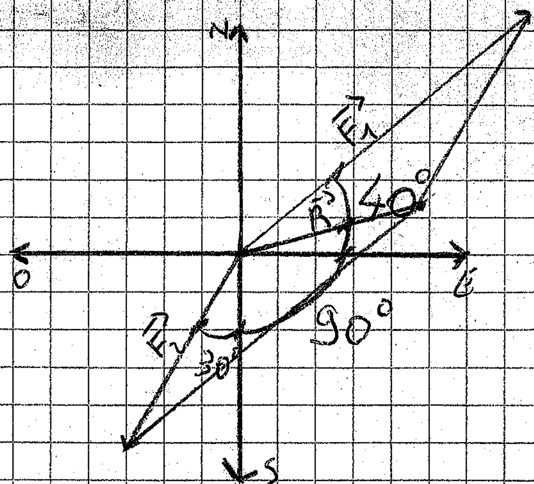
① $\vec{F_1} = 10\,N \quad \vec{F_2} = 6\,N$

SCALA -> 1 cm = 2 N



$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2\vec{F_1}\cdot\vec{F_2}\cdot\cos\alpha}$$

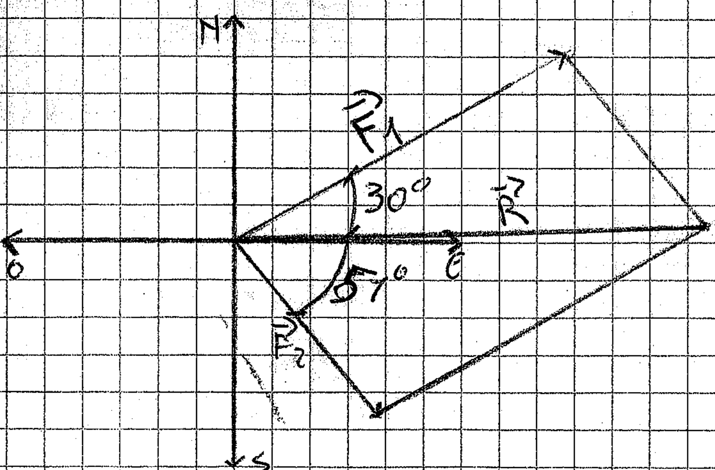
$$= \sqrt{100 + 36\cancel{0} + 2\cdot 10\cdot 6\cdot(0,9397)}$$

$$= \sqrt{136 + 120\cdot(-0,9397)}$$

$$= \sqrt{136 - 112,764} = \sqrt{23,236} = \boxed{4,8\,N}$$

② $\vec{F_1} = 10\,N \quad \vec{F_2} = 6\,N$

SCALA -> 1 cm = 2 N



$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2\vec{F_1}\cdot\vec{F_2}\cdot\cos\alpha}$$

$$= \sqrt{100 + 36 + 2\cdot 10\cdot 6\cdot 0,16}$$

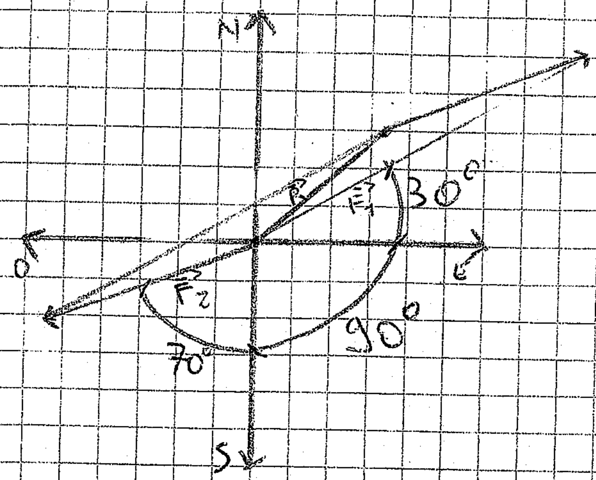
$$= \sqrt{136 + 120\cdot 0,16}$$

$$= \sqrt{136 + 19,2} = \sqrt{155,2} = \boxed{12,5\,N}$$

5

(3) $\vec{F_1} = 10\,N$ $\vec{F_2} = 6\,N$

SCALA -> 1cm =



$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2\,\vec{F_1} \cdot \vec{F_2} \cdot \cos\alpha}$$

$$= \sqrt{100 + 36 + 2 \cdot 10 \cdot 6 \cdot (-0{,}98)}$$

$$= \sqrt{136 + 120 \cdot (-0{,}98)}$$

$$= \sqrt{136 - 117{,}6} = \sqrt{18{,}4} = \boxed{4{,}3\ N}$$

UNITA DI MISURA FORZA

NEWTON (N) FORZE

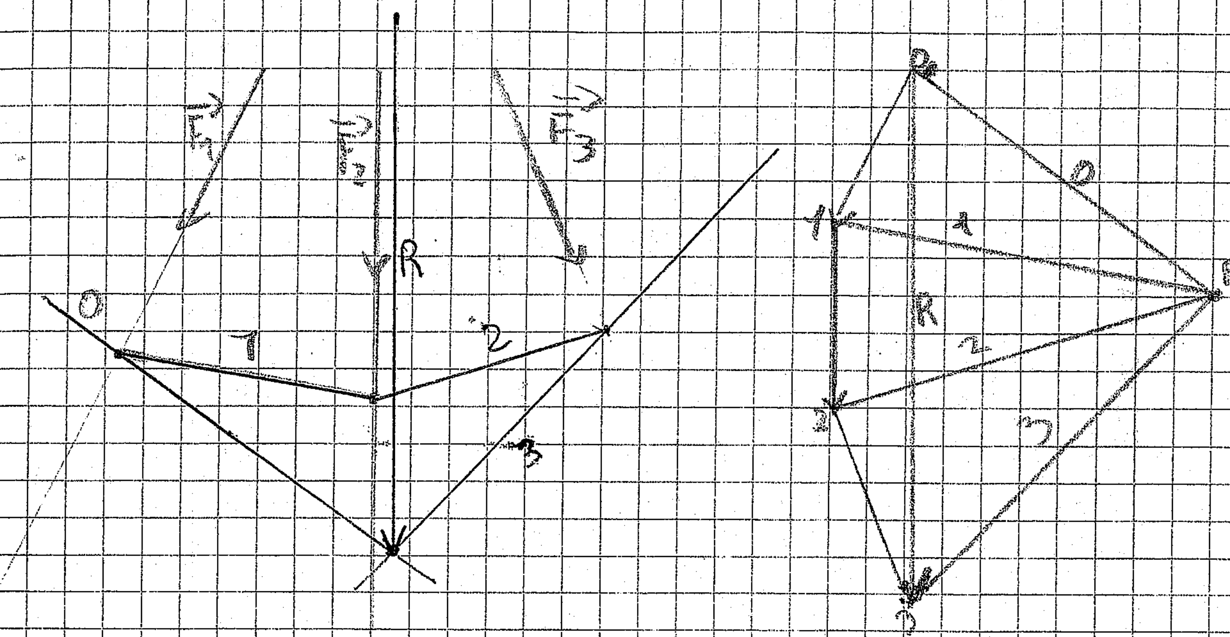
DECIMA PARTE DEL Kg ~~9,81 N 0,98~~
$1\,N = 0{,}981\,Kg \Rightarrow 0{,}1\,Kg$
$1\,Kg \Rightarrow 9{,}81\,N = 10\,N$

1 daN (deca)	10 N
1 KN	1000 N
1 MN	1000000 N
1 GN	1 000 000 000 N

1 N	~~=~~ 0,1
1 da N	= 10 N
1 KN	= 1000 N
1 MN	= 1000000 N
1 GN	= 1000000000 N

POLIGONO FUNICOLARE

È un sistema di forze (serie di vettori con direzioni e inclinazioni varie), serve per sapere quanto vale e da dove passa la risultante.



POLIGONO FUNICOLARE

COS'È

Costruzione grafica: serve per trovare, con procedimento unico, il vettore risultante $\overline{R}$ *di due o più vettori del piano; nel caso di due soli vettori è preferibile il metodo del parallelogramma*

L'alternativa al metodo del poligono funicolare è l'applicazione ripetututa del metodo del parallelogramma. Questa alternativa, però, non è efficace perché è lunga (per sommare 5 vettori devo applicare il metodo 4 volte) e rende il foglio illeggibile (a causa delle numerose linee di costruzione).

VETTORE

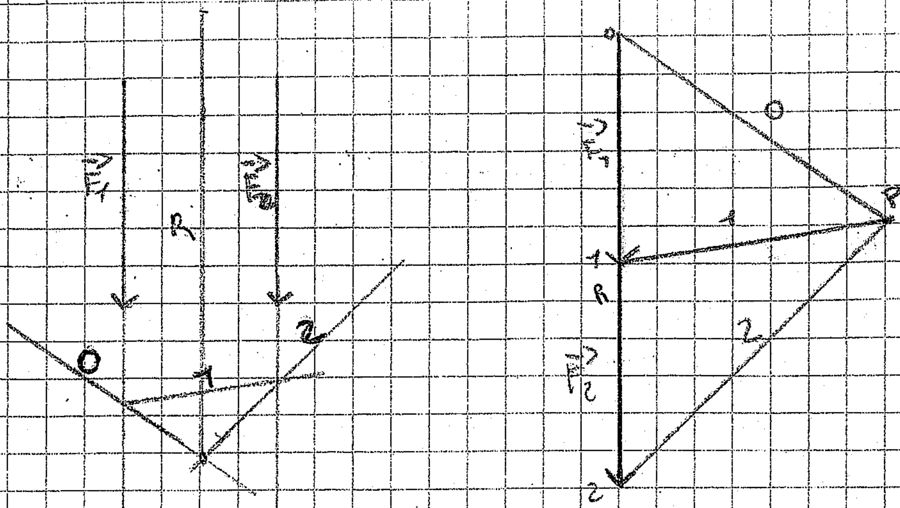
Un **vettore** rappresenta graficamente una "grandezza vettoriale", cioè una grandezza che, per essere completamente definita, necessita di intensità, direzione e verso (Per esempio lo spostamento è un vettore, la temperatura non è un vettore). Un vettore si disegna con una freccia. Va sempre indicata sul disegno la scala per la lettura dell'intensità.

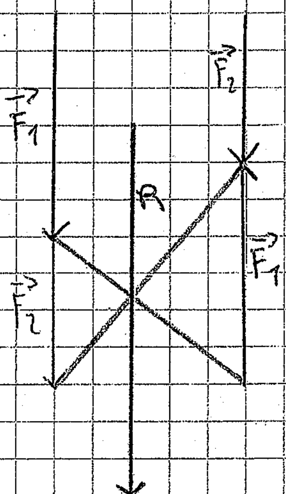
VETTORE RISULTANTE o RISULTANTE

Il vettore **risultante**, detto anche "la risultante", è quello da solo ha lo stesso effetto fisico dei vettori del sistema dato. Pertanto esso può essere usato come sostituto dell'intero sistema di vettori. (Esempio: spostamento di un passeggero rispetto al ponte della nave composto con il vettore spostamento della nave danno il vettore risultante, che è lo spostamento effettivo del passeggero rispetto al mare)

METODO (con riferimento alla figura)

1. Tracciare le rette d'azione dei vettori e assegnare il nome ai vettori e alle loro rette d'azione (a,b,c....)
2. Sulla parte destra del foglio costruire il poligono dei vettori (con il metodo cosiddetto "punta coda")
3. La risultante $\overline{R}$ (provvisoria) inizia nel punto di applicazione **P** del primo vettore e termina nella punta **Q** dell'ultimo vettore
4. Scegliere un punto **H** a destra (a distanza "ottimale") e tracciare le "funi" (sempre una più dei vettori): la terza fune, III, per esempio, collega il punto H con l'inizio del vettore tre
5. Dare i nomi alle funi (con numero romano) e cerchiare la prima e l'ultima fune
6. In alto a sinistra rispetto al sistema dei vettori originali scegliere un punto **J** , a distanza "ottimale"
7. Dal punto **J** condurre la prima fune (parallela a quella disegnata a destra !) fino ad incontrare la retta del primo vettore nel punto **A**. Scrivere il numero romano della fune.
8. Dal punto **A** condurre la seconda fune fino ad incontrare la retta del secondo vettore nel punto **B**. Scrivere il numero romano della fune.
9. Si prosegue analogamente con i punti **C**, **D** .... fino a che l'ultima fune non ha nulla da incontrare
10. La prima e ultima fune, contrassegnate, vengono fatte intersecare nel punto **K**
11. Nel punto **K** si conduce la retta parallela alla retta della risultante $\overline{R}$, trovata precedentemente
12. La risultante $\overline{R}$ può essere applicata in qualsiasi punto di questa retta





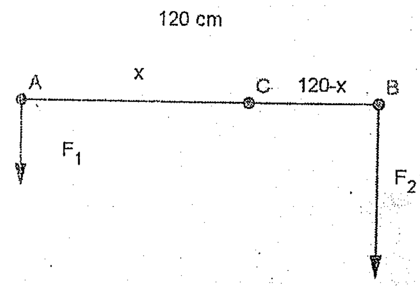
$\vec{F}_1 = 100\,N$

$\vec{F}_2 = 80\,N$

$\vec{R} = \vec{F}_1 + \vec{F}_2 = 100N + 80N = 180N$

Esercizi CLASSE 3B ITG ALTAMURA 06/Ottobre/2016

Due forze parallele $\vec{F}_1$ e $\vec{F}_2$ hanno modulo rispettivamente di 30 N e 50 N. La distanza tra le loro rette di azione è pari a 120 cm. Determina il punto di applicazione C della forza risultante e il suo modulo quando $\vec{F}_1$ e $\vec{F}_2$ sono concordi e il punto di applicazione C' della forza risultante e il suo modulo quando $\vec{F}_1$ e $\vec{F}_2$ sono discordi. Quanto valgono , infine, i momenti delle forze $\vec{F}_1$ e rispettivamente ai punti C e C'?



F1 e F2 concordi

F1= 30 N F2= 50 N d=120 cm

F=F1+F2=30+50=80N

Il centro C si trova all'interno delle due forze e si x = C A

$$F_1b_1 = F_2b_2 \quad 30x = 50(120-x) \quad 30x = 6000-50x \quad 80x = 6000 \quad x = \frac{6000}{80} = 75cm$$

$$b_1 = 75cm \quad b_2 = 45cm.$$

$$M = F_1b_1 = F_2b_2 = 0,75 \cdot 30 = 22,5Nm$$

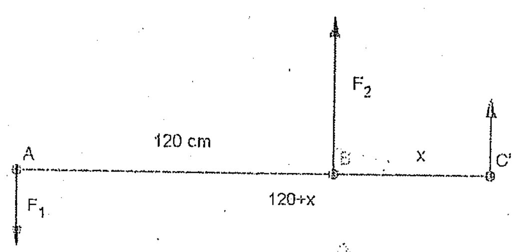
F1 e F2 discordi

Il centro C' si trova all'esterno di F2 e sia x = C'B

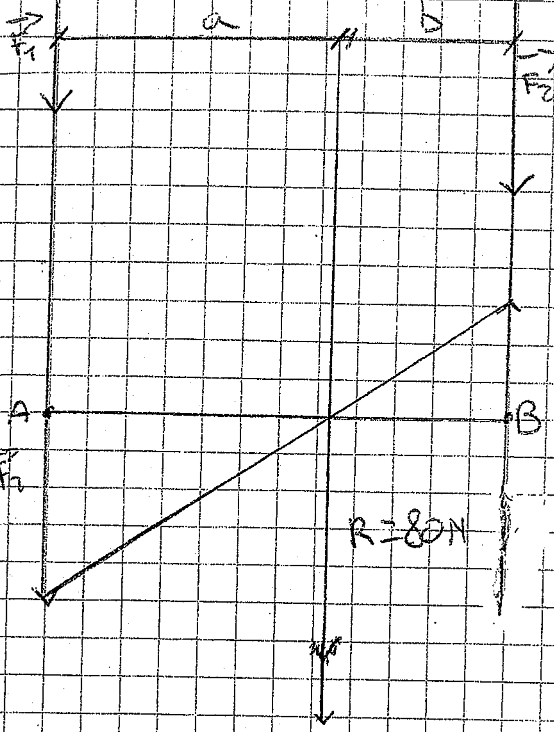
F=F2-F1=50-30=20N

$$F_1b_1 = F_2b_2 \quad (120+x)30 = 50(x) \quad 360+x30 = 50x \quad 360 = 20x \quad x = 180cm$$

$$b_1 = 300cm \quad b_2 = 180cm \quad M = F_1b_1 = F_2b_2 = 3 \cdot 30 = 90Nm$$



F1 e F2 DISCORDI



DATI

$\vec{F_1} = 30N$

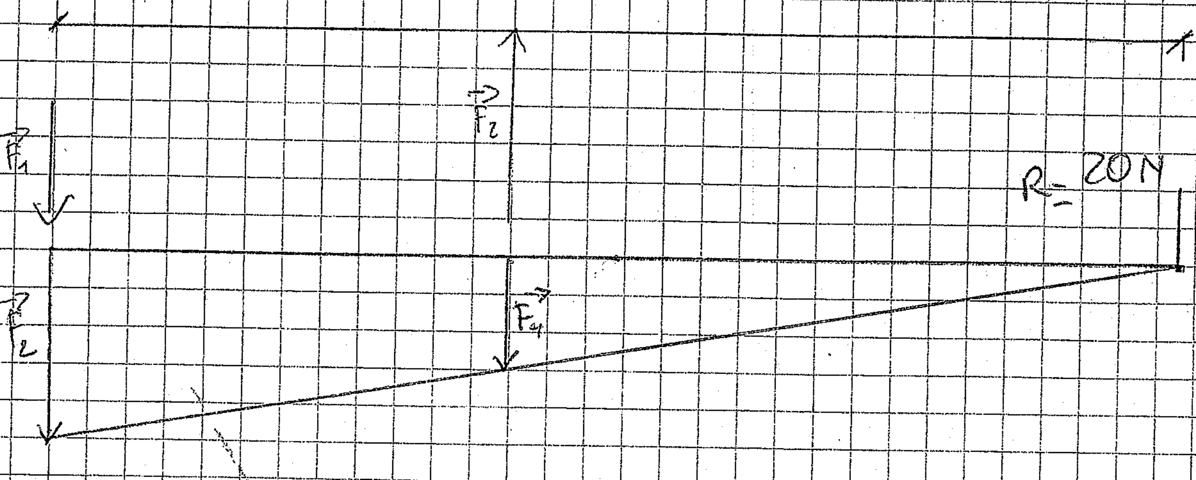
$\vec{F_2} = 50N$

$d = 120\,cm$

$F_2 \cdot a = F_1 \cdot b$

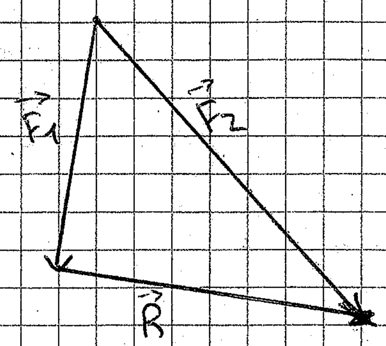
$b = \dfrac{F_1 \cdot d}{R} \quad a = \dfrac{F_2 \cdot d}{R}$

$b = \dfrac{30 \cdot 120}{80} = 45\,cm \quad a = \dfrac{50 \cdot 120}{80} = 75\,cm$



$a = \dfrac{F_1 \cdot d}{\underbrace{F_2 - F_1}_{R}}$

DIFFERENZA DI VETTORI



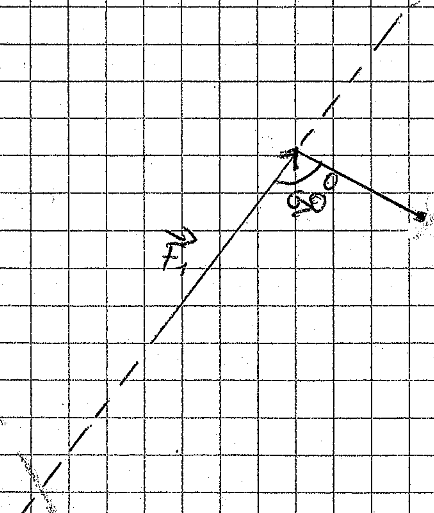
$$\vec{R} = \vec{F_1} - F_2$$

I MOMENTI

Il momento è una grandezza vettoriale definito come il prodotto tra forza per braccio

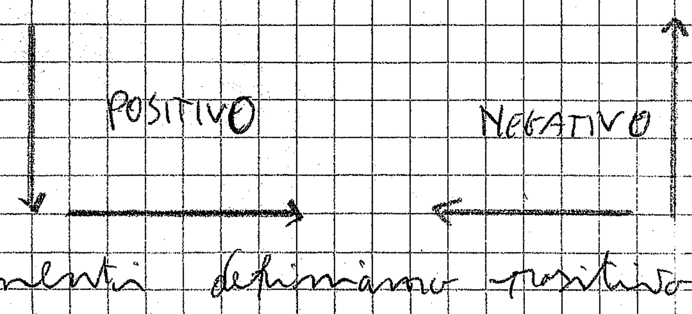
$$\boxed{M = \vec{F} \cdot b = N \cdot m}$$

Qualsiasi forza applicata in un punto produce un momento come in un punto qualsiasi in un piano.



I MOMENTI POSSONO ESSERE POSITIVI O NEGATIVI

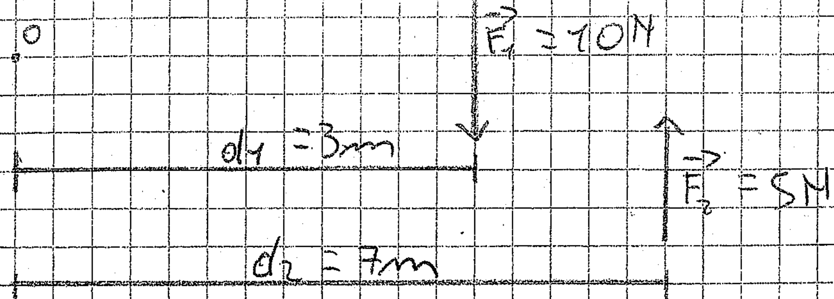
PER CONVENZIONE DEFINIAMO LE FORZE



Per i momenti definiamo positivo il momento orario e ~~positivo~~ negativo quello antiorario.

$M = \sum M$

Sommatoria



$M = F_1 \cdot d_1 + (-F_2 \cdot d_2)$

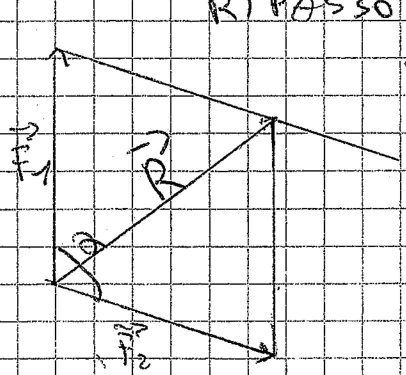
$M = 10 \cdot 3 - 5 \cdot 7 = 30 - 35 = -5 \text{ N} \cdot m$

COMPITO 26 OTTOBRE

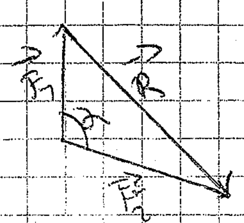
VETTORI, SOMME E DIFFERENZE DI VETTORI, SCOMPOSIZIONE DEI VETTORI, TEOREMA DI PITAGORA E CARNOT, FORZE PARALLELE CONCORDI E DISCORDI, POLIGONO FUNICOLARE, MOMENTI (VARIGNON)

RIPASSO

SOMMA E CARNOT



$\vec{R} = \sqrt{\vec{F}_1^2 + \vec{F}_2^2 + 2F_1 \cdot F_2 \cdot \cos\alpha}$



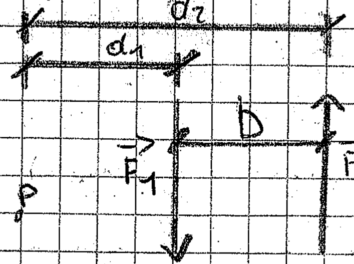
$\vec{R} = |\vec{F}_1| - |\vec{F}_2|$

DIFFERENZA E CARNOT

$\vec{R} = \sqrt{\vec{F}_1^2 + \vec{F}_2^2 - 2\vec{F}_1 \cdot \vec{F}_2 \cdot \cos\alpha}$

~~[illegible]~~ MOMENTI DI UNA COPPIA

Il momento di una coppia non varia rispetto a qualsiasi punto del piano.



$$\vec{M}_P = +\vec{F}_1 \cdot d_1 - \vec{F}_2 \cdot d_2$$

$$M_P = F(d_2 - d_1)$$

$$M = F \cdot b$$

Una coppia di Forza è costituita da 2 forze con lo stesso modulo e ~~dis~~ verso opposto.

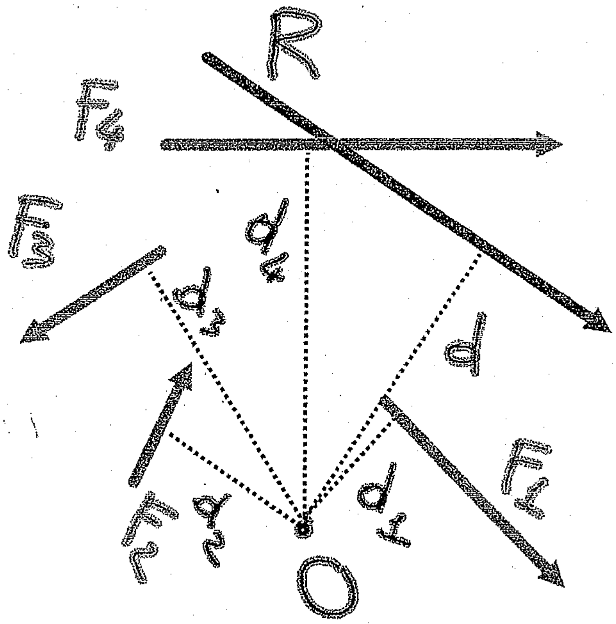
Una coppia di Forza rispetto a qualsiasi punto nel piano mi darà un momento che sarà sempre uguale a $M_P = F \cdot b$, dove F è il modulo della Forza

Teorema di Varignon 3ª B

Dato un sistema di forze complanari e scelto un punto nel piano, si può calcolare il momento di ciascuna forza e determinare il momento risultante ; ma i singoli momenti e il momento risultante devono soddisfare il teorema di Varignon, per il quale:

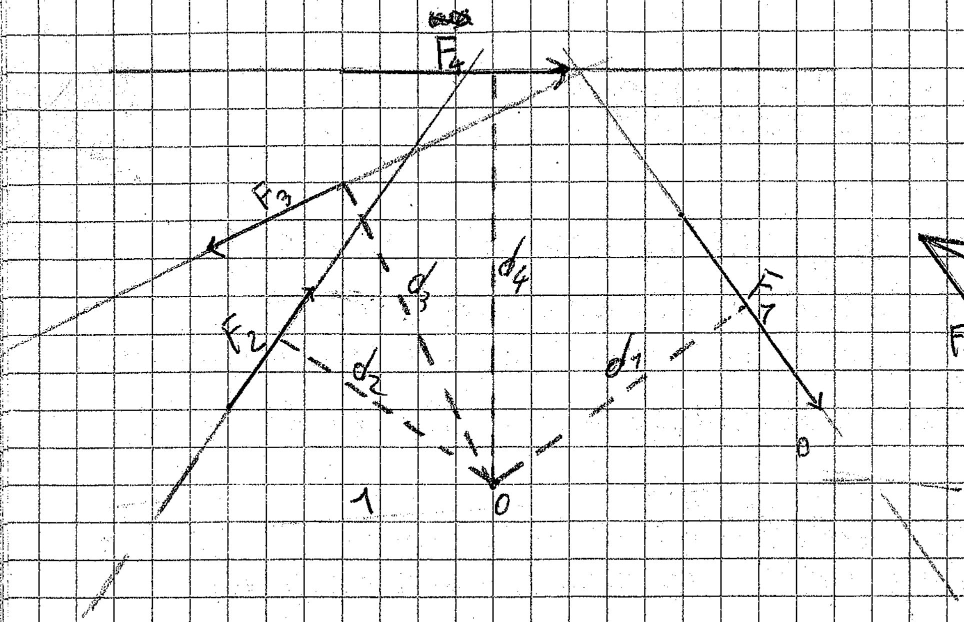
"In un sistema di forze complanari (sullo stesso piano) il momento della risultante , rispetto a un punto "O" qualsiasi nel piano, è uguale alla somma algebrica dei momenti delle singole forze rispetto al piano stesso."

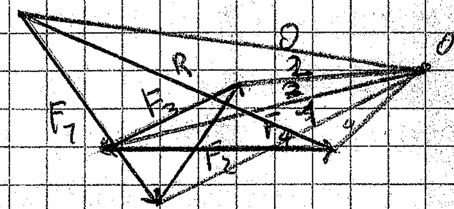
es. $F_1 \times d_1 \ldots$



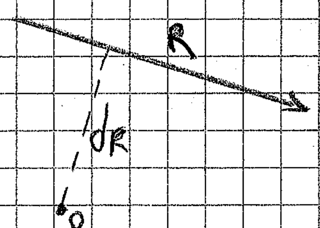
$$F_1 \times d_1 + F_2 \times d_2 + F_3 \times d_3 + F_4 \times d_4 = R \times d$$

$$d = \frac{F_1 \times d_1 + F_2 \times d_2 + F_3 \times d_3 + F_4 \times d_4}{R}$$

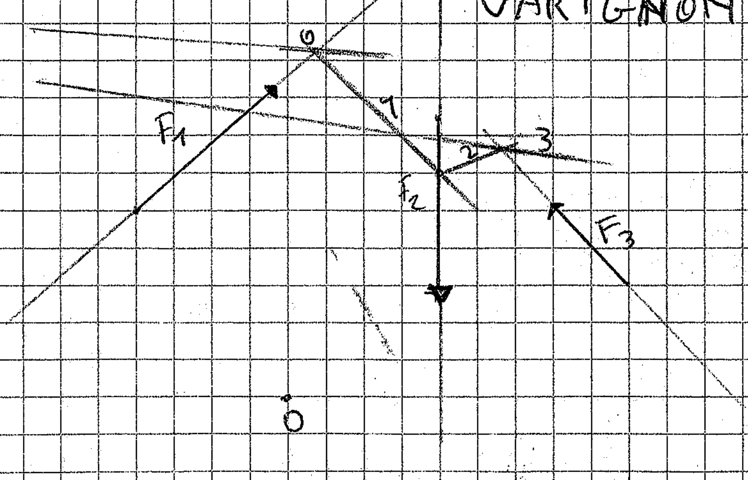




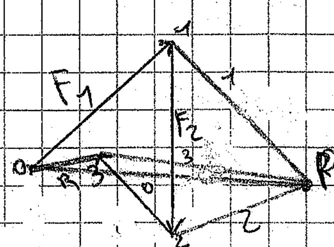
$$M_0 = (F_1 \cdot d_1) + (F_2 \cdot d_2) + (F_3 \cdot d_3) + (F_4 \cdot d_4) = R \cdot d_R$$



VARIGNON



1N = 1cm



R = ?

Passa R = ?

M_0 = ?

$$M_0 = R \cdot d$$

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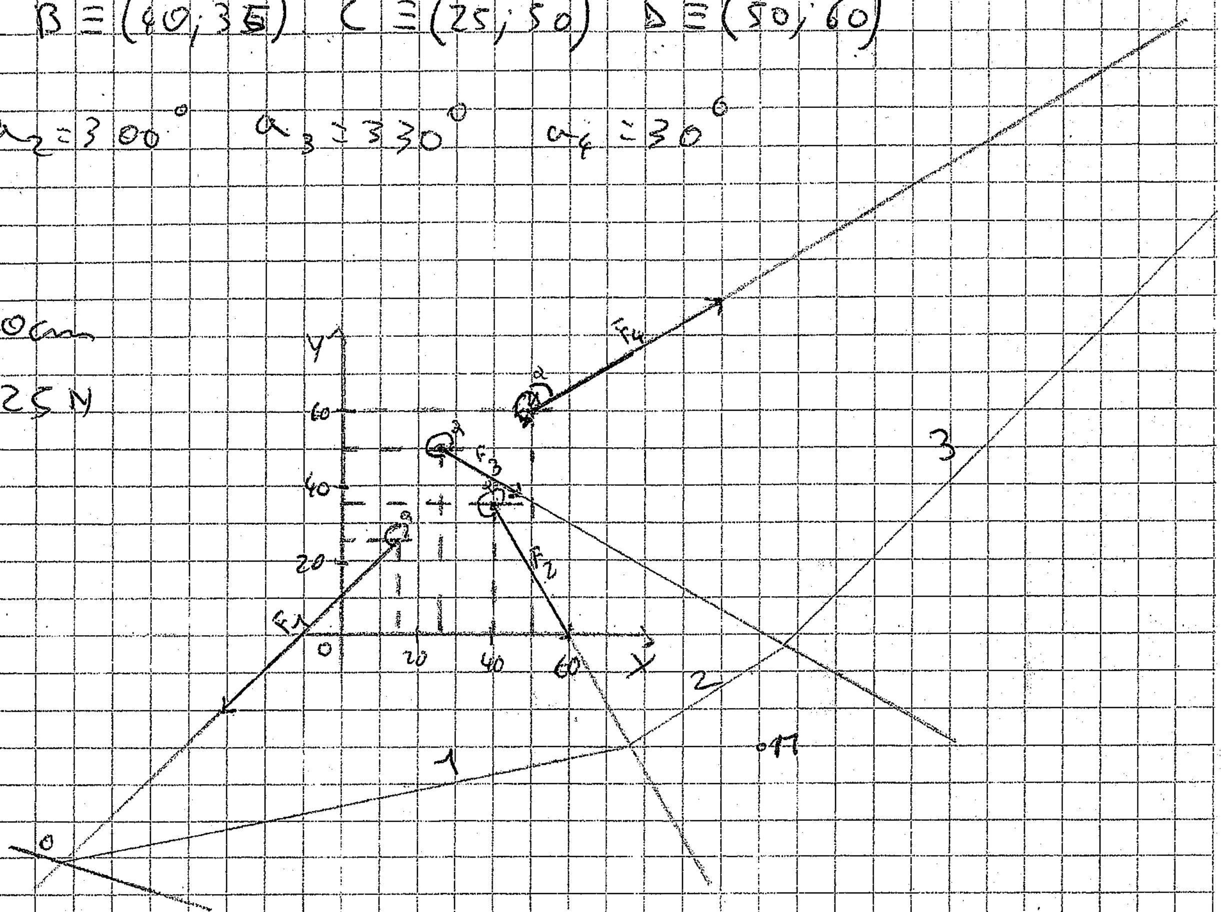
$F_1 = 80N \quad F_2 = 50N \quad F_3 = 30N \quad F_4 = 60N$

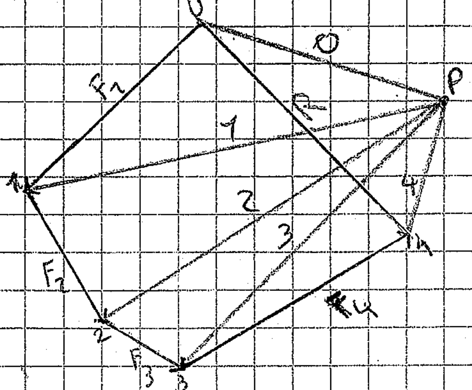
$A \equiv (15;25) \quad B \equiv (40;35) \quad C \equiv (25;50) \quad D \equiv (50;60)$

$\alpha_1 = 225° \quad \alpha_2 = 300° \quad \alpha_3 = 330° \quad \alpha_4 = 30°$

1cm ≙ 20cm

1cm ≙ 25N





COMPONENTI ORIZZONTALI

$F_{1x} = F_1 \cdot \cos\alpha_1 = 80 \times \cos 225° = -56,57 N$

$F_{2x} = F_2 \cdot \cos\alpha_2 = 50 \times \cos 300° = +25,00 N$

$F_{3x} = F_3 \cdot \cos\alpha_3 = 30 \times \cos 330° = +25,98 N$

$F_{4x} = F_4 \cdot \cos\alpha_4 = 60 \times \cos 30° = +51,96 N$

$R_x = \Sigma F_x = +46,37 N$

COMPONENTI VERTICALI

$F_{1y} = F_1 \cdot \mathrm{sen}\,\alpha_1 = 80 \cdot \mathrm{sen}\, 225° = -56,57 N$

$F_{2y} = F_2 \cdot \mathrm{sen}\,\alpha_2 = 50 \cdot \mathrm{sen}\, 300° = -43,30 N$

$F_{3y} = F_3 \cdot \mathrm{sen}\,\alpha_3 = 30 \cdot \mathrm{sen}\, 330° = -15,00 N$

$F_{4y} = F_4 \cdot \mathrm{sen}\,\alpha_4 = 60 \cdot \mathrm{sen}\, 30° = +30,00 N$

$R_y = \Sigma F_y = +84,37 N$

18

MODULO DELLA RISULTANTE

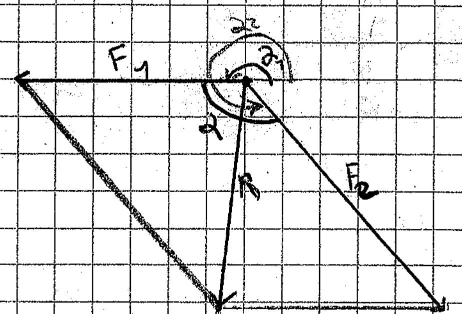
$R = \sqrt{R_x^2 + R_y^2} = \sqrt{46,47^2 + (-84,87)^2} \simeq 96,76\,N$

FORMA CON L'ORIZZONTALE UN ANGOLO DI:

$a = \operatorname{arctg} \frac{R_y}{R_x} = \operatorname{arctg} = \frac{-84,87}{+46,37} \approx \operatorname{arctg}(-1,83) \approx 61°,35$

es. 1 pag 33 fotocopia.

$1\,cm = 20\,N$



$R = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \cdot \cos\alpha} = \sqrt{9 + 16 + 24 \cdot (-0,6428)} = \sqrt{25 - 15,4272} =$

$= \sqrt{9,5728} = 3,09\,N$

$R = 3,09\,N \cdot 20\,N = \boxed{61,88\,N}$

$\boxed{\beta = 262°,03}$

n. 2

$F_1 = 12\ KN$ $F_2 = 8\ KN$ $F_3 = 18\ KN$

$A \equiv (40, 70)$ $B \equiv (20, -50)$ $C \equiv (-60, -20)$

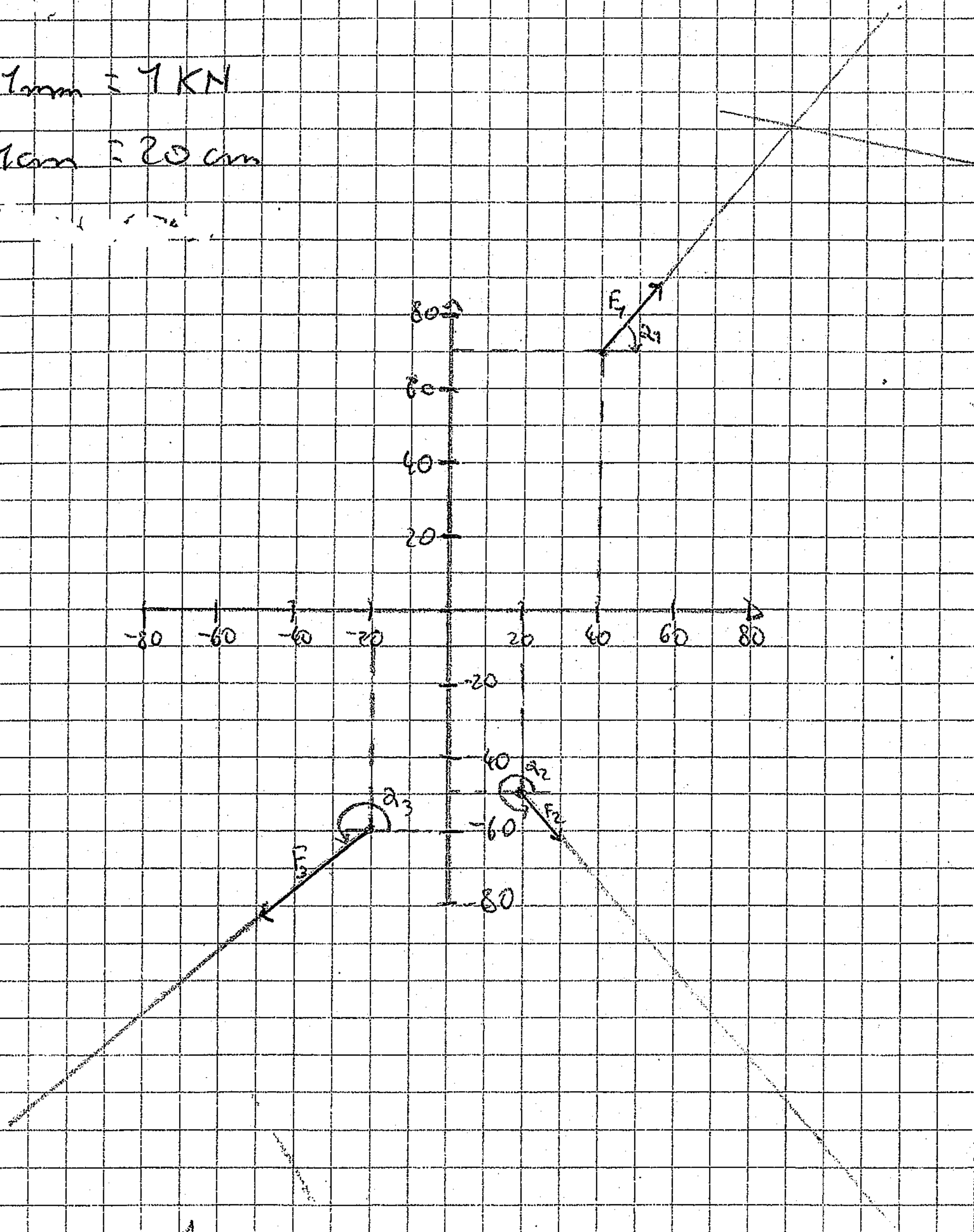
$a_1 = 50°$ $a_2 = 310°$ $a_3 = 220°$

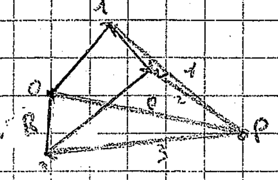
$R = ?$

$a_R = ?$

1mm = 1 KN

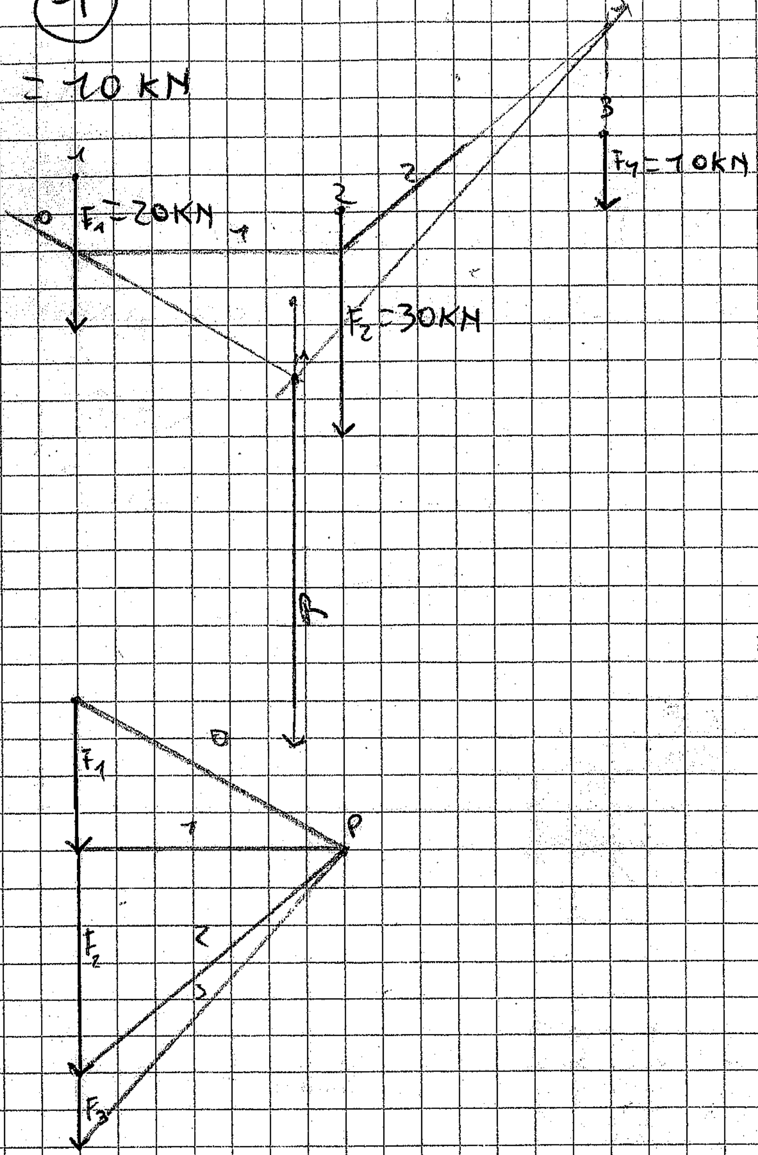
1cm = 20 cm

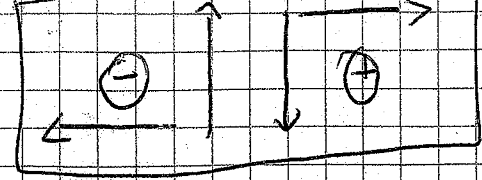




Es ①

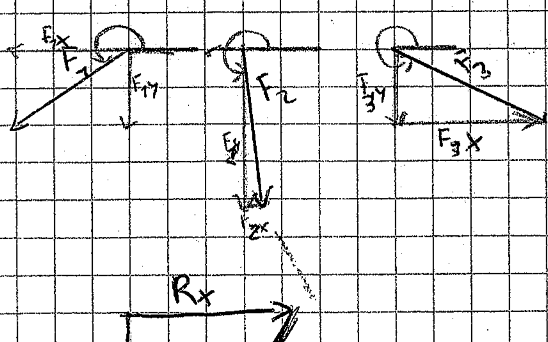
1cm = 10 KN

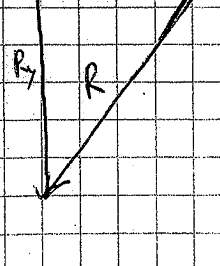




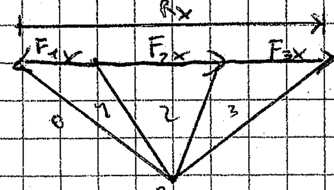
$\vec{R} = F_1 + F_2 + F_3 = 20 + 30 + 10 = 60k$

Es ②

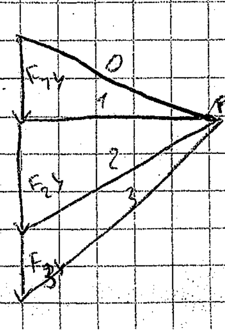


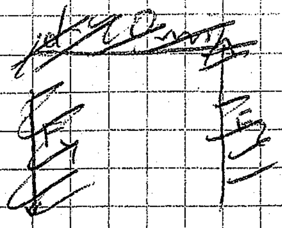


$R_x = \sum F_x = -F_{1x} + F_{2x} + F_{3x}$



$R_y = \sum F_y = F_{1y} + F_{2y} + F_{3y}$





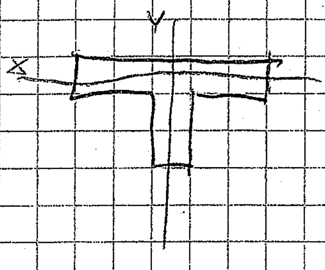
GEOMETRIA DELLE MASSE

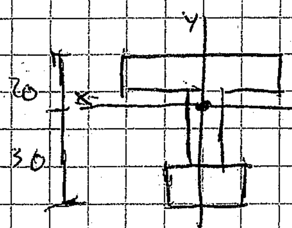
1) BARICENTRO (G)
2) MOMENTO STATICO
3) MOMENTO DI INERZIA

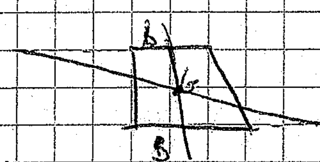
IL BARICENTRO

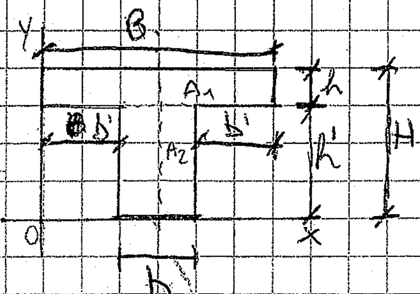
è un luogo geometrico (punto) che fa parte della sagoma in cui è contentrata tutta la materia dell'oggetto

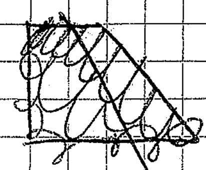
Il baricentro si trova sull'asse











Il momento statico (S) è uguale all'area della figura moltiplicata alla distanza del baricentro (G)

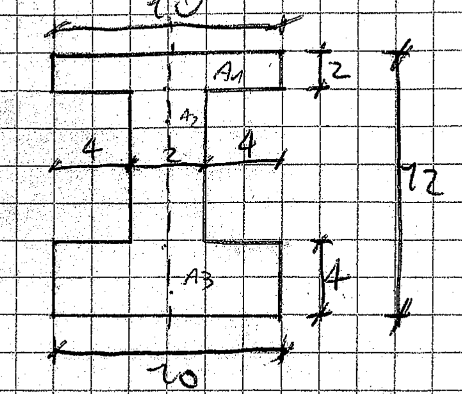
$$S_x = A_1 \cdot Y_G \qquad Y_G = \frac{S_x}{A}$$

$$S_Y = A_2 \cdot X_G$$

$A_1 = B \cdot h$

$A_2 = b \cdot h'$

$$S_x = \left(A_1 \cdot \frac{h}{2} + h'\right) + \left(A \cdot \frac{h'}{2}\right)$$



$S_x = A \cdot Y_G$

$\cancel{S_y = A \cdot X_G}$

$A_1 = 2 \cdot 10 = 20\ cm^2$

$A_2 = 2 \cdot 6 = 12\ cm^2$

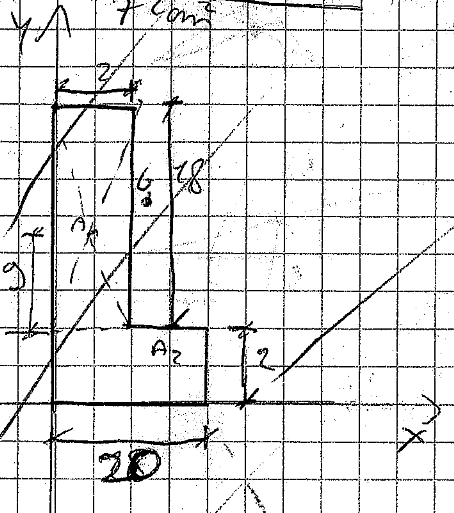
$A_3 = 4 \cdot 10 = 40\ cm^2$

$\cancel{S_x = (20 + 12\ cm^2 + 40\ cm^2)(4 + 8 + 1)}$

$\cancel{Y_G}$

$S_x (20\ cm^2 \cdot 11) + (12\ cm^2 \cdot (6+1)) + (40\ cm^2 \cdot 2) = 220\ cm^3 + 84\ cm^3 + 80\ cm^3 = 384\ cm^3$

$G = \frac{S_x}{A} = \frac{384\ cm^3}{72\ cm^2} = 5{,}3\ cm$



$G = ?$

$X_G = ?$

$Y_G = ?$

$S_x = A_t \cdot Y_G$

$S_y = A_t \cdot X_G$

$A_1 = 18 \cdot 2 = 36\ cm^2$

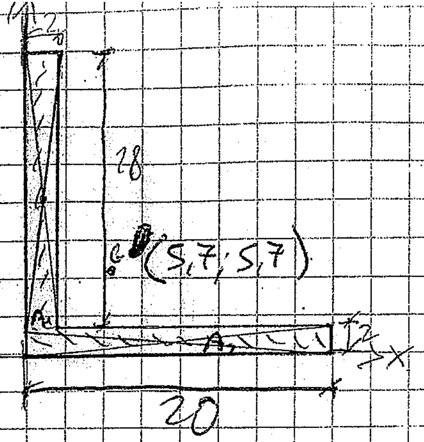
$A_2 = 20 \cdot 2 = 40\ cm^2$

$A_t = 36\ cm^2 + 40\ cm^2 = 76\ cm^2$

$S_x = 36\ cm^2 \cdot 11\ cm = 396\ cm^3$

$S_y = 40\ cm^2 \cdot 1\ cm = 40\ cm^3$

$Y_G = \frac{396\ cm^3}{76\ cm^2} = 5{,}2\ cm$ $\quad X_G = \frac{40}{76} = 0{,}53\ cm$



$G = ?$

$X_G = ?$

$Y_G = ?$

$S_x = A_t \cdot Y_G$

$S_y = A_t \cdot X_G$

$A_1 = 18 \cdot 2 = 36\ cm^2$

$A_2 = 20 \cdot 2 = 40\ cm^2$

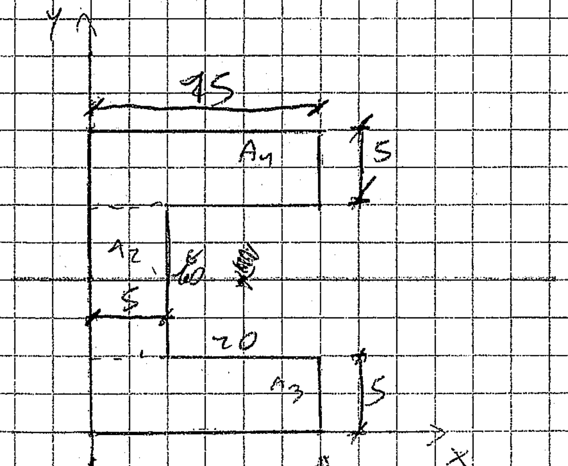
$A_t = 36 + 40 = 76\ cm^2$

$S_x = (36\ cm^2 \cdot 11\ cm) + (40\ cm^2 \cdot 1\ cm) = 436\ cm^3$

$X_G = \frac{436\ cm^3}{76\ cm^2} = 5,7\ cm$

$S_y = (36\ cm^2 \cdot 1\ cm) + (40\ cm^2 \cdot 10\ cm) = 436\ cm^3$

$Y_G = \frac{436\ cm^3}{76\ cm^2} = 5,7\ cm$



$G(10,25 ; 10)$

*

$S_y = (5 \cdot 15) \cdot 7,5 + (15 \cdot 5) \cdot 7,5 + (5 \cdot 10) \cdot 2,5 =$
$562,5 + 562,5 + 125 = 1250$

$X_G = \frac{1250}{200} = 6,25$

$A_1 = 15 \cdot 5 = 75\ cm^2$

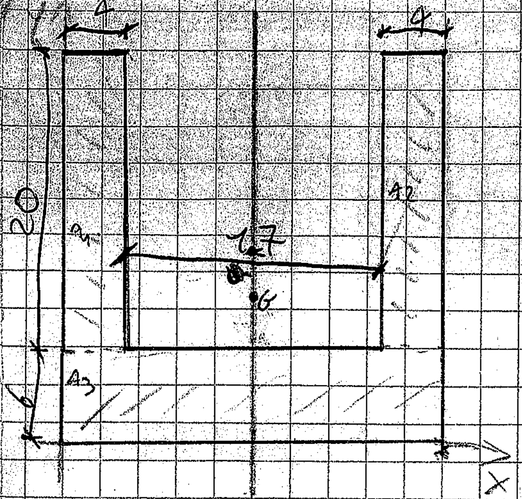
$A_2 = 5 \cdot 10 = 50\ cm^2$

$A_3 = 5 \cdot 15 = 75\ cm^2$

$A_t = 75\ cm^2 + 75\ cm^2 + 50\ cm^2 = 200\ cm^2$

$S_x = (75\ cm^2 \cdot 17,5\ cm) + (50\ cm^2 \cdot 10\ cm) + (75\ cm^2 \cdot 2,5\ cm) = 1312,5\ cm^3 +$
$+ 500\ cm^3 + 187,5\ cm^3 = 2000\ cm^3$

$Y_G = \frac{2000\ cm^3}{200\ cm^2} = 10\ cm$ *



$G(12,5 ; 9,71)$

$A_1 = 4 \cdot 20 = 80\ cm^2$

$A_2 = 4 \cdot 20 = 80\ cm^2$

$A_3 = 6 \cdot 25 = 150\ cm^2$

$A_G = 150\ cm^2 + 80\ cm^2 + 80\ cm^2 = 310\ cm^2$

$S_y = (80\ cm^2 \cdot 2\ cm) + (80\ cm^2 \cdot 23\ cm) + (150\ cm^2 \cdot 12,5\ cm) = 160\ cm^3 + 1840\ cm^3 +$
$+ 1875\ cm^3 = 3875\ cm^3$

$X_G = \frac{3875\ cm^3}{310\ cm^2} = 12,5\ cm$

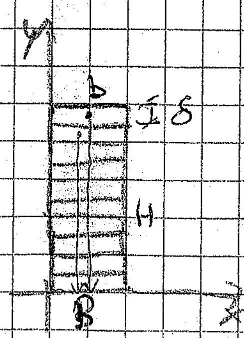
$S_x = (20 \cdot 4) \cdot 16 + (20 \cdot 4) \cdot 16 + (25 \cdot 6)\ 3 = 1280 + 1280 + 450 = 3010\ cm^3$

$Y_G = \frac{3010\ cm^3}{310} = 9,71\ cm$

MOMENTO D'INERZIA

$I_{xG} = A \cdot d_y^2 = cm^2 \cdot cm^2 = cm^4$

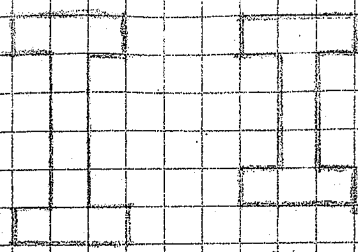
$I_{yG} = A \cdot d_x^2 = \frac{1}{12} B \cdot H^3$



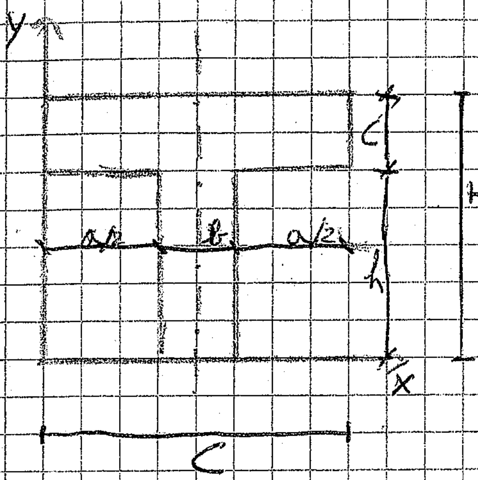
$I_x = \Sigma A_i \cdot d^2$

$I_x = \frac{1}{3} B \cdot H^3$

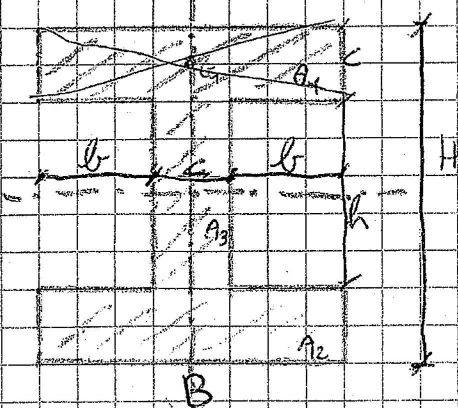
$I_y = \frac{1}{3} H \cdot B^3$



$G(x, y)$



$$I_{xG} = \left(\frac{C \cdot h^3}{12}\right) - 2\left(\frac{1}{12}\, a \cdot h^3\right)$$



$$I_{xG} = \left(\frac{1}{12} \cdot B \cdot H^3\right)$$

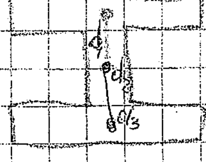
$$I_{xG} = \left[\frac{1}{12} \cdot B \cdot C^3 + A_1 \left(\frac{h}{2} + \frac{c}{2}\right)^2 + \frac{1}{12} \cdot c \cdot h^3\right]$$

$A_1 = C \cdot B$

$A_2 = C \cdot B$

$A_3 = C \cdot h$

MOMENTO DI TRASPOSIZIONE

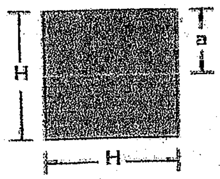
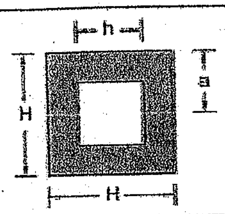
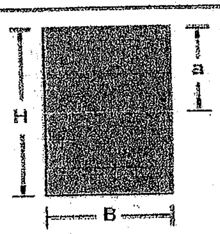
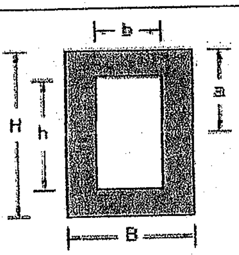
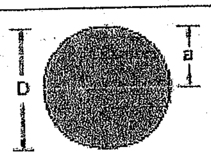
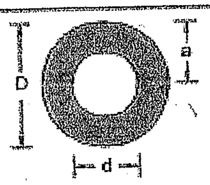
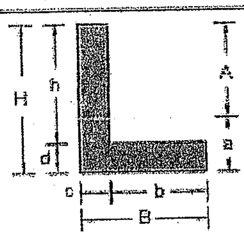
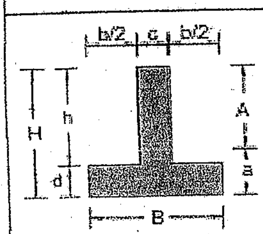


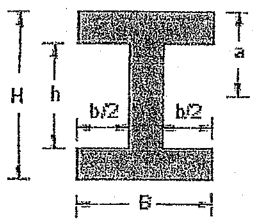
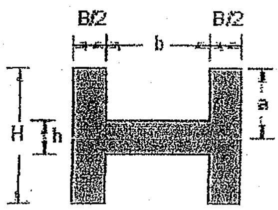
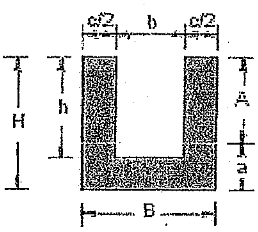
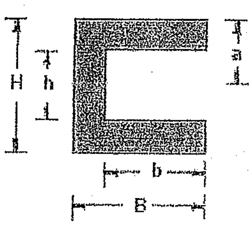
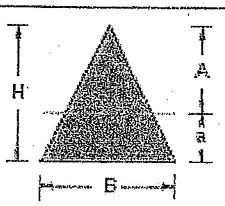
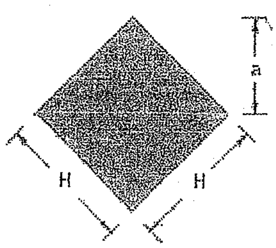
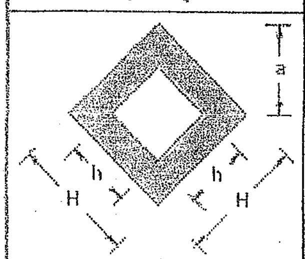
$A_1 \cdot d_1^2$

$A_2 \cdot d_2^2$

$A_3 \cdot d_3^2$

Tabella figure geometriche piane

Sezione	Area della sezione A	Distanza dal baricentro a	Momento di inerzia J	Modulo di resistenza W
	cm^2	cm	cm^4	cm^3
	H^2	$\frac{H}{2}$	$\frac{H^4}{12}$	$\frac{H^3}{6}$
	$H^2 - h^2$	$\frac{H}{2}$	$\frac{H^4 - h^4}{12}$	$\frac{H^4 - h^4}{6\,H}$
	$B \cdot H$	$\frac{H}{2}$	$\frac{B \cdot H^3}{12}$	$\frac{B \cdot H^2}{6}$
	$BH - bh$	$\frac{H}{2}$	$\frac{1}{12} \cdot (BH^3 - bh^3)$	$\frac{1}{6H} \cdot (BH^3 - bh^3)$
	$\frac{\pi \cdot D^2}{4}$	$\frac{D}{2}$	$\frac{\pi \cdot D^4}{64}$	$\frac{\pi \cdot D^3}{32}$
	$\frac{\pi \cdot (D^2 - d^2)}{4}$	$\frac{D}{2}$	$\frac{\pi \cdot (D^4 - d^4)}{64}$	$\frac{\pi \cdot (D^4 - d^4)}{32 \cdot D}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{I}{A}$ $W_a = \frac{I}{a}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{J}{A}$ $W_a = \frac{J}{a}$

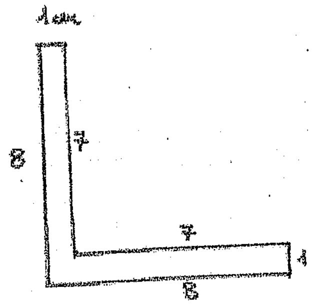
Sezione	Area della sezione A	Distanza dal baricentro a	Momento di inerzia J	Modulo di resistenza W
	cm^2	cm	cm^4	cm^3
	$BH - bh$	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$
	$BH - bh$	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{I}{A}$ $W_a = \frac{I}{a}$
	$BH - bh$	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$
	$\frac{B \cdot H}{2}$	$A = \frac{2H}{3}$ $a = \frac{H}{3}$	$\frac{B \cdot H^3}{36}$	$W_A = \frac{B \cdot H^2}{24}$ $W_a = \frac{B \cdot H^2}{12}$
	H^2	$\frac{H}{2} \cdot \sqrt{2}$	$\frac{H^4}{12}$	$\frac{H^3}{6\sqrt{2}}$
	$H^2 - h^2$	$\frac{H}{2} \cdot \sqrt{2}$	$\frac{H^4 - h^4}{12}$	$\frac{H^4 - h^4}{6H\sqrt{2}}$

I = raggio di inerzia = $\sqrt{\frac{\text{Momento di inerzia}}{\text{Area della sezione}}} = \sqrt{\frac{J}{A}}$

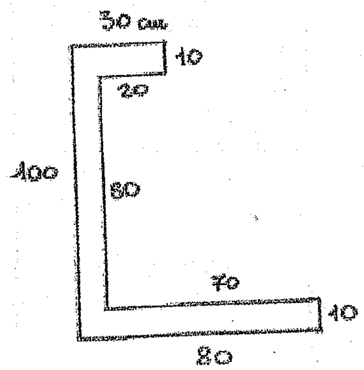
ESERCIZI DI RIEPILOGO

Determinare le coordinate dei baricentri delle seguenti sezioni.

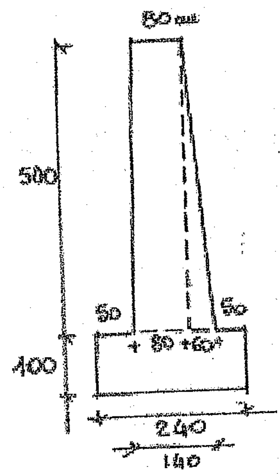
1)



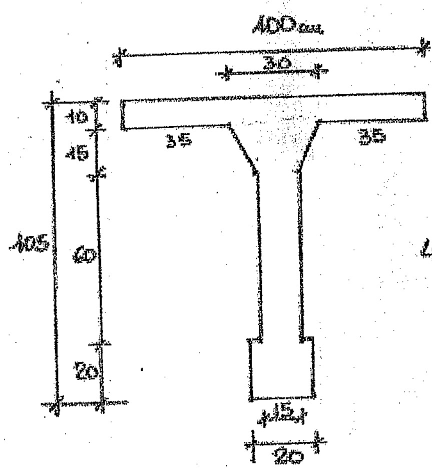
2)



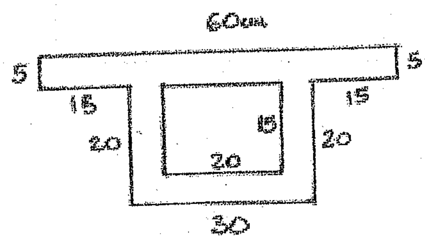
3)



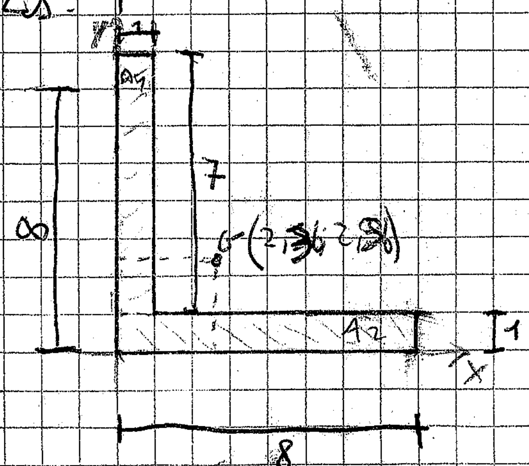
4)



5)



Es. 1



$G = ?$

$x_G = ?$

$y_G = ?$

$S_x = A_t \cdot y_G$

$S_y = A_t \cdot x_G$

8x $A_1 = 7 \cdot 1 = 7\ cm^2$

$A_2 = 8 \cdot 1 = 8\ cm^2$

$A_t = 7 + 8 = 15\ cm^2$

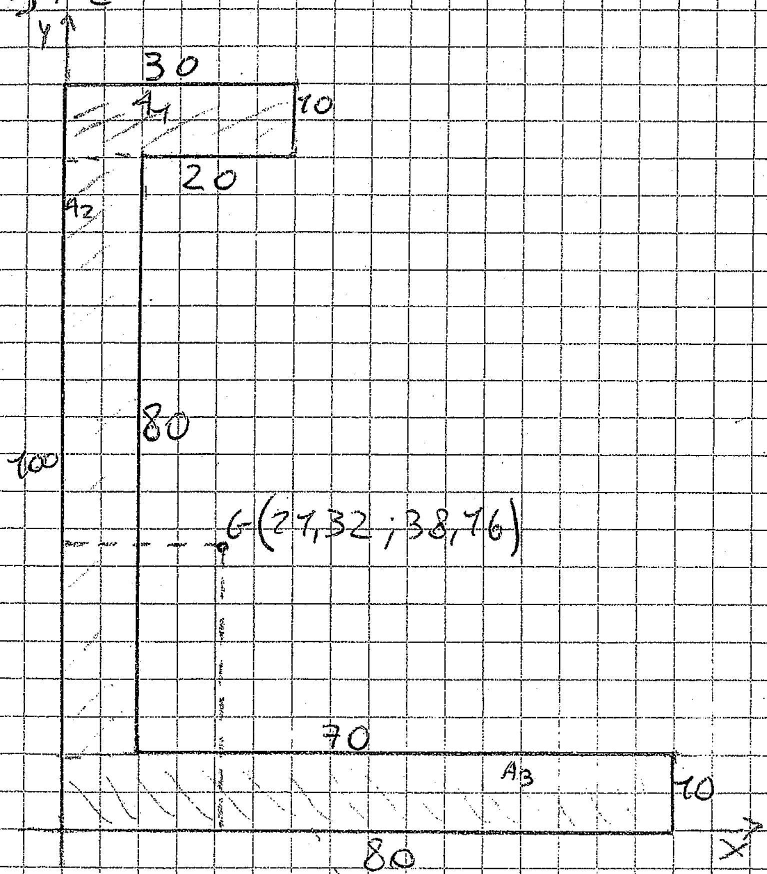
$S_x = (7 \cdot 4,5) + (8 \cdot 0,5) = 31,5\ cm^3 + 4\ cm^3 = 35,5\ cm^3$

$Y_G = \dfrac{35,5\ cm^3}{15\ cm^2} = 2,36\ cm$

$S_y = (8 \cdot 0,5) + (7 \cdot 4,5) = 4\ cm^3 + 31,5\ cm^3 = 35,5\ cm^3$

$X_G = \dfrac{35,5\ cm^3}{15\ cm^2} = 2,36\ cm$

Es. 2



G = ?

X_G = ?

Y_G = ?

$A_1 = 30 \cdot 10 = 300\ cm^2$

$A_2 = 80 \cdot 10 = 800\ cm^2$

$A_3 = 80 \cdot 10 = 800\ cm^2$

$A_t = 300\ cm^2 + 800\ cm^2 + 800\ cm^2 = 1900\ cm^2$

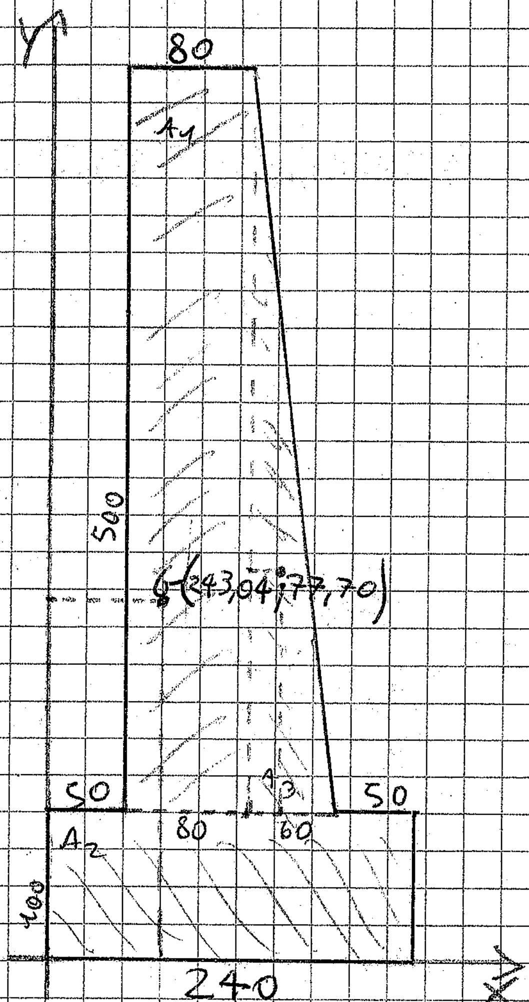
$S_x = (300 \cdot 95) + (800 \cdot 50) + (800 \cdot 5) = 28500\ cm^3 + 40000\ cm^3 + 4000\ cm^3 = 7250$

$Y_G = \dfrac{72500\ cm^3}{1900\ cm^2} = 38,16\ cm$

$S_y = (300 \cdot 15) + (800 \cdot 5) + (800 \cdot 40) = 4500\ cm^3 + 4000\ cm^3 + 32000\ cm^3 =$
$= 40500\ cm^3$

$X_G = \dfrac{40500\ cm^3}{1900\ cm^2} = 21{,}32\ cm$

es 3



$G = ?$

$X_G = ?$

$Y_G = ?$

$A_1 = 80 \cdot 500 = 40000\ cm^2$

$A_2 = 240 \cdot 100 = 24000\ cm^2$

$A_3 = \dfrac{60 \cdot 500}{2} = 15000\ cm^2$

$A_G = 40000 + 24000 + 15000 = 79000\ cm^2$

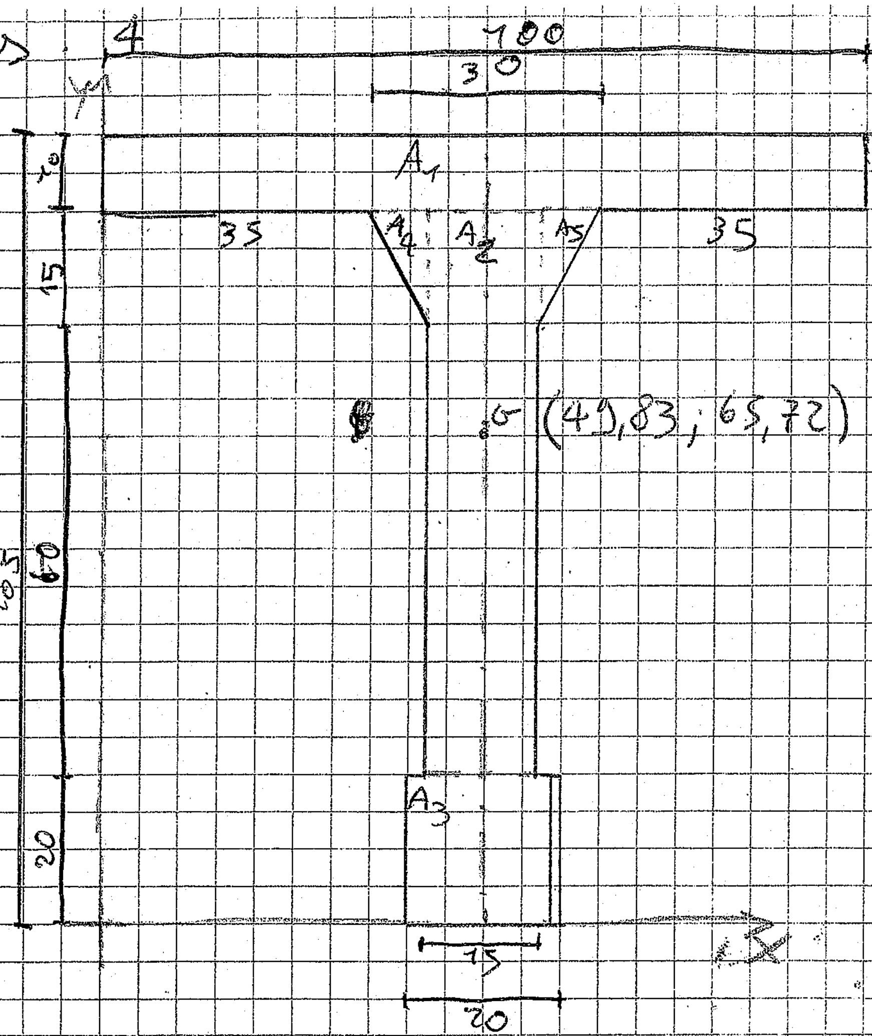
$S_x = (40000 \cdot 350) + (24000 \cdot 50) + (15000 \cdot 266{,}66) = 14000000 + 1200000 + 39999$
$= 19199900\ cm^3$

$Y_G = \dfrac{19199900\ cm^3}{79000\ cm^2} = 243{,}04\ cm$

$S_y = (40000 \cdot 90) + (2400 \cdot 120) + (15000 \cdot 150) = 3600000 + 288000 + 2250000$
$= 6138000\ cm^3$

$X_G = \dfrac{6138000\ cm^3}{79000\ cm^2} = 77{,}70\ cm$

es 4



$G = ?$

$X_G = ?$

$Y_G = ?$

$A_1 = 100 \cdot 10 = 1000\ cm^2$

$A_2 = 15 \cdot 60 = 900\ cm^2$

$A_3 = 20 \cdot 20 = 400\ cm^2$

$A_4 = \frac{7{,}5 \cdot 15}{2} = 56{,}25\ cm^2$

$A_5 = \frac{7{,}5 \cdot 15}{2} = 56{,}25\ cm^2$

$A_t = 1000 + 900 + 400 + 56{,}25 + 56{,}25 = 2412{,}5\ cm^2$

$S_x = (1000 \cdot 100) + (900 \cdot 50) + (400 \cdot 10) + (56{,}25 \cdot 85) + (56{,}25 \cdot 85) =$

$= 100000 + 45000 + 4000 + 4781{,}25 + 4781{,}25 = 158562{,}5\ cm^3$

$Y_G = \frac{158562{,}5\ cm^3}{2412{,}5\ cm^2} = 65{,}72\ cm$

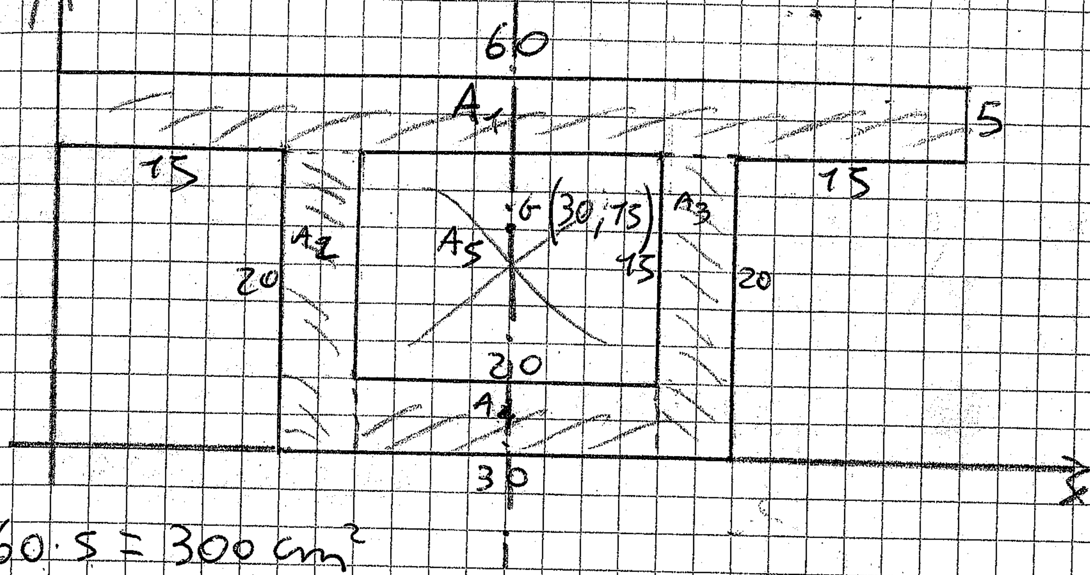
$S_y = (1000 \cdot 50) + (900 \cdot 50) + (400 \cdot 52{,}5) + (56{,}25 \cdot 37{,}5) + (56{,}25 \cdot 37{,}5) =$

$= 50000 + 45000 + 21000 + 2109{,}375 + 2109{,}375 = 120218{,}75$

$X_G = \frac{120218{,}75\ cm^3}{2412{,}5\ cm^2} = 49{,}83\ cm$

32

es. 5



$G = ?$

$X_G = ?$

$Y_G = ?$

$A_1 = 60 \cdot 5 = 300\ cm^2$

$A_2 = 20 \cdot 5 = 100\ cm^2$

$A_3 = 20 \cdot 5 = 100\ cm^2$

$A_4 = 20 \cdot 5 = 100\ cm^2$

$A_5 = 20 \cdot 15 = 300\ cm^2$

$A_G = 300 + 100 + 100 + 100 = 600\ cm^2$

$S_x = (300 \cdot 22{,}5) + (100 \cdot 10) + (100 \cdot 10) + (100 \cdot 2{,}5) = 6750 + 1000 + 1000 + 2$

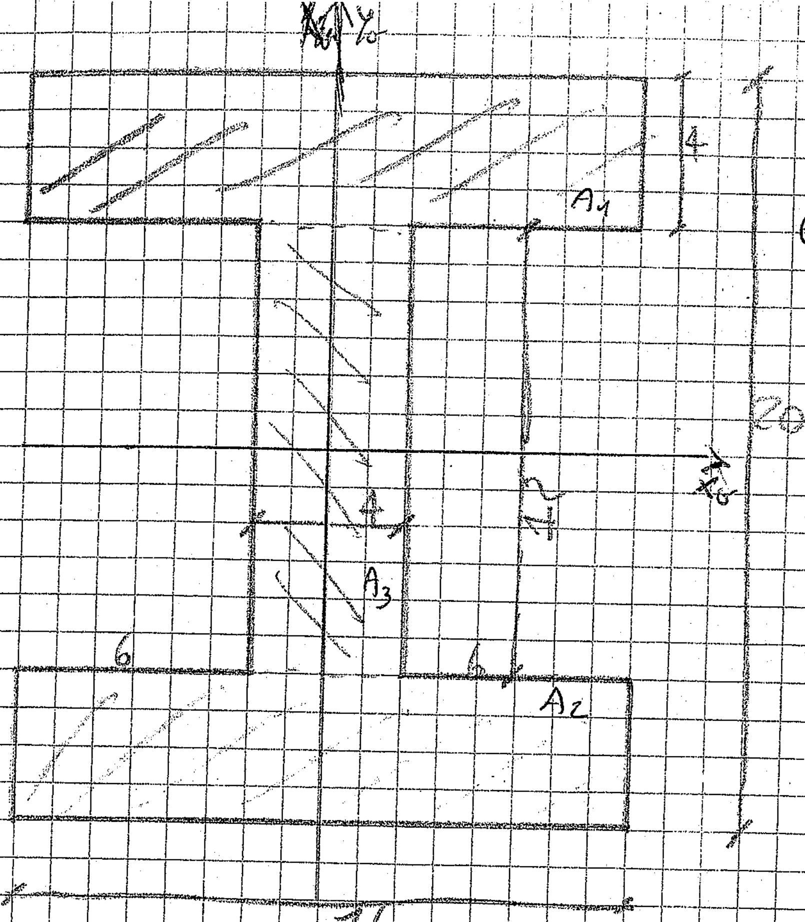
$= 9000\ cm^3$

$$Y_G = \frac{9000\ cm^3}{600\ cm^2} = 15\ cm$$

$S_y = (300 \cdot 30) + (100 \cdot 17{,}5) + (100 \cdot 42{,}5) + (100 \cdot 30) = 9000 + 1750 + 4250$

$+ 3000 = 18000$

$$X_G = \frac{1800}{600} \frac{cm^3}{cm^2} = 30\ cm$$

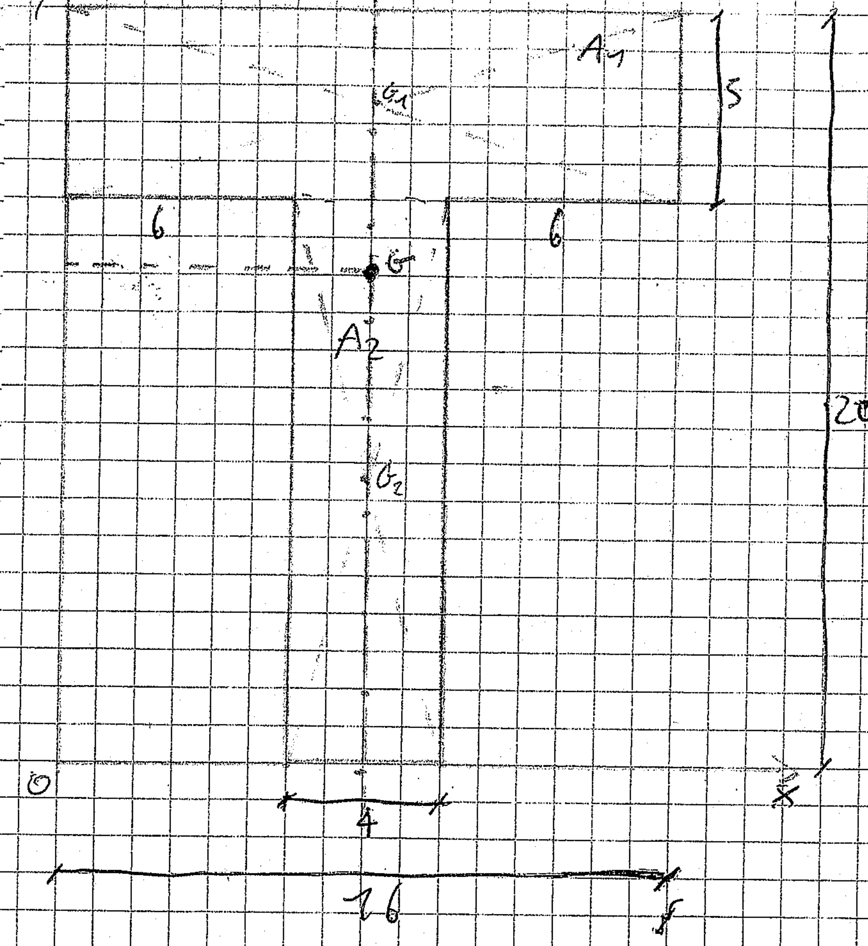


$G(8;10)$

$$I_{x_G} = \left[\left(\frac{1}{12}\cdot 16\cdot 4^3\right) + \left(64\cdot 8^2\right)\right]\cdot 2 + \left(\frac{1}{12}\cdot 4\cdot 12^3\right) + \cancel{\left(\underbrace{48}_{A_3}\cdot 0\right)}$$

$$\left[85{,}3 + 4096\right]\cdot 2 + 576 = 8938{,}6$$

$$\frac{1}{12}\cdot 16\cdot 20^3 - 2\left(\frac{1}{12}\cdot 6\cdot 12^3\right) = 8938{,}6$$



$G(x;y) = ?$

I_{xG}?

I_{yG}?

$A_1 = 5 \cdot 16 = 80 cm^2$

$A_2 = 4 \cdot 15 = 60 cm^2$

$A_G = 60 + 80 = 140 cm^2$

$S_x = (80 \cdot 17{,}5) + (60 \cdot 7{,}5) = 1400 + 450 = 1850 cm^3$

$Y_G = \frac{1850 cm^3}{140 cm^2} = 13{,}21 cm$

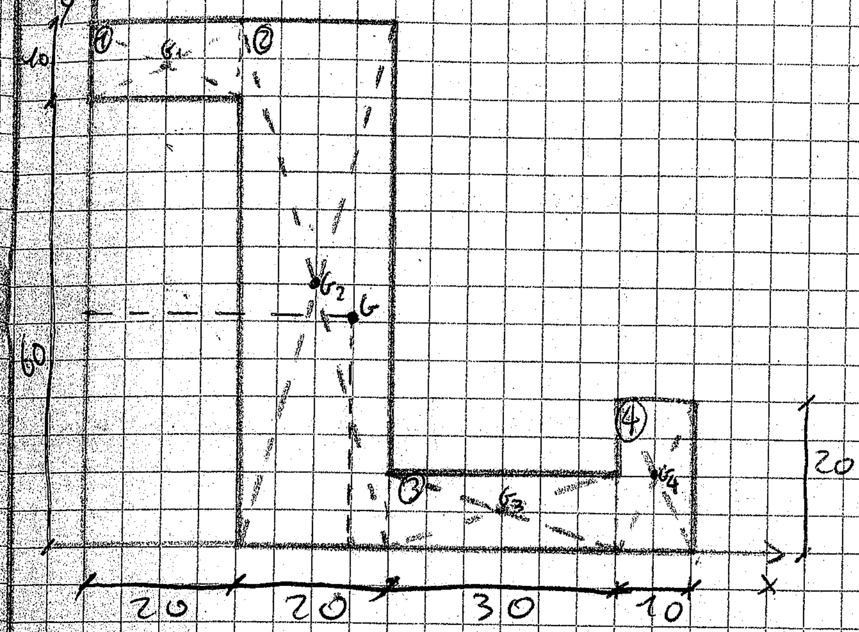
$S_y = (80 \cdot 8) + (60 \cdot 8) = 640 + 480 = 1120 cm^3$

$X_G = \frac{1120 cm^3}{140 cm^2} = 8 cm$

$I_{xG} = \left(\frac{1}{12} \cdot 16 \cdot 5^3\right) + \left(80 \cdot 4{,}29^2\right) + \left(\frac{1}{12} \cdot 4 \cdot 15^3\right) + \left(60 \cdot 5{,}71^2\right) =$

$166{,}67 + 1472{,}33 + 1125 + 1956{,}25 = 4720{,}25 cm^4$

$\cancel{I_{yG} = \left(\frac{1}{12} \cdot 16 \cdot 5^3\right) + \left(80 \cdot 8^2\right) + \left(\frac{1}{12} \cdot 4 \cdot 15^3\right) + \left(60 \cdot 8^2\right) = 166{,}67 + 5120 + 1125 + 38\ldots = 10251{,}67 cm^4}$



$G(x_G ; y_G)$

$A_{TOT} = A_1 + A_2 + A_3 + A_4 = 200 + 1400 + 300 + 200 = 2100 \, cm^2$

$S_{x_{TOT}} = S_x^{(1)} + S_x^{(2)} + S_x^{(3)} + S_x^{(4)} = (A_1 \cdot d_1) + (A_2 \cdot d_2) + (A_3 \cdot d_3) + (A_4 \cdot d_4) =$

$= (200 \cdot 65) + (1400 \cdot 35) + (300 \cdot 5) + (200 \cdot 10) = 13000 + 49000 + 1500 + 2000$

$= 65500 \, cm^3$

$Y_G = \frac{65500 \, cm^3}{2100 \, cm^2} = \cancel{28,62 \, cm} \; 31,19 \, cm$

$S_{y_{TOT}} = (200 \cdot 10) + (1400 \cdot 30) + (300 \cdot 55) + (200 \cdot 75) = 2000 + 42000 + 16500 + 150$

$= 75500 \, cm^3$

$X_G = \frac{75500 \, cm^3}{2100 \, cm^2} = 35,95 \, cm$

$I_{x_G} = \left(\frac{1}{12} \cdot 20 \cdot 10^3\right) + \left(200 \cdot 33,81^2\right) + \left(\frac{1}{12} \cdot 20 \cdot 70^3\right) + \left(1400 \cdot 3,81^2\right) + \left(\frac{1}{12} \cdot 30 \cdot 10^3\right) +$

$+ \left(300 \cdot 26,19^2\right) + \left(\frac{1}{12} \cdot 10 \cdot 20^3\right) + \left(200 \cdot 21,19^2\right) = 1666,67 + 228623,22 + 571666,6$

$+ 20322,54 + 2500 + 205774,$

$+ 6666,67 + 89803,22 =$

$= 1127023,82$

~~EQUAZIONI DI EQUILIBRIO~~

TRAVI

GLI ELEMENTI STRUTTURALI SI DIVIDONO IN:

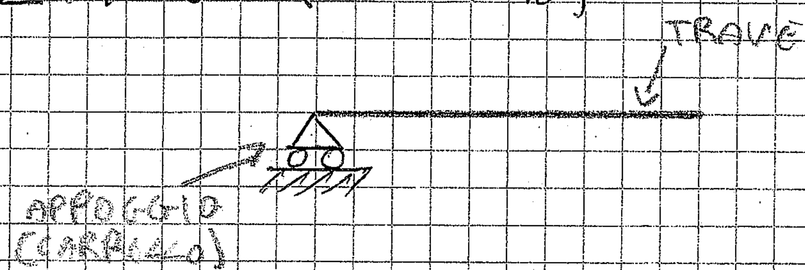
- LINEARE (TRAVE)
- BIDIMENSIONALE

Un corpo è in equilibrio se soddisfa 3 equazioni:

$\Sigma F_x = 0$ (SPOSTAMENTO ORIZZONTALE)

$\Sigma F_y = 0$ (SPOSTAMENTO VERTICALE)

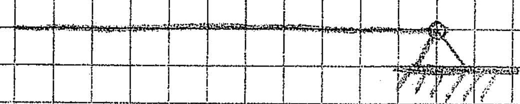
$\Sigma M = 0$ (ROTAZIONE)



① VINCOLO
APPOGGIO o CARRELLO

LA cerniera ~~[illegible]~~ azzera le forze x e y ma la rotazione no.

② VINCOLO
CERNIERA



L'incastro è un vincolo che toglie tutti ~~tre~~ ~~le~~ i gradi di libertà

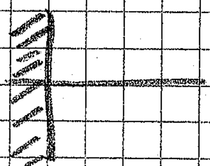
③ VINCOLO
INCASTRO

LE TRAVI POSSONO ESSERE
~~STRUTTURE~~ = labile ~~[illegible]~~, isostatica, iperstatica

n = numero travi
a = appoggi
c = cerniere
i = incastro.

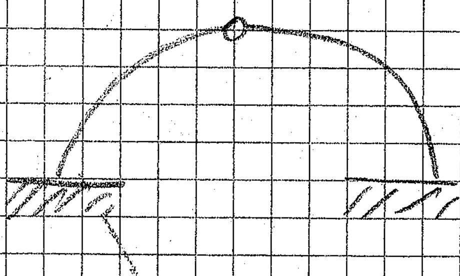
$$3n \;\overset{\leq}{\geq}\; \sum \text{vincoli } (a+c+i)$$

Se è < è una trave ipostatica
Se è = è una trave isostatica
Se è > è una trave ~~iperstatica~~ labile.



$3 \cdot 1 = a \cdot 0 + c \cdot 0 + i \cdot 1 \cdot 3$

$3 = 3$ isostatica

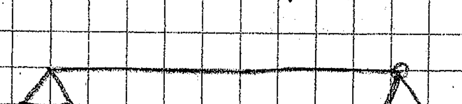


$3 \cdot 2 = a \cdot 0 + 1 \cdot c + 2 \cdot i$

$6 = 2 + 6$

$6 = 8$

$6 < 8$ ipostatica



$3 \cdot 1 = 1 \cdot 1 + 1 \cdot 2 + i \cdot 0 \Rightarrow 3 = 1 + 2 \Rightarrow 3 = 3$ isostatica

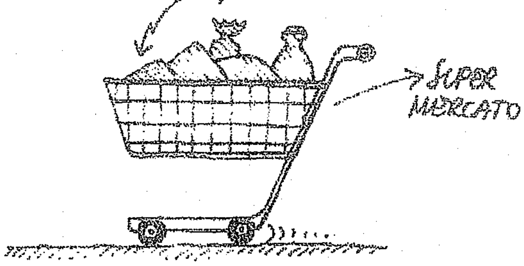
LEZIONE 3°B

ITG NERVI/2016

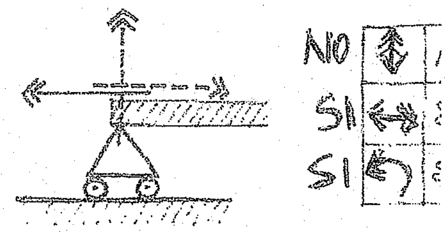
VINCOLI

Gli esempi pratici

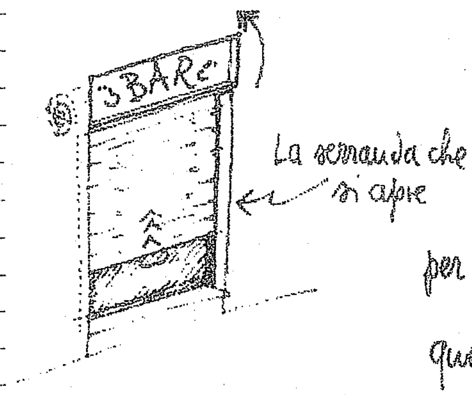
La schematizzazione statica



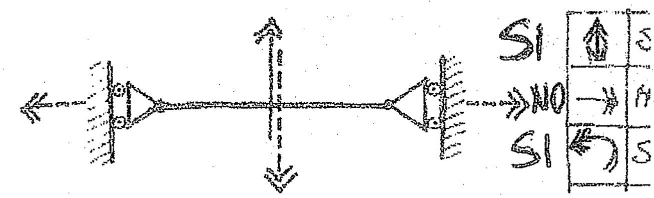
per ogni ruota questo è il vincolo!

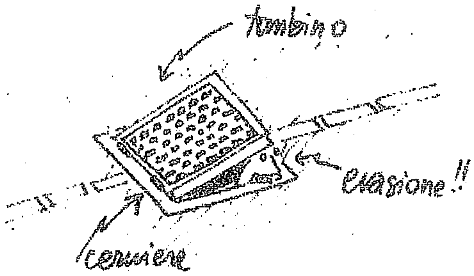


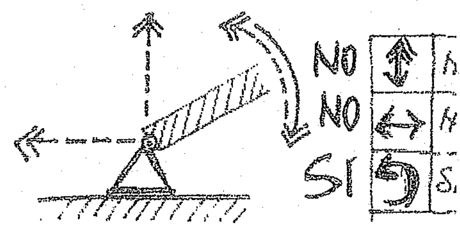
ABOLISCE
BLOCCA N° 1 GRADO DI LIBERTÀ



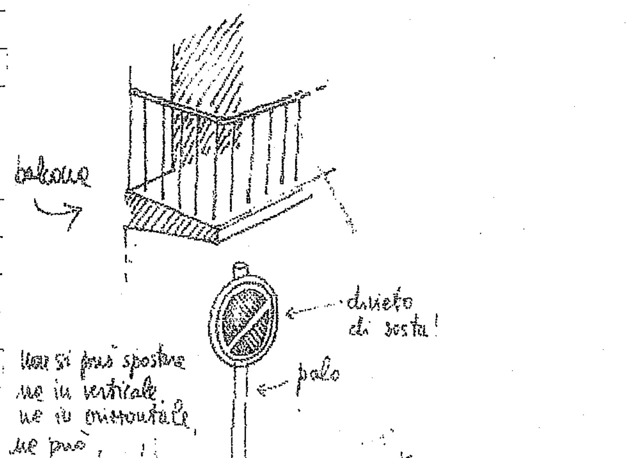
per ogni stecca del bandone questo è il vincolo!

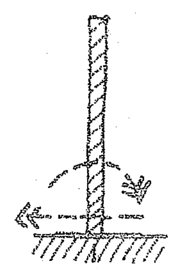


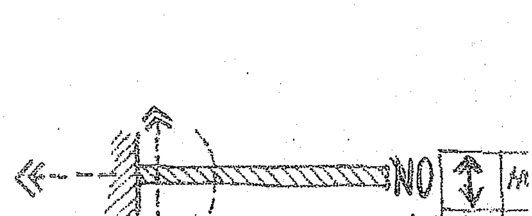




ABOLISCE
BLOCCA N° 2 GRADI DI LIBERTÀ





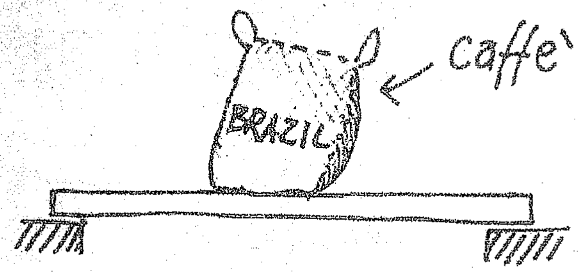


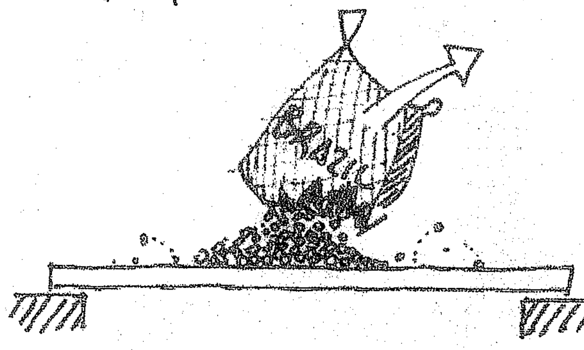
questo è per il palo!

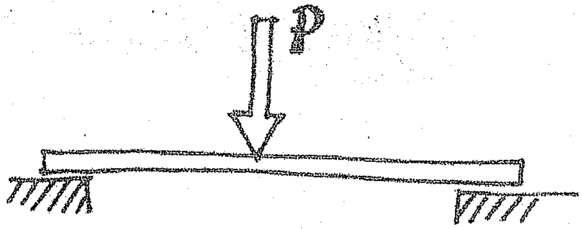
questo è per il balcone!

BLOCCA N° 3 GRADI DI LIBERTÀ

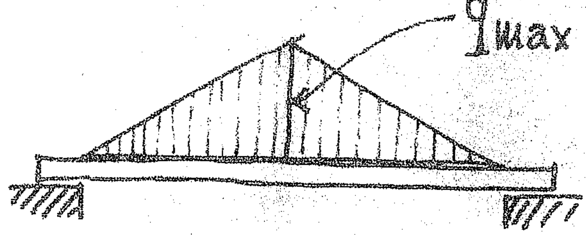
TIPI DI CARICHI TRAVI



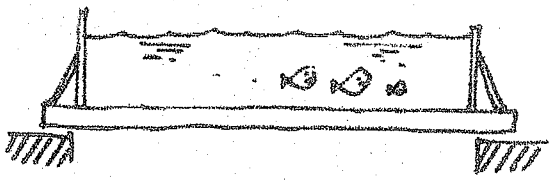


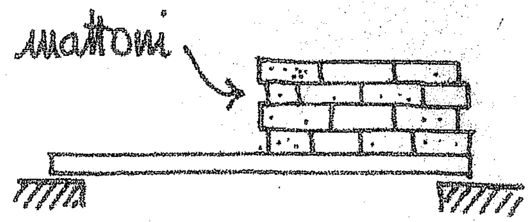


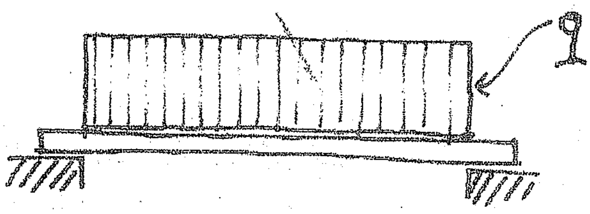
Carico concentrato



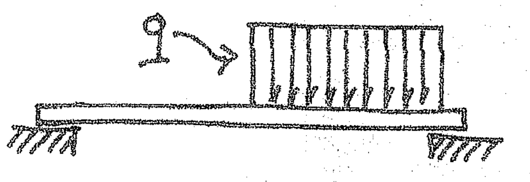
carico distribuito
(CON LEGGE TRIANGOLARE)



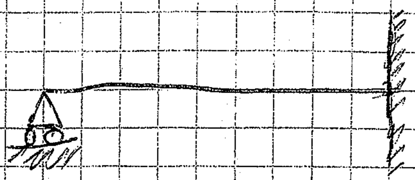




Carico distribuito uniformemente
(CON LEGGE COSTANTE)

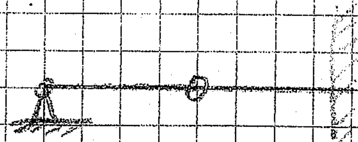


carico distribuito
(CON LEGGE COSTANTE)
su porzione di trave



$3 = 1 \cdot 1 + 0 \cdot 2 + 1 \cdot 3$

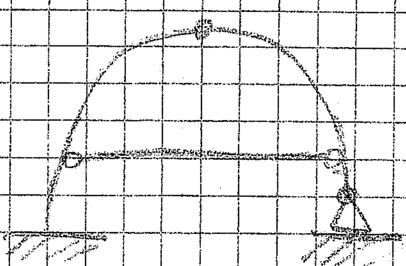
$3 < 4$ IPERSTATICA



$3 \cdot 2 = \sum 0 \cdot 1 + 2 \cdot 2 + 1 \cdot 3$

$6 = \sum 0 + 4 + \cancel{3}$

$6 < \cancel{7}$ IPERSTATICA



$3 \cdot \cancel{3} = \sum 0 + \overset{8}{\cancel{4}} + 1 \cdot 3$

$9 = \sum \cancel{7}\,11$

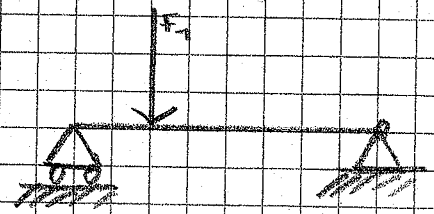
$9 < \cancel{7}\,11$

I CARICHI

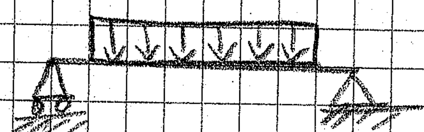
~~Peso~~

Carico concentrato

Carico distribuito uniformemente

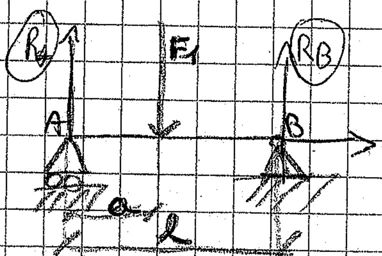


① Carico concentrato



② Carico uniformemente distribuito

EQUILIBRIO DELLA STATICA



DATI

$F_1 = 80\ KN$

$l = 5\ m$

$a = 2\ m$

$R_A = ?$

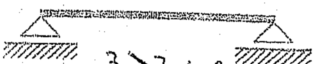
$R_B = ?$

R (REAZIONE)

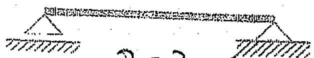
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases}$$

$$\begin{cases} \sum F_y = F_1 + RA + R_B = 0 \\ \sum M_A = R_A \cdot 0 + F_1 \cdot a + R_B \cdot l = \end{cases}$$

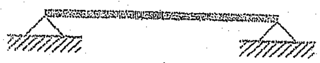
Effettuare il computo dei vincoli per le strutture qui di seguito riportate.

7


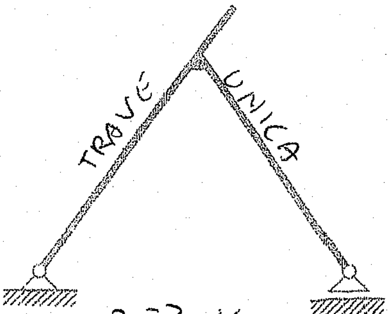
3 > 2 LABILE

8


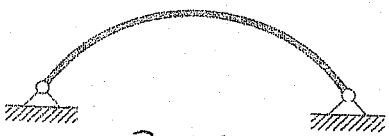
3 = 3 ISOSTATICA

9


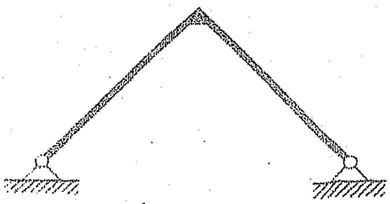
3 < 4 IPERSTATICA

10


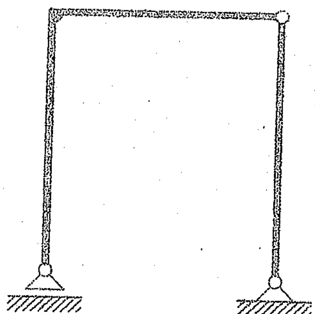
3 = 3 ISOSTATICA

11


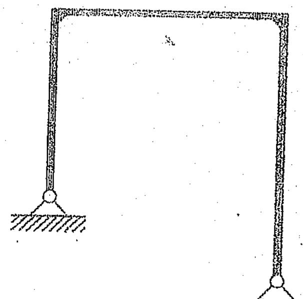
3 < 4 IPERSTATICA

12


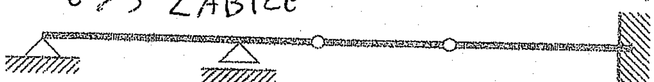
3 < 4 IPERSTATICA

14


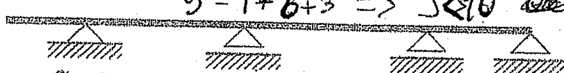
6 > 5 LABILE

15


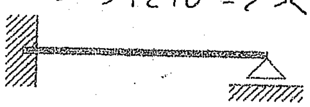
3 = 3 ISOSTATICA

16


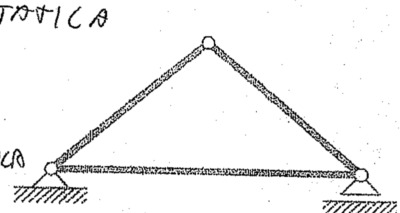
9 = 1 + 6 + 3 => 9 < 10 ~~LABILE~~ IPERSTATICA

17


3 = 3 + 2 + 0 => 3 < 5 IPERSTATICA

18


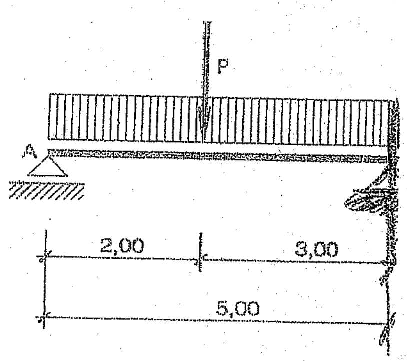
3 = 1 + 0 + 3 => 3 < 4 IPERSTATICA

19


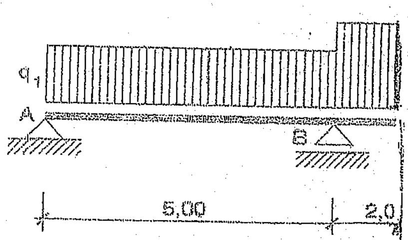
6 = 1 + 4 => 6 > 5 LABILE

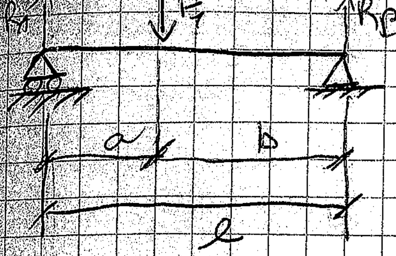
Calcolare le reazioni vincolari delle travi e dei portali qui di seguito riportati.

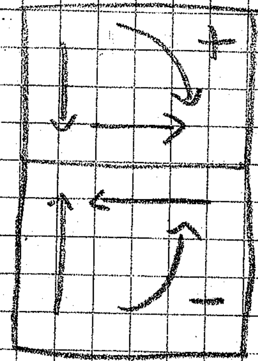
20 Procedimento ~~grafico~~ analitico;
$P = 8$ kN, $q = 6$ kN/m.



21 Procedimento analitico;
$q_1 = 4$ kN/m, $q_2 = 6$ kN/m.







DATI

$F_1 = 80 KN$

$l = 5 m$

$a = 2 m$

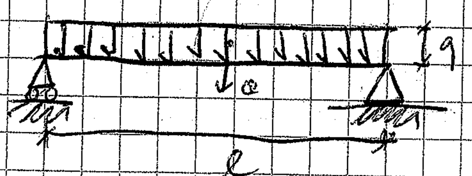
$b = 3 m$

$R_A = ?$ -48

$R_B = ?$ -32

$$\begin{cases} 80KN - R_A - R_B = 0 \\ R_A \cdot 0 + 80 KN \cdot 2m - R_B \cdot 5 m = 0 + 160 - 5R_B = 0 \end{cases}$$

$$\begin{cases} 80 KN - R_A - R_B = 0 \\ \frac{\cancel{-5}RB}{\cancel{-5}} = \frac{-\cancel{160}^{32}}{\cancel{-5}} = -32 \end{cases} \quad \begin{cases} 80KN - R_A - 32 = 0 \\ R_B = -32 \end{cases} \quad \begin{cases} R_A = -80+32 = -48 \\ R_B = -32 \end{cases}$$



DATI

$l = 3 m$

$q = 20 Kn/ml$

$Q = 60 Kn/ml$

~~ES~~

~~DATI~~

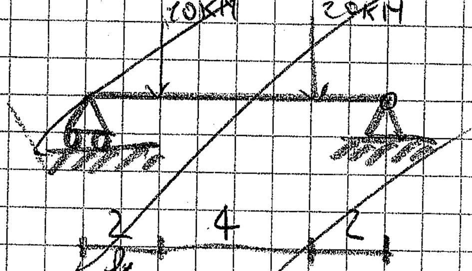
~~$F_1 = 10 KN$~~

~~$F_2 = 20 KN$~~

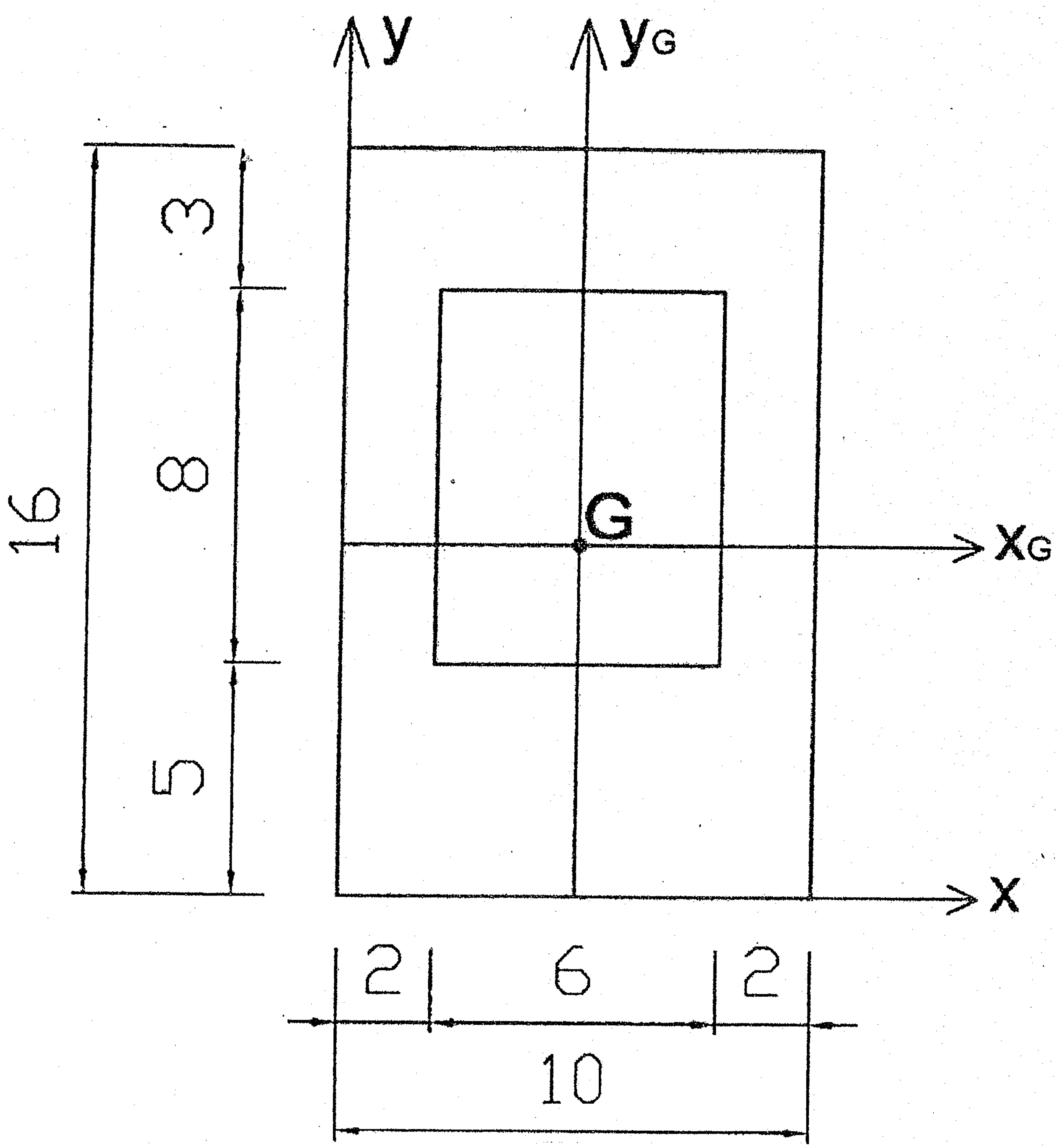
~~$l_1 = 2 m$~~

~~$l = 8 m$~~

~~$R_A = ?$~~



Determinare il nocciolo centrale di inerzia della seguente sezione

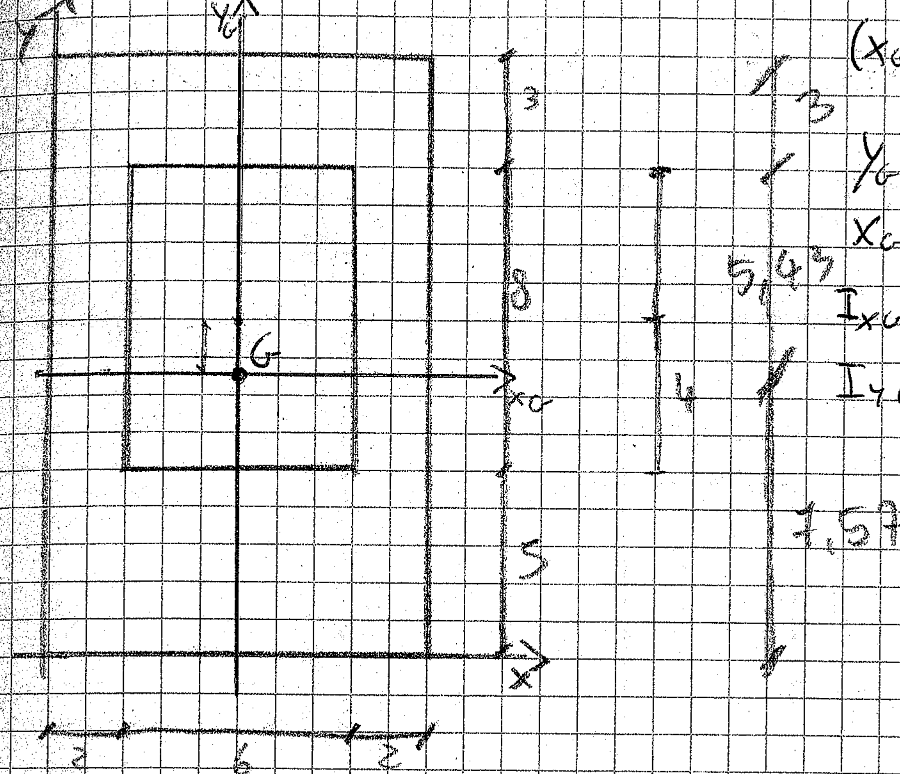


Dopo aver determinato le coordinate del baricentro :

X_G = 5,0 cm
Y_G = 7,57 cm

Dopo aver determinato i momenti principali di inerzia :

I_{xG} = 3091,18 cm^4
I_{yG} = 1189,33 cm^4



$(X_G ; Y_G)$ $(I_{xG} ; I_{YG})$

$Y_G = 7{,}57\ cm$

$X_G = 5{,}0\ cm$

$I_{xG} = 3088{,}76\ cm^4$

$I_{YG} = 1189{,}33\ cm^4$

$A_1 = 3 \cdot 10 = 30\ cm^2$

$A_2 = 8 \cdot 2 = 16\ cm^2$

$A_3 = 8 \cdot 2 = 16\ cm^2$

$A_4 = 5 \cdot 10 = 50\ cm^2$

$A_G = 50 + 30 + 16 + 16 = 112\ cm^2$

$S_x = (30 \cdot 14{,}5) + (16 \cdot 9) + (16 \cdot 9) + (50 \cdot 2{,}5) = 435 + 144 + 144 + 125 = 848\ cm^3$

$Y_G = \frac{848\ cm^3}{112\ cm^2} = 7{,}57\ cm$

~~$S_Y = (30 \cdot 5) + (16 \cdot 1) + (16 \cdot 9) + (50 \cdot 5) - (48 \cdot 5) = 150 + 16 + 144 + 250 - 240 =$~~

~~$(30 \cdot 5) + (16 \cdot 1) + (16 \cdot 9) + (80 \cdot 8) =$~~

$I_{xG} = \left(\frac{1}{12} \cdot 10 \cdot 16^3\right) + (160 \cdot 0{,}43^2) - \left[\left(\frac{1}{12} \cdot 6 \cdot 8^3\right) + (48 \cdot 1{,}43^2)\right] =$

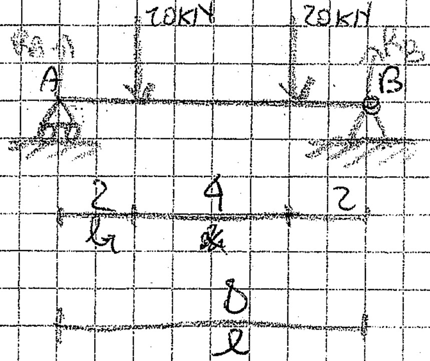
~~$= 3413{,}33 + 29{,}58 - [256 + 98{,}15] = 3442{,}91 - 354{,}15 = 3088{,}76$~~

$= 3413{,}33 + 29{,}58 - [256 + 98{,}15] = 3442{,}91 - 354{,}15 =$

$= 3088{,}76\ cm^4$

$I_{YG} = \left(\frac{1}{12} \cdot 16 \cdot 10^3\right) - \left(\frac{1}{12} \cdot 8 \cdot 6^3\right) = 1333{,}33 - 144$
$= 1189{,}33\,cm^4$

Es



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases}$$

DATI:

$F_1 = 10\ KN$

$F_2 = 20\ KN$

$l_1 = 2\ m$

$l = 8\ m$

$R_A = ?$

$R_B = ?$

DIAGR. TAGLIO

DIAGR. MOMENTO

$$\begin{cases} \sum F_y = F_1 - R_A + F_2 - R_B \\ \sum M_A = R_A \cdot 0 + F_1 \cdot l_1 + F_2 \cdot 6 - R_B \cdot l \end{cases}$$

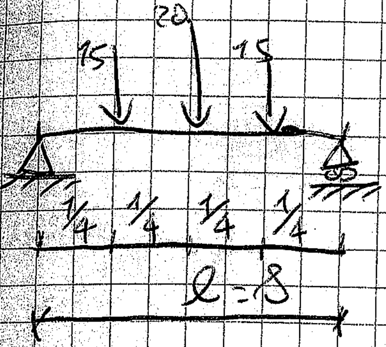
$$\begin{cases} \sum F_y = 10 - R_A + 20 - R_B \\ \sum M_A = 0 + 20 + 120 - R_B \cdot 8 = 0 \end{cases}$$

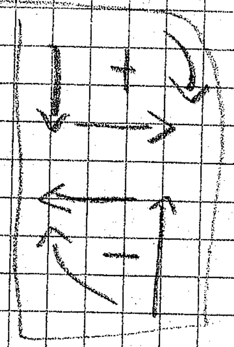
$$\begin{cases} \sum F_y = 30 - R_A - R_B \\ \sum M_A = \frac{140 - R_B}{8} = 0 \end{cases} \quad R_B = \frac{140}{8} = 17{,}5\,KN$$

$$\begin{cases} \sum F_y = 30 - R_A - 17{,}5\,KN \\ R_B = -17{,}5\,KN \end{cases}$$

$$\begin{cases} R_A = -12{,}5\,KN \\ R_B = -17{,}5\,KN \end{cases}$$

Es caso





DATI

$F_1 = 15\ kN$

$F_2 = 20\ kN$

$F_3 = 15\ kN$

$l = 8\ m$

$l_1 = 2\ m$

$R_A = ? - 22,5\ kN$

$R_B = ? - 27,5\ kN$

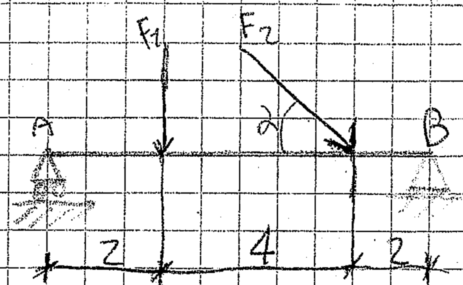
$$\begin{cases} \sum F_y = F_1 + F_2 + F_3 - R_A - R_B \\ \sum M_A = R_A \cdot 0 + F_1 \cdot 2 + F_2 \cdot 4 + F_3 \cdot 6 - R_B \cdot 8 \end{cases}$$

$$\begin{cases} \sum F_y = 15 + 20 + 15 - R_A - R_B \\ \sum M_A = 0 + 30 + \cancel{\quad} 80 + 90 - R_B \cdot 8 \end{cases}$$

$$\begin{cases} \sum F_y = 50 - R_A - R_B \\ \sum M_A = \frac{200 - R_B \cdot \cancel{8}}{8} \Rightarrow R_B = \frac{200}{8} = \cancel{\quad} - 25\ kN \end{cases}$$

$$\begin{cases} \sum F_y = 50 - R_A - 25 \\ R_B = -25\ kN \end{cases}$$

$$\begin{cases} R_A = -25\ kN \\ R_B = -25\ kN \end{cases}$$



$F_1 = 30\,KN$

$F_2 = 40\,KN$

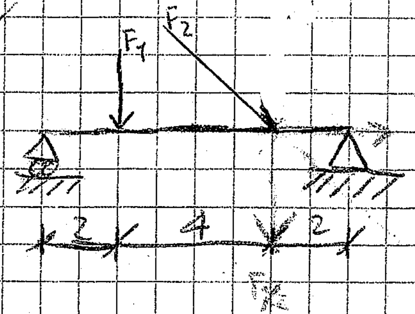
$\downarrow$

$F_{y_2} = ?\ 28{,}28\,KN$

$F_{x_2} = ?\ 28{,}28\,KN$

$R_A = ?$

$R_B = ?$



$F_{y_2} = F_2 \cdot \text{sen}\,45° = 40KN \cdot 0{,}707106781 = 28{,}28\,KN$

$F_{x_2} = F_2 \cdot \cos 45° = 40KN \cdot 0{,}707106781 = 28{,}28\,KN$

$$\begin{cases} \Sigma F_x = F_{x_2} + R_{xB} = 0 \\ \Sigma F_y = F_1 + F_{y_2} - R_A - R_B \\ \Sigma M_A = R_A \cdot 0 + F_1 \cdot 2 - F_{y_2} \cdot 6 - R_B \cdot 8 + F_{x_2} \cdot 0 + R_x \cdot 0 \end{cases}$$

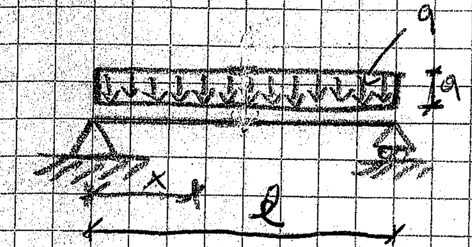
$$\begin{cases} \Sigma F_x = 28{,}28\,KN + R_{xB} = 0 \\ \Sigma M_A = 0 + (30 \cdot 2) - (28{,}28 \cdot 6) - R_B \cdot 8 \\ \Sigma F_y = 30 + 28{,}28 - R_A - R_B \end{cases}$$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 58{,}28 - R_A - R_B \\ \Sigma M_A = 60 + 169{,}69 - R_B \cdot 8 \end{cases}$$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 58{,}28 - R_A - R_B \\ R_B = \frac{229{,}69}{8} = 28{,}71\,KN \end{cases} \qquad \begin{cases} \Sigma F_x = 0 \\ R_A = 58{,}28 - 28{,}71 = 29{,}57 \\ R_B = 28{,}71\,KN \end{cases}$$

50

CARICO UNIFORMEMENTE DISTRIBUITO.

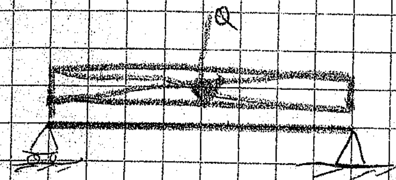


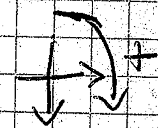
ENTITÀ DI CARICO $\to q = 300$ N/ml

$l = 6$ m

$Q = q \cdot l = 300 \cdot 6 = 1800$

$x \begin{cases} 0 \\ l = 6 \end{cases}$

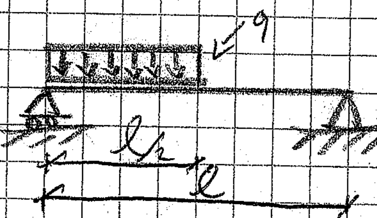




$$\begin{cases} \sum F_x = 0 \to = 0 \\ \sum F_y = 0 \Rightarrow R_A + (q \cdot l) + R_B = 0 \\ \sum M_A = 0 \Rightarrow R_A \cdot 0 + \left(q \cdot l \cdot \frac{l}{2}\right) - R_B \cdot l = 0 \end{cases}$$

$$\sum M_A = R_B \cancel{l} = \frac{1}{2} q \cdot l^{\cancel{2}} \Rightarrow R_B = \frac{1}{2} q \cdot l$$

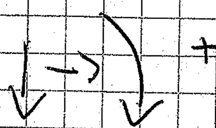
es casa



~~q = 80 kN~~

$q = 500$ kN/ml

$l = 4$ m



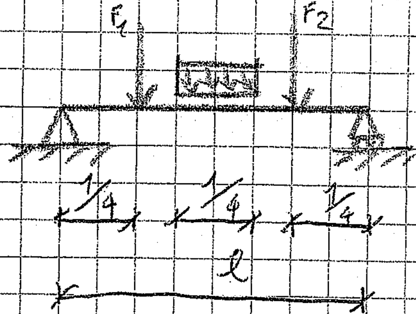
$$\begin{cases} \sum F_y = R_A + \left(q \cdot \frac{l}{2}\right) + R_B \\ \sum M_A = R_A \cdot 0 + \left(q \cdot \frac{l}{2} \cdot \frac{l}{4}\right) - R_B \cdot l \end{cases}$$

$$\begin{cases} \sum F_y = R_A + (500 \cdot 2) + R_B \\ \sum M_A = 0 + (500 \cdot 2 \cdot 1) - R_B \cdot 4 \end{cases}$$

$$\begin{cases} \sum F_y = R_A + 1000 + R_B \\ R_B = +\frac{1000}{4} = +250 \text{ KN} \end{cases}$$

$$\begin{cases} R_A = +1000 - 250 = +750 \text{ KN} \\ R_B = +250 \text{ KN} \end{cases}$$

Es. 2



$l = 8\,m$

$F_1 = 10\,KN$

$F_2 = 20\,KN$

$q = 300\,\frac{KN}{ml}$

$$\begin{cases} \Sigma F_y = R_A + F_1 + \left(q \cdot \frac{l}{4}\right) + F_2 + R_B \\ \Sigma M_A = R_A \cdot 0 + F_1 \cdot 2 + \left(q \cdot \frac{l}{4} \cdot 4\right) + F_2 \cdot 6 - R_B \cdot 8 \end{cases}$$

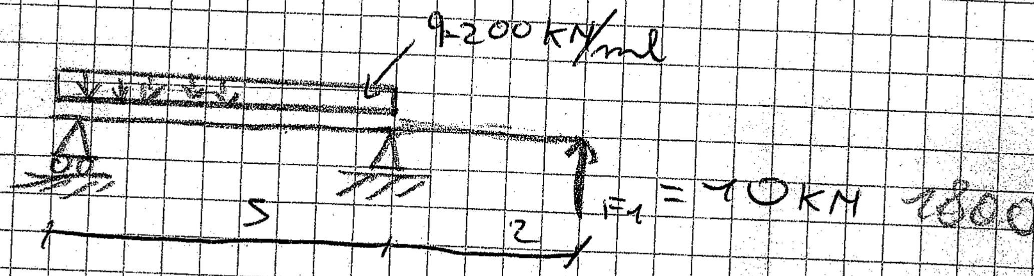
$$\begin{cases} \Sigma F_y = R_A + 10 + (300 \cdot 2) + 20 + R_B \\ \Sigma M_A = 0 + 20 + 2400 + 120 - R_B \cdot 8 \end{cases}$$

$$\begin{cases} \Sigma F_y = R_A + 10 + 600 + 20 + R_B \\ \cancel{\Sigma M_A} \; R_B = \frac{-2540}{8} = -317{,}5\,KN \end{cases}$$

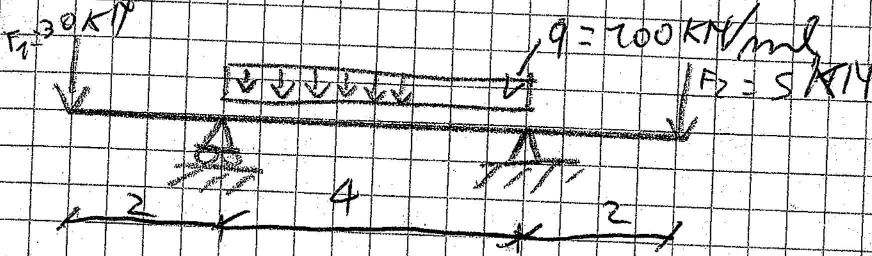
$$\begin{cases} R_A = 10 + 600 + 20 - 317{,}5 = -312{,}5\,KN \\ R_B = -317{,}5\,KN \end{cases}$$

TRAVI A SBALZO

①



②



① $\begin{cases} \Sigma F_y = (200 \cdot 5) \cancel{+4} \, 10 + R_A + R_B \\ \Sigma M_A = (\cancel{R_A \cdot 0} + (200 \cdot 5 \cdot 2,5) - 10 \cdot 7 - R_B \cdot 7 \end{cases}$

$\begin{cases} \Sigma F_y = 1000 - 10 + R_A + R_B \\ \Sigma M_A = 2500 - 70 - R_B \cdot 7 \end{cases}$

$\begin{cases} \Sigma F_y = 990 + R_A + R_B \\ R_B = \frac{-2430}{7} = -347,14 \end{cases}$

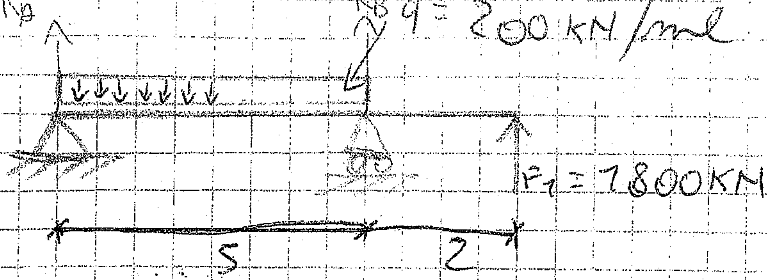
486

$\begin{cases} -R_A = 990 - \cancel{347,14} = \cancel{642,86 \text{ KN}} -504 \text{ KN} \\ R_B = \cancel{-347,14 \text{ KN}} -486 \text{ KN} \end{cases}$

② $\begin{cases} \Sigma F_y = 30 + R_A + (100 \cdot 4) + R_B + 5 \\ \Sigma M_A = (30 \cdot 2) + \cancel{R_A \cdot 0} + (100 \cdot 4 \cdot 2) - R_B \cdot 4 + (5 \cdot 6) \end{cases}$

$\begin{cases} \Sigma F_y = 30 + R_A + 405 + R_B \\ \Sigma M_A = -60 + 800 - R_B \cdot 4 + 30 \end{cases}$

$\begin{cases} -R_A = 435 + R_B \\ R_B = \frac{770}{4} = -192,5 \end{cases}$ $\begin{cases} -R_A = 435 - 192,5 = -242,5 \\ R_B = -192,5 \end{cases}$



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = R_A + (5 \cdot 200) + R_B - 1800 \\ \sum M_A = \cancel{R_A \cdot 0} + (5 \cdot 200 \cdot 2{,}5) - R_B \cdot 5 - 1800 \cdot 7 \end{cases}$$

$$\begin{cases} \sum F_y = R_A + 1000 + R_B - 1800 \\ \sum M_A = 2500 - 12600 - R_B \cdot 5 \end{cases}$$

$$\begin{cases} \sum F_y = R_A - 800 + R_B \\ R_B = \frac{-10100}{-5} = 2020 \end{cases}$$

$$\begin{cases} R_A = 800 - 2020 = \boxed{-1220 \text{ KN}} \\ R_B = \boxed{2020 \text{ KN}} \end{cases}$$

1) GLI ORGANISMI STRUTTURALI IN UNA COSTRUZIONE

Ogni costruzione si compone di elementi tra loro assemblati

elementi < di finitura / strutturali

2) LA PROGETTAZIONE DEGLI ELEMENTI STRUTTURALI

a) Individuazione dello schema statico
b) Analisi dei carichi
c) Risoluzione dello schema statico con i carichi (reazioni vincolari)
d) Determinazione delle caratteristiche di sollecitazione
e) Analisi tensionale con progetto e verifiche degli elementi strutturali

3) UNITA' DI MISURA

FORZE $\longrightarrow$ N (Newton) $\quad 1\ Kg = 9,81\ N$

$1\ Kg = 1\ daN$

TENSIONI $\longrightarrow$ $1\ Pa = \frac{1\ N}{m^2}$ (Pascal)

$$1 \frac{Kg}{cm^2} = 10 \frac{N}{cm^2} = 0,10 \frac{N}{mm^2} = 0,10\ MPa$$

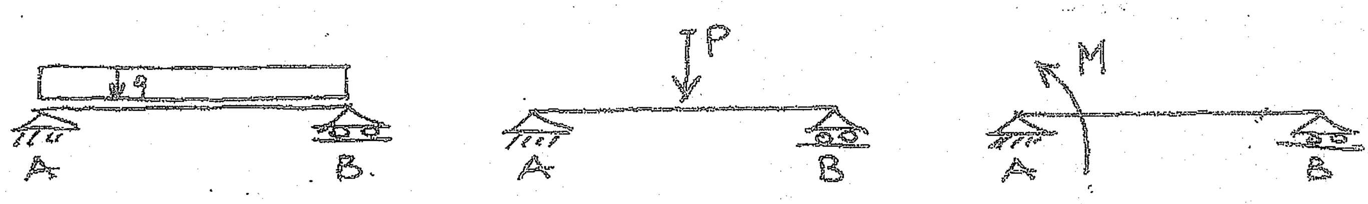
$$1\ MPa = \frac{10^6\ N}{m^2} = 1 \frac{N}{mm^2} = 10 \frac{Kg}{cm^2}$$

4) LE STRUTTURE ISOSTATICHE

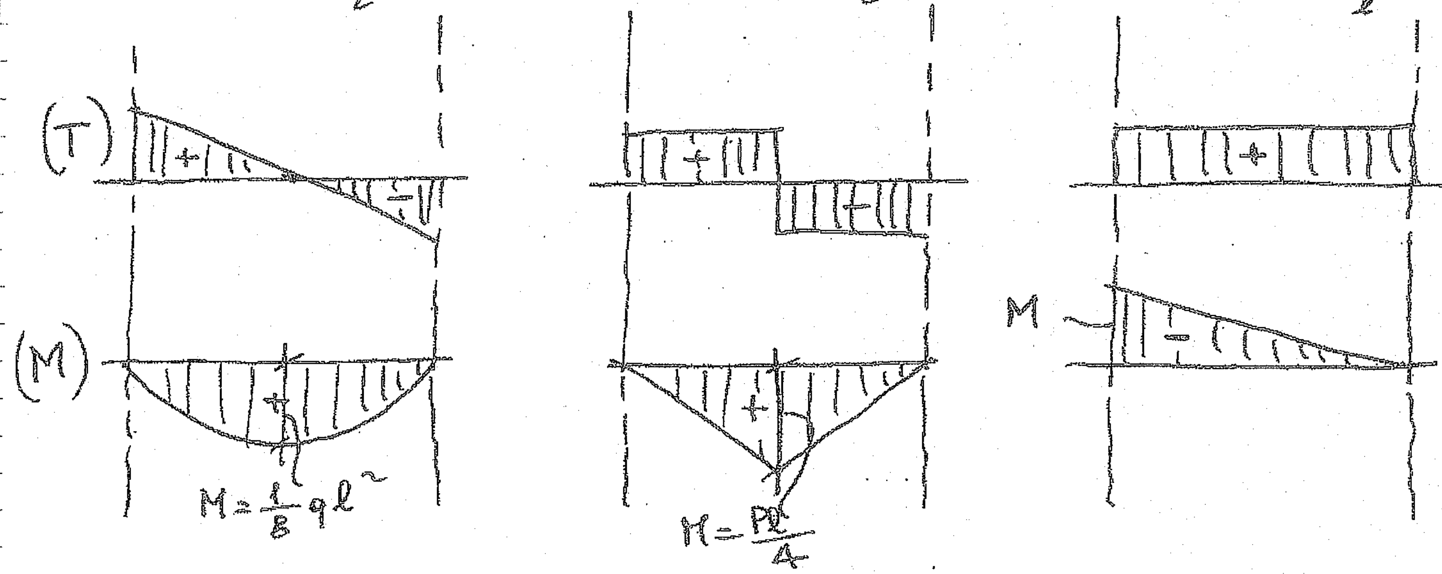
Le equazioni fondamentali della statica dei sistemi rigidi

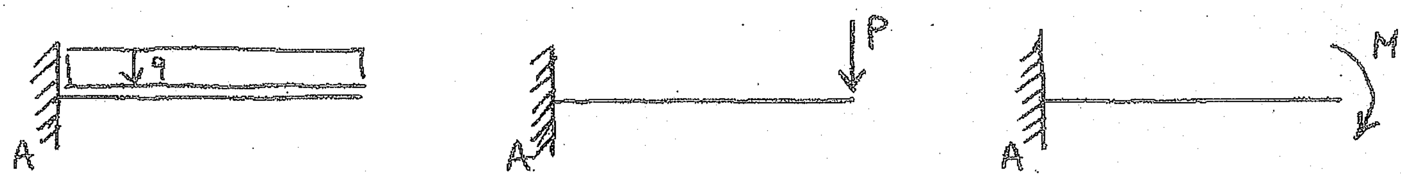
$$\begin{cases} \Sigma F_o = 0 \\ \Sigma F_v = 0 \\ \Sigma M = 0 \end{cases}$$

4) RIEPILOGO STRUTTURE ISOSTATICHE



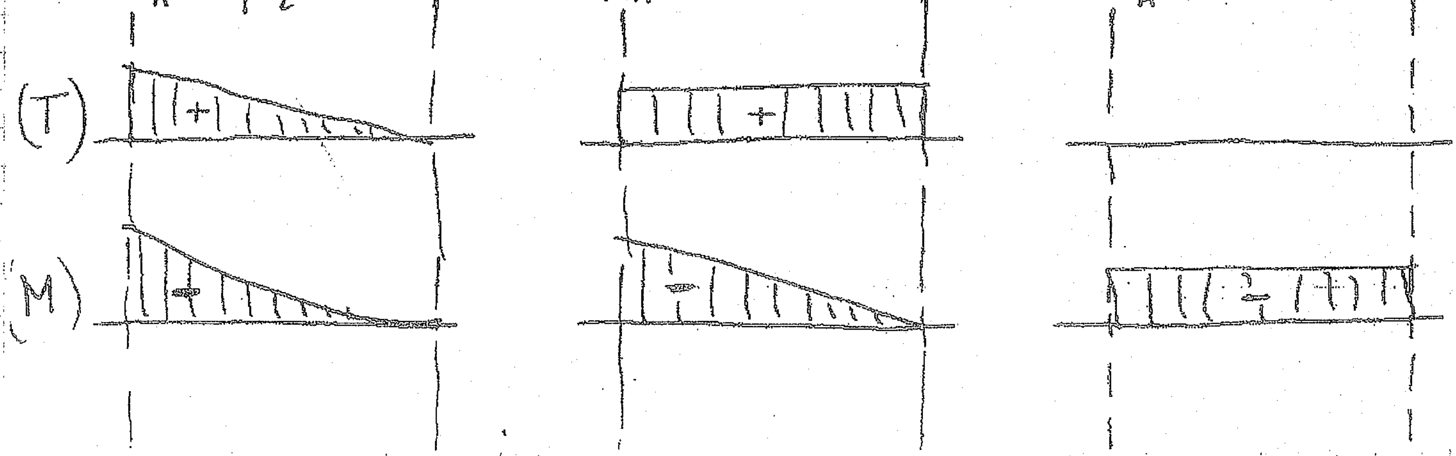
$R_A = R_B = \frac{ql}{2}$ $\qquad$ $R_A = R_B = \frac{P}{2}$ $\qquad$ $R_A = -R_B = \frac{M}{l}$

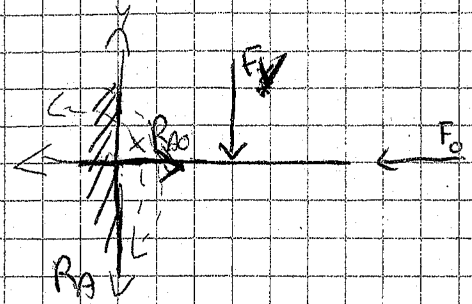




$R_A = ql$ $\qquad$ $R_A = P$ $\qquad$ $R_A = 0$

$M_A = q\frac{l^2}{2}$ $\qquad$ $M_A = Pl$ $\qquad$ $M_A = M$





1) isost ?

$3 \cdot n = 1 \cdot a + 2 \cdot c + 3 \cdot i$

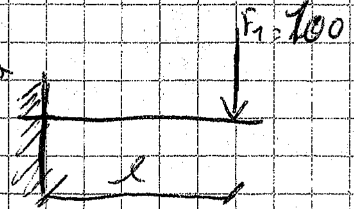
$3 \cdot 1 = 0 + 0 + 3$

$3 = 3$ isost

2) convenzione

3) equazioni

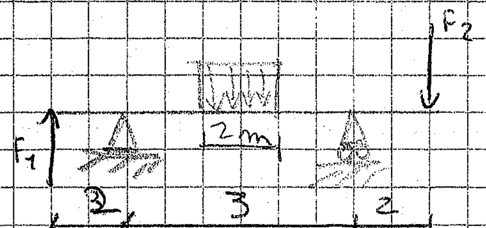
$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases}$$



$$\begin{cases} \Sigma F_y = R_A + F_1 = 0 \\ \Sigma M_A = M_A + F_1 \cdot l = 0 \end{cases}$$

$$\begin{cases} R_A = -F_1 \\ M_A = -F_1 \cdot l \end{cases}$$

1)

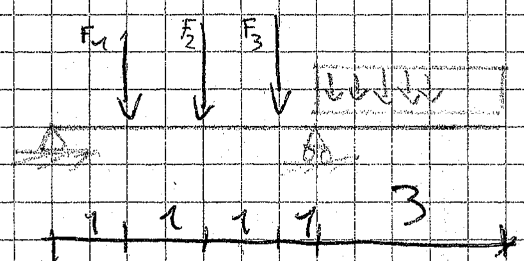


$F_1 = 100\,kN$

$F_2 = 20\,kN$

$q = 50\,kN/ml$

2)



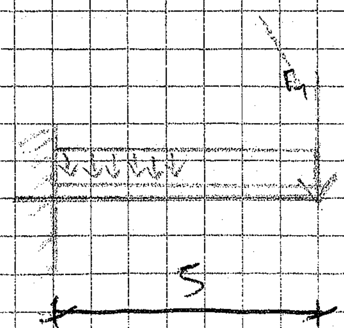
$F_1 = 10\,kN$

$F_2 = 20\,kN$

$F_3 = 30\,kN$

$q = 25\,kN/ml$

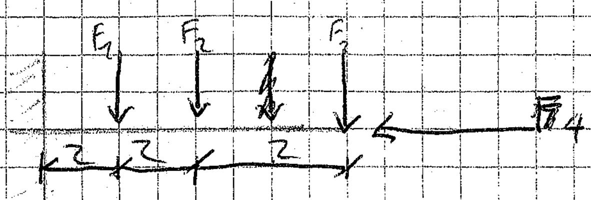
3)



$q = 10\,kN/ml$

$F_1 = 24\,kN$

4)



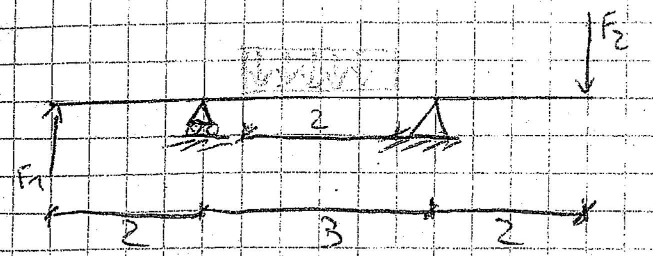
$F_1 = 15\,kN$

$F_2 = 10\,kN$

$F_3 = 5\,kN$

$F_4 = 20\,kN$

1)



$F_1 = 100\ kN$

$F_2 = 20\ kN$

$q = 50\ kN/ml$

$\Sigma F_x =$ non ci sono forze orizzontali $= 0$

$\Sigma F_y = F_1 + R_A + (q \cdot l) + R_B + F_2$

$\Sigma M_A = (F_1 \cdot 2) + (R_A \cdot 0) + (q \cdot l \cdot 1{,}5) \mp (R_B \cdot 3) \mp F_2 \cdot 5$

$\Sigma F_y = \cancel{200} + R_A \cancel{+100} + R_B + \cancel{100}\ 120$

$\Sigma M_A = +200 + 0 + 150 - R_B \cdot 3 + \cancel{100}\ 100$

$\Sigma F_y = \cancel{200}\ 120 + R_A + R_B$

$\Sigma M_A = \cancel{300}\ 450 - R_B \cdot 3$

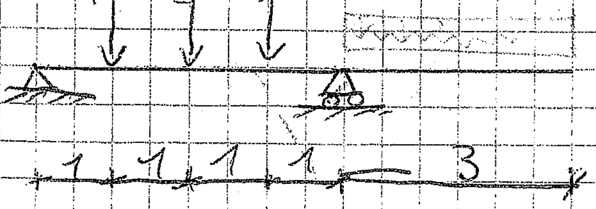
$\Sigma F_y = 120 + R_A + R_B$

$R_B = \frac{450}{3} = \cancel{100}\ \cancel{130}\ 150$

$-R_A = 120 + 150 = -270\ kN$

$R_B = +150\ kN$

n. 2



$F_1 = 10\ kN$

$F_2 = 20\ kN$

$F_3 = 30\ kN$

$q = 25\ kN/ml$

$\Sigma F_y = 0$

$\Sigma F_x = F_1 + F_2 + F_3 + (q \cdot l) + R_A + R_B$

$\Sigma M_A = R_A \cdot 0 + (F_1 \cdot 1) + (F_2 \cdot 2) + (F_3 \cdot 3) + (R_B \cdot 4) + (q \cdot 3 \cdot 5{,}5)$

$\Sigma F_x = 10 + 20 + 30 + 75 + R_A + R_B$

$\Sigma M_A = 0 + 10 + 40 + 90 \mp R_B \cdot 4 + 412{,}5$

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$\Sigma F_x = 135 + R_A + R_B$

$\Sigma M_A = 552{,}5 - R_B \cdot 4$

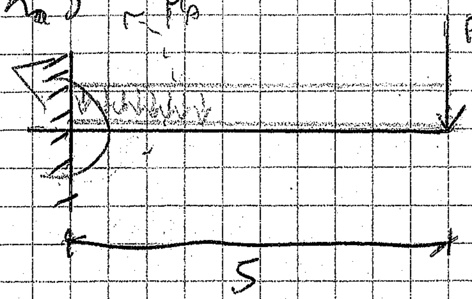
$\Sigma F_x = 135 + R_A + R_B$

$R_B = \frac{552{,}5}{4} = 138{,}13$

$-R_A = 135 + 138{,}13 = -273{,}13\ kN$

$R_B = 138{,}13\ kN$

3) n. 3



$F_1 = 24\,kN$

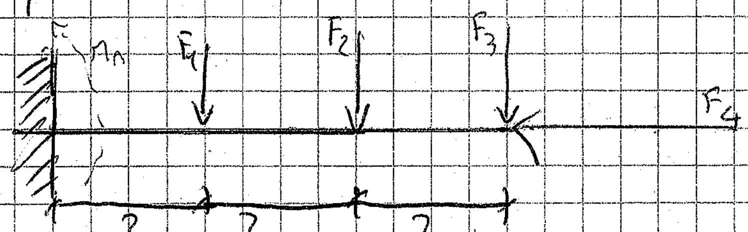
$q = 10\,kN/ml$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = R_A + (q \cdot l) + F_1 \\ \Sigma M_A = +M_A + (q \cdot l \cdot l/2) + (F_1 \cdot l) \end{cases}$$

$$\begin{cases} \Sigma F_y = R_A + 50 + 24 \\ \Sigma M_A = +M_A + 125 + 120 \end{cases}$$

$$\begin{cases} R_A = 74 = -74\,kN \\ M_A = 245 = -245\,kN/ml \end{cases}$$

n. 4



$F_1 = 15\,kN$

$F_2 = 10\,kN$

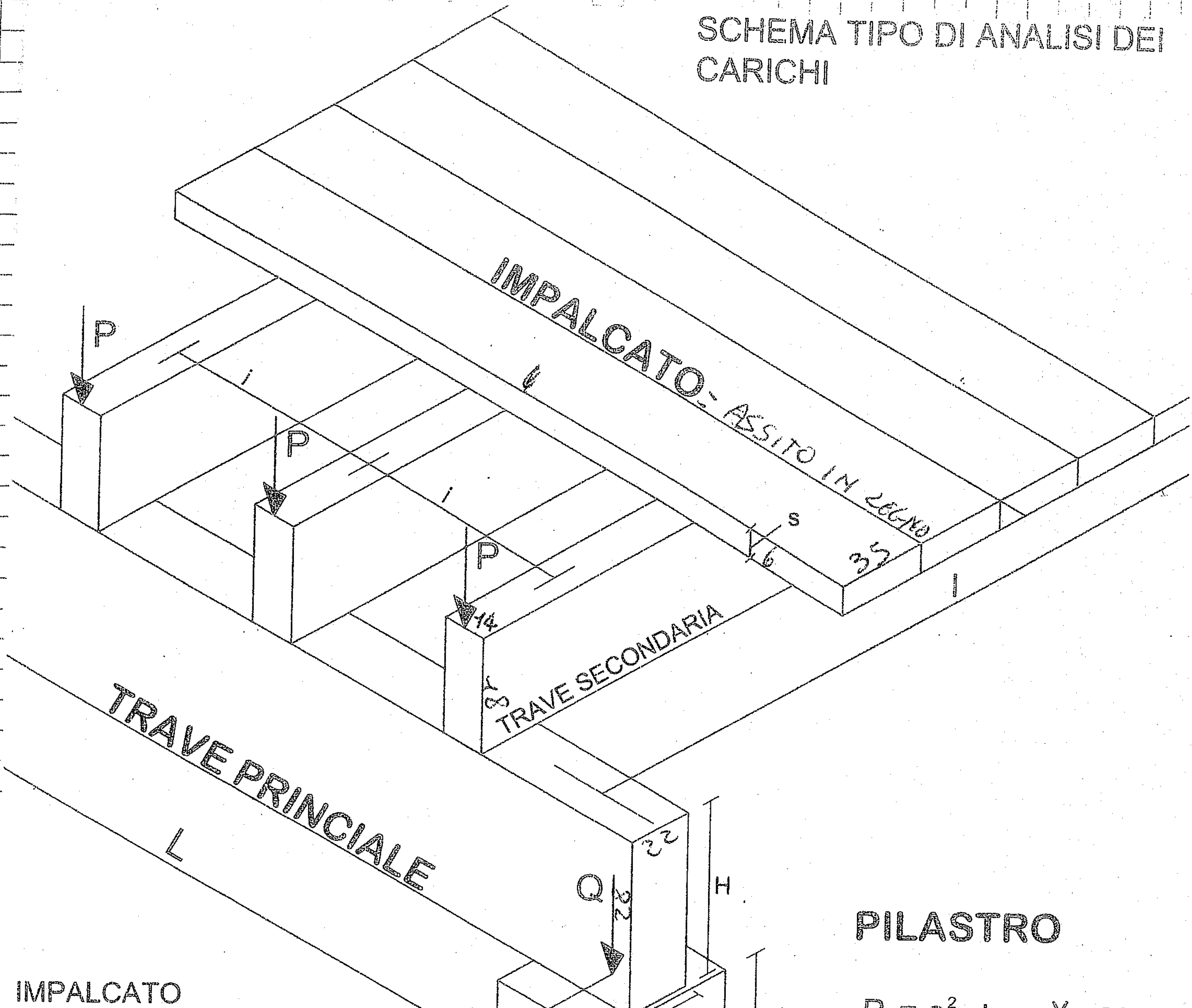
$F_3 = 5\,kN$

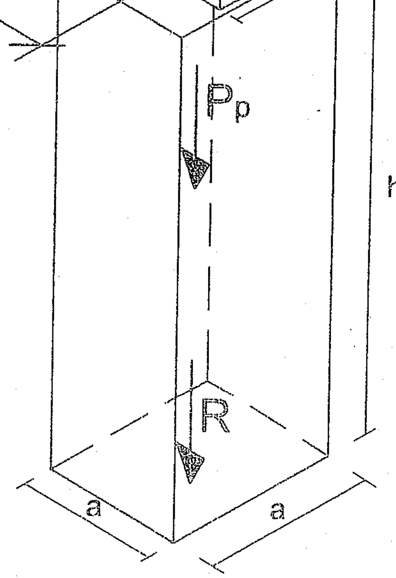
$F_4 = 20\,kN$

$$\begin{cases} \Sigma F_x = 20\,kN - R_{xB} = 0 \\ \Sigma F_y = R_A + 15 + 10 + 5 \\ \Sigma M_A = +M_A + (F_1 \cdot 2) + (F_2 \cdot 4) + (F_3 \cdot 6) \end{cases}$$

$$\begin{cases} R_{xB} = 20\,kN \\ R_A = -30\,kN \\ M_A = 30 + 40 + 30 = -100\,kN/ml \end{cases}$$

SCHEMA TIPO DI ANALISI DEI CARICHI



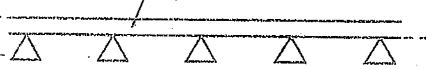


PILASTRO

$$P_p = a^2 \cdot h_1 \cdot \gamma_p \ \text{[Kg]}$$

$$R = Q + P_p \ \text{[Kg]}$$

IMPALCATO



sovraccarico accidentale Kg/m^2

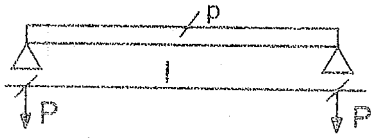
peso proprio impalcato

$$(s \cdot 1 \cdot 1)\,\frac{m^3}{\cancel{m^2}} \times \gamma\ \frac{Kg}{m^3} = \frac{Kg}{m^2}$$

---

q = sovr. acc. + p.p.i [Kg/m^2]

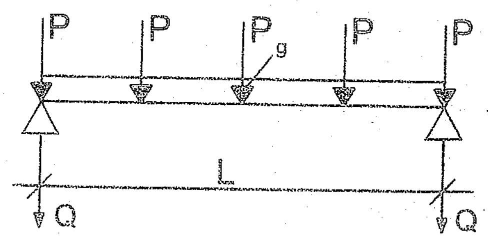
TRAVE SECONDARIA



$$p = q \cdot i \ \text{[Kg/m]} + b \cdot h \cdot 1 \cdot \gamma \ \text{[Kg/m]}$$

$$P = \frac{p \cdot l}{2} \ \text{[Kg]}$$

TRAVE PRINCIPALE



$$P = \frac{p \cdot l}{2} \ \text{[Kg]}$$

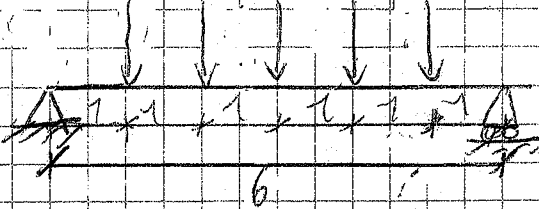
$$g = B \cdot H \cdot 1 \cdot \gamma \ \text{[Kg/m]}$$

$$Q = \frac{\Sigma P}{2} + \frac{g \cdot L}{2} \ \text{[Kg]}$$

$L = 6\ m$

$i = 1\ m$

$\gamma = 850\ Kg/m^3$



$P_{TOT} = 325\ Kg$ $\begin{cases} P_1 = 325 \\ P_2 = '' \\ P_3 = '' \\ P_4 = '' \\ P_5 = '' \end{cases}$

$P_A = 1 \cdot 1 \cdot 0{,}06 \cdot 850 = 51\ Kg$

$P_{TS} = 0{,}14 \cdot 0{,}18 \cdot 1 \cdot 850 = 21{,}42\ Kg$

$P_{TP} = 0{,}22 \cdot 0{,}28 \cdot 1 \cdot 850 = 52{,}36\ Kg$

$\approx 125\ Kg/ml$

$P_{esercizio\ persone} = 200\ Kg/m^2 = 200\ Kg/ml$

$P_{TOT} = 125 + 200 = 325\ Kg/ml$

$P = 325 \cdot 10 = 3250\ N \Rightarrow 3{,}25\ KN$

$$\begin{cases} \Sigma F_y = R_A + P_1 + P_2 + P_3 + P_4 + P_5 + R_B \\ \Sigma F|_A = R_A \cdot 0 + (P_1 \cdot 1) + (P_2 \cdot 2) + (P_3 \cdot 3) + (P_4 \cdot 4) + (P_5 \cdot 5) - (R_B \cdot 6) \end{cases}$$

$$\begin{cases} \Sigma F_y = R_A + 1625 + R_B \\ \Sigma M_A = 0 + 325 + 650 + 975 + 1300 + 1625 - R_B \cdot 6 \end{cases}$$

$$\begin{cases} -R_A = 1625 + R_B \\ -R_B = \frac{4875}{6} = -812{,}5\ KN \end{cases} \qquad \begin{cases} +R_A = -8{,}12\ KN \\ R_B = -8{,}12\ KN \end{cases}$$

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