APPUNTI E CONCETTI BASE PER LA MATERIA DI COSTRUZIONI PER GLI ALUNNI 3°B ITG "NERVI" - ALTAMURA

Si può dire che, in generale, lo scopo della **scienza** e della **tecnica** delle costruzioni è quello di stabilire le condizioni di **sicurezza** e di **funzionalità** delle strutture.

Nel corso di costruzioni degli istituti per geometri si raggiungono le competenze necessarie a dimensionare alcuni semplici elementi di una struttura.

Questo risultato si può ottenere esercitandosi nello studio delle **azioni** agenti sull'intera struttura e sui singoli elementi strutturali, e nell'analisi delle **sollecitazioni** e delle **tensioni** interne presenti in ognuna delle sue sezioni.

Le azioni sulle costruzioni sono costituite essenzialmente da **forze** e **momenti**, il cui studio è previsto nei corsi di fisica del biennio (cinematica, statica).

Innanzitutto: cos'è un elemento strutturale? È una parte (elemento) della struttura che può essere studiato singolarmente. Sono elementi strutturali le fondazioni, le murature, i pilastri, le travi e tutte quelle parti dell'edificio senza le quali non si terrebbe in piedi. Non sono elementi strutturali gli infissi (porte e finestre), i muri divisori, i pavimenti ed i rivestimenti, et cetera.

Gli elementi strutturali sono rappresentati in modo schematico con un segmento proporzionale alla loro lunghezza. Con questa rappresentazione **bidimensionale** i possibili movimenti si riducono a due **traslazioni** (orizzontali e verticale), dovute a delle forze, ed ad una **rotazione**, dovuta ad una coppia di forze (momento).

I collegamenti degli elementi strutturali tra loro e con il mondo esterno sono simboleggiati dai vincoli.

I più comuni sono:

il **carrello** impedisce la traslazione orizzontale

la **cerniera** impedisce sia la traslazione orizzontale che quella verticale

l'**incastro** impedisce le due traslazioni nel piano e la rotazione

Naturalmente una trave viene progettata in modo che non si muova affatto. I vincoli quindi devono essere disposti in modo da impedire tutti i possibili movimenti. Se ciò accade la struttura è detta **isostatica** (o **iperstatica**), altrimenti la struttura è detta **ipostatica** o **labile**.

Studieremo le principali strutture isostatiche: la trave a mensola, la trave appoggiata (con e senza sbalzi), la trave Gerber, l'arco a tre cerniere.

Inizialmente ci limiteremo ad individuare le sole forze esterne (azioni e reazioni vincolari). Passeremo quindi ad analizzare le azioni interne (sollecitazioni e tensioni). Infine proveremo a dimensionare e verificare alcuni semplici elementi strutturali.

$q = 34$ kN/m in kN/cm = 3400 kN/cm → $\frac{34 \cdot kN}{cm \cdot 100} = 0{,}34$ kN

ERRATA CORRIGE

1 Kg ⇒ 10 N

1 Kg ⇒ 0,01 N

M = 2800 kg·m ⇒ kN·m → 2800 kg·m = 28 kN·m

MKS ⇒ S.I.

ESEMPI

26.000 kg·m = 260 kN·m

30.000 kg ⇒ 30.000 N

1 daN ⇒ 10 N

1 kN ⇒ 1000 N → $N\,10^3$

1 MN ⇒ 1.000.000 N ⇒ 10^6 N

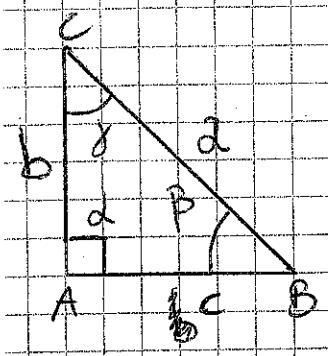
ESERCIZI PAG. 6

3) 26 kN/m in N/cm ⇒ $\frac{26 \cdot 1.000}{100} = 260$ N/cm

4) 85 kg/cm² in N/mm² ⇒ $\frac{85 \cdot 10}{100} = 8{,}50$ N/mm²

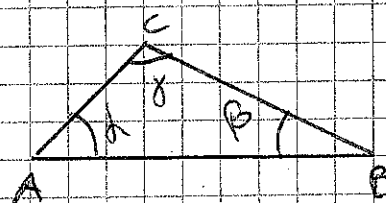
5) 42 kN in N ⇒ 42·1000 kN = 42.000 N

6) 32 kN in N/mm ⇒ $32 \cdot 10^6$ N = 32.000.000 N



$\sqrt{a^2 + b^2} = c$

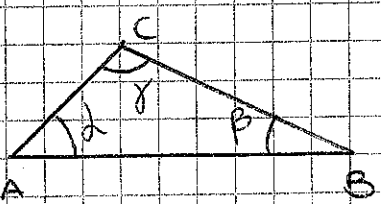
$\boxed{c^2 = a^2 + b^2}$



FORMULA DI CARNOT

$c^2 = b^2 + a^2 - 2b \cdot a \cdot \cos \alpha$

$= 10^2 + 15^2 - 2 \cdot 10 \cdot 15 \cdot 0{,}34 = 223$



$AB^2 = AC^2 + BC^2 - 2AC \cdot BC \cdot \cos \gamma$

OPPURE

$a^2 = b^2 + c^2 - 2b \cdot c \cdot \cos \alpha$

$c^2 = b^2 + a^2 - 2b \cdot a \cdot \cos \gamma$

$= a \cdot \cos \beta + b \cdot \cos \alpha$

2D. PROIEZIONE $\Rightarrow a = b \cdot \cos \gamma + c \cdot \cos \beta$

$= c \cdot \cos \alpha + a \cdot \cos \gamma$

ARICHI → CONCENTRATI
→ DISTRIBUITI

$= m \cdot g$

$= m \cdot a$

$p = \gamma \cdot h$

peso

$F = R$

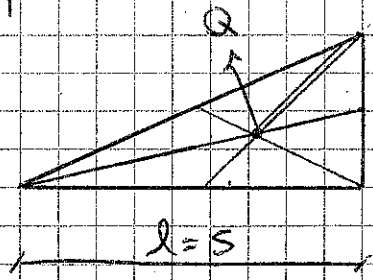
$F - R = 0$

$= 65 \cdot 9{,}81 = 637{,}65 \Rightarrow 638\ N \rightarrow 0{,}638\ N$

$N = 1\ kg \cdot 1\ m / s^2$

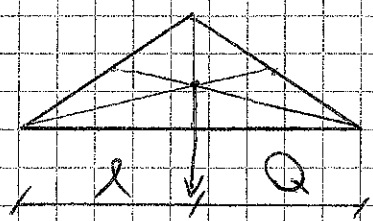
$q = N/mm$

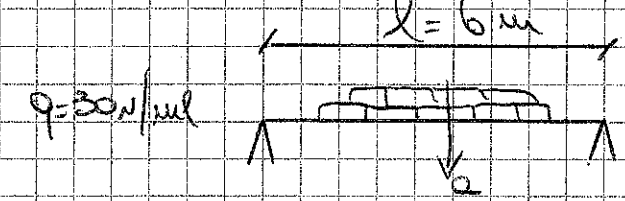
$q = 0$



$$Q = \frac{q \cdot l}{2} = \frac{100 \cdot 5}{2} = 250\ N/mm$$

$q = 40\ N/mm$

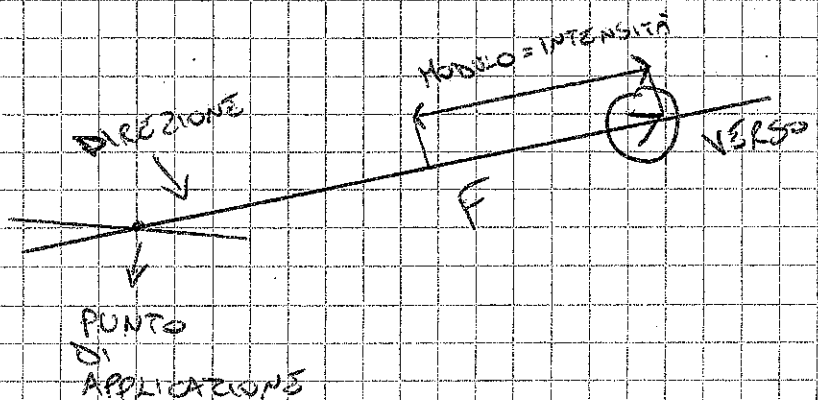




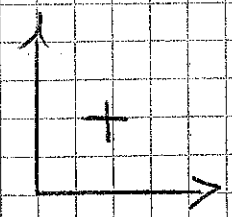
$$Q = q \cdot l = 30 \cdot 60 = 180\ N/mm$$

GRANDEZZA VETTORIALE:

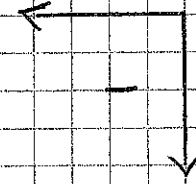
1) PUNTO DI APPLICAZIONE;
2) INTENSITÀ;
3) DIREZIONE;
4) VERSO;



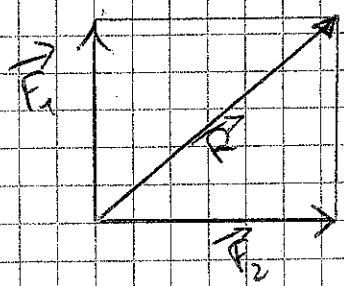
FORZA VERSO L'ALTO E A DESTRA, SEMPRE POSITIVO:



FORZA VERSO IL BASSO E A SINISTRA, SEMPRE NEGATIVO:



EQUILIBRIO: QUANDO LA SOMMATORIA DELLE FORZE APPLICATE A QUEL CORPO È = a 0.

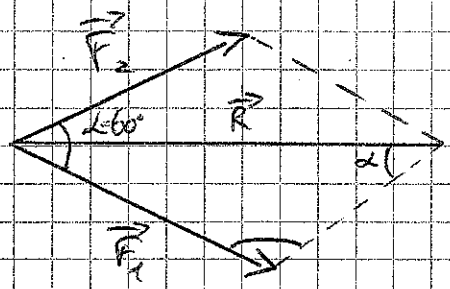


$\vec{F_1} = 10\ N$

$\vec{F_2} = 8\ N$

$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2} = \sqrt{10^2 + 8^2} = \sqrt{164} = 12{,}8\ N$

ESEMPIO DI FORZE E RISOLUZIONI



FORMULA

$$\vec{R} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 - 2 F_1 \cdot F_2 \cdot \cos\beta} =$$

$$= \sqrt{20^2 + 10^2 - 2 \cdot 20 \cdot 10 \cdot \cos\beta} =$$

$$= \sqrt{400 + 100 - 400 \cdot (-0{,}5)} =$$

$$= \sqrt{500 + 200} = \sqrt{700} = 26{,}46 \text{ N}$$

DATI:

$\vec{F_1} = 20\,N$

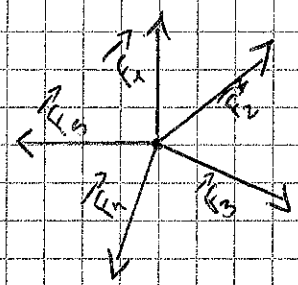
$\vec{F_2} = 10\,N$

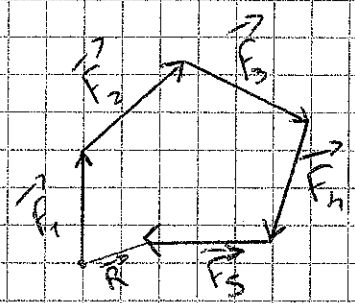
$\angle = 60°$

$\beta = 120°$

$\alpha = 30°$

$\gamma = 30°$





IL MOMENTO È IL PRODOTTO DI UNA FORZA PER UNA DISTANZ
QUESTA DISTANZA È PERPENDICOLARE ALLA FORZA E VIEN
CHIAMATA "BRACCIO". VIENE INDICATO CON M.

$M = F \cdot d = [N \cdot m]$

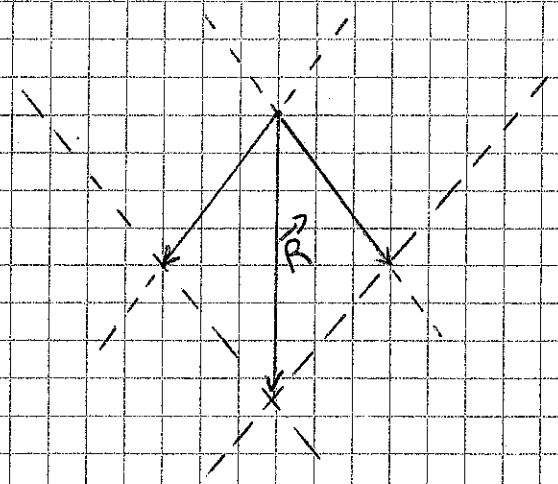
$M = F \cdot b$

λ SI CHIAMA CONDUCIBILITÀ TERMICA DEL MATERIALE AVENTE UN CERTO SPESSORE (W/m K) VIENE CHIAMATA TRASMITTANZA TERMICA (U) DI UN PACCHETTO DI STRATO (W/m²

BIOEDILIZIA

1. EFFICIENZA ENERGETICA, NORMATIVA DI RIFERIMENTO, BIOEDILIZIA BIOARCHITETTURA, PROGETTAZIONE.

LA BIOEDILIZIA = BIO → FAVOREVOLE ALLA VITA.



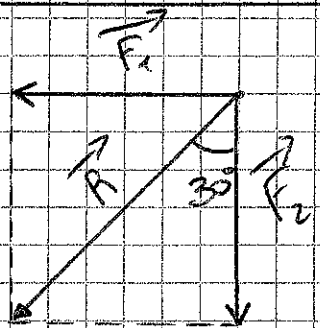
FORMULE:

$\vec{R} = \vec{F_1} + \vec{F_2}$ = TECNICA

$|R| + |F_1| + |F_2|$ = MATEMATICA

$R = F_1 + F_2$ = FISICA

$\vec{R} = \sqrt{F_1^2 + F_2^2 - 2 F_1 F_2 \cdot \cos\alpha}$



$\vec{F_2} = 17{,}40\ N$

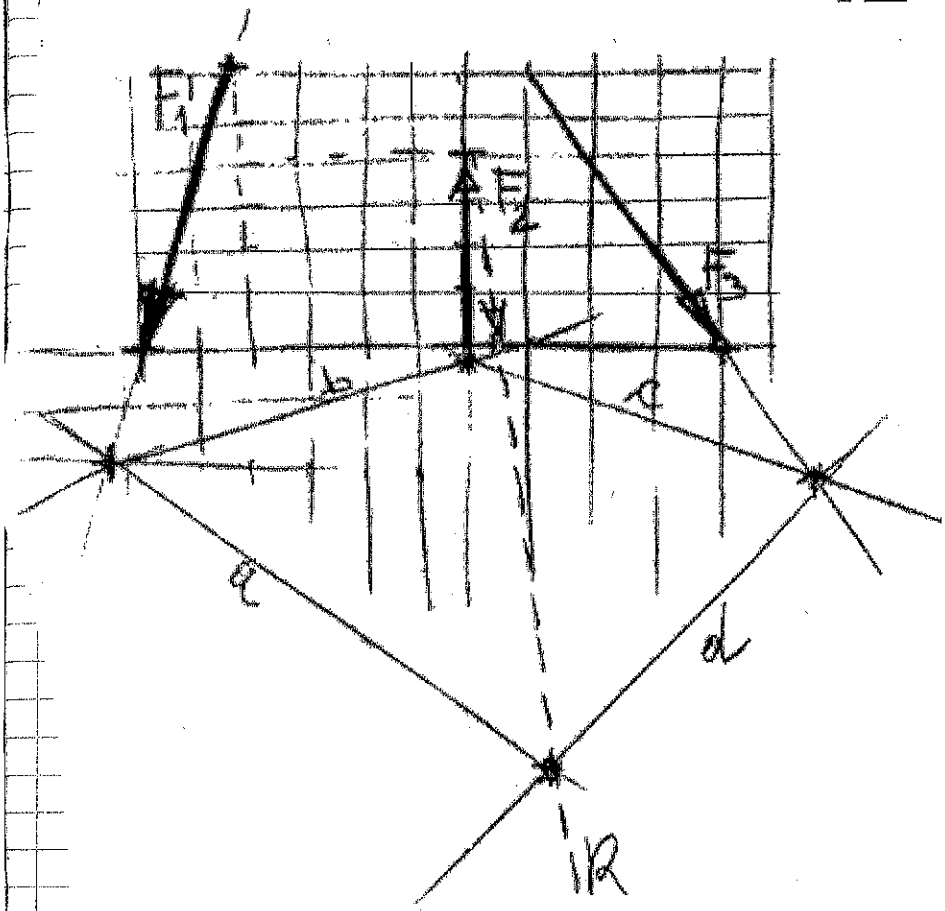
$\vec{F_1} = 10\ N$

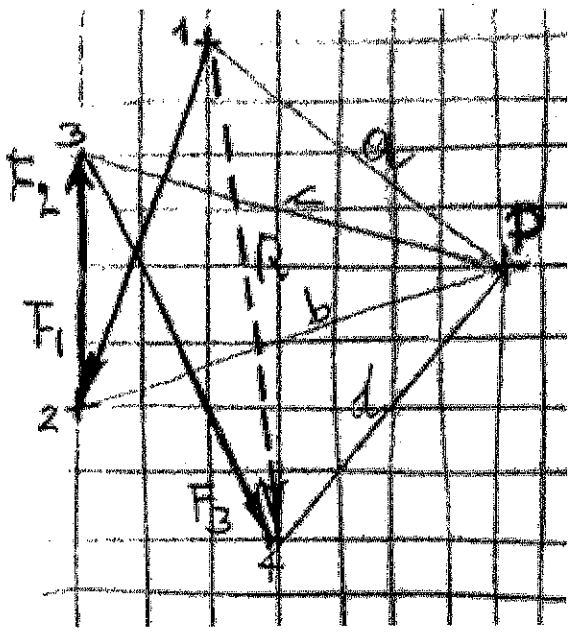
$\vec{R} = \vec{F_1} + \vec{F_2}$

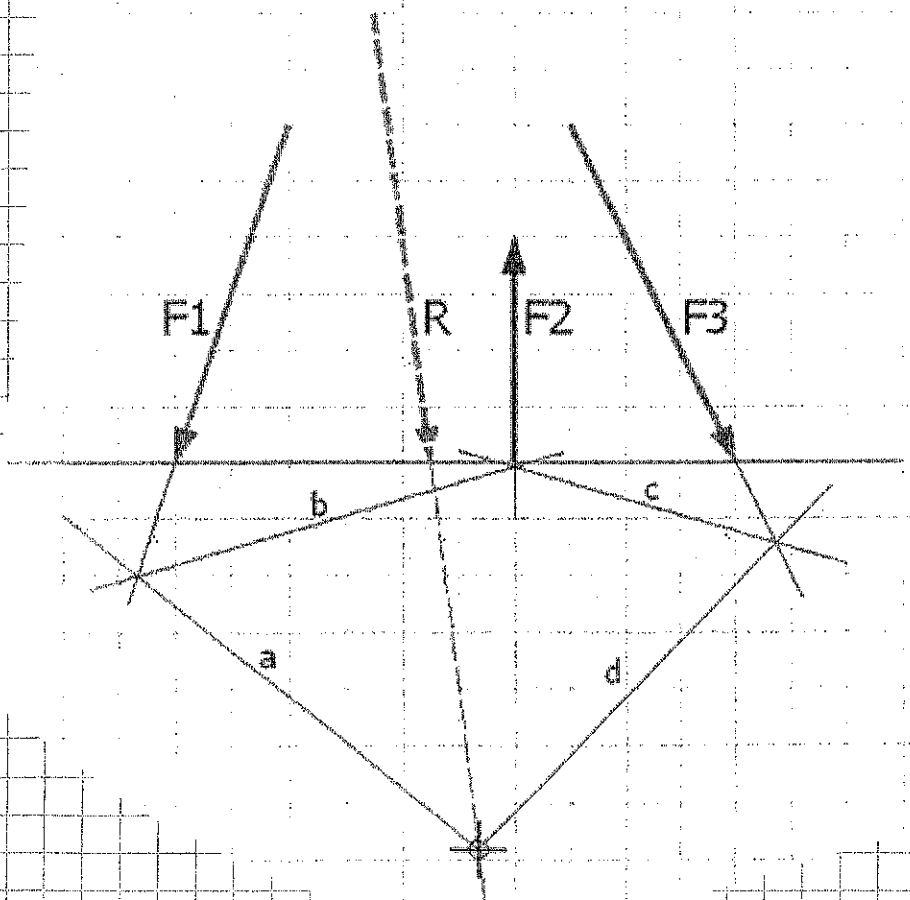
$\vec{F_2} = \vec{R} \cdot \cos(30°)$

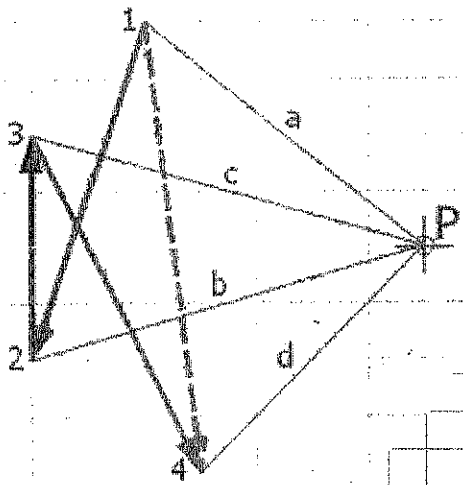
$\vec{F_1} = \vec{R} \cdot \sin(30°)$

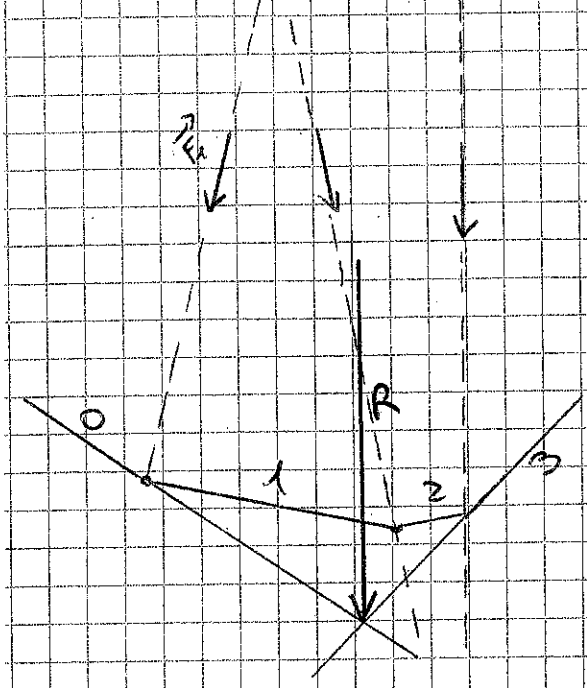
IL POLIGONO FUNICOLARE

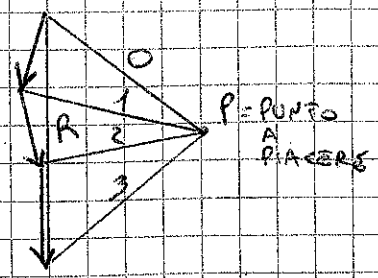


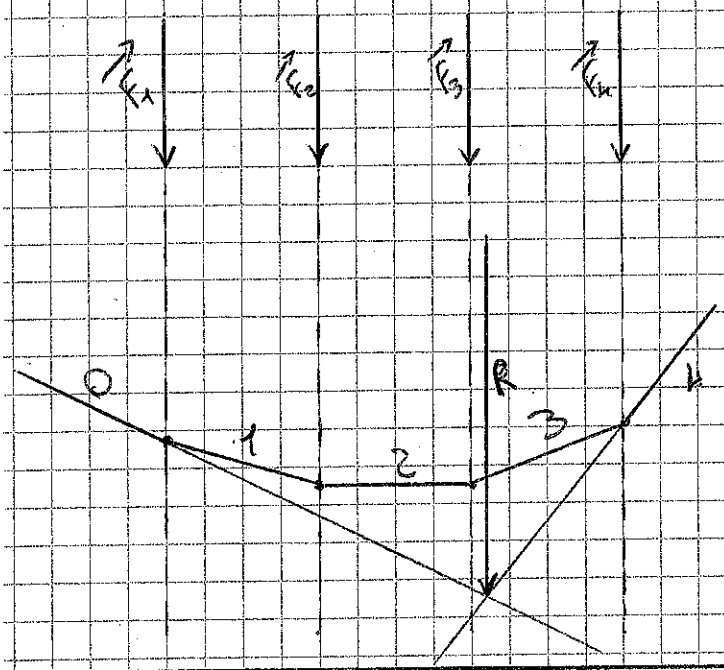


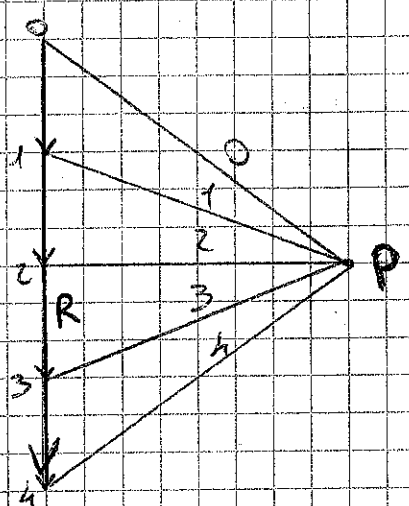


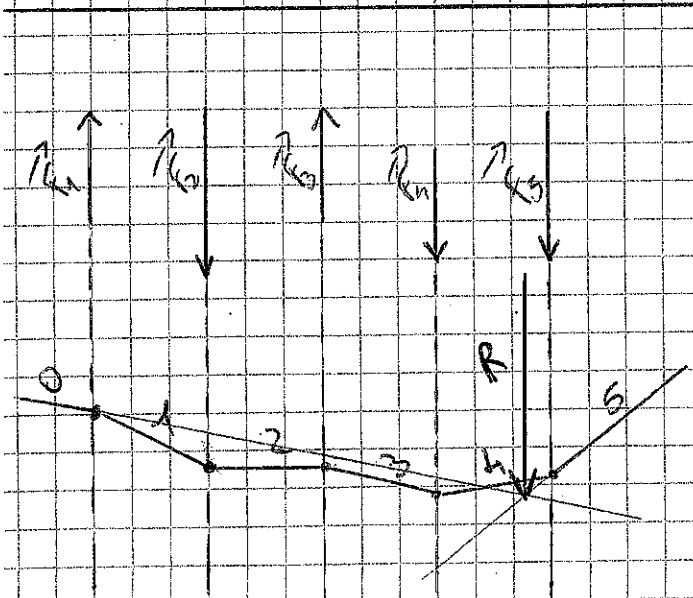


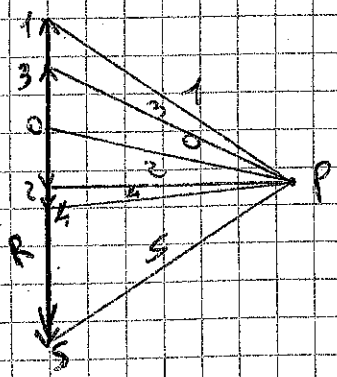

R
0
1
2
3


0
1
R
2
3
P - PUNTO A PIACERE

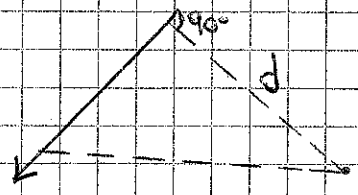

R
0
1
2
3
4


0
1
2
R
3
4
P


R
0
1
2
3
4
5


1
3
0
2
R
4
5
P

DETERMINAZIONE DEI MOMENTI IN UN SISTEMA DI VETTORI



$M = F \cdot d$

$M = F \cdot (b)$

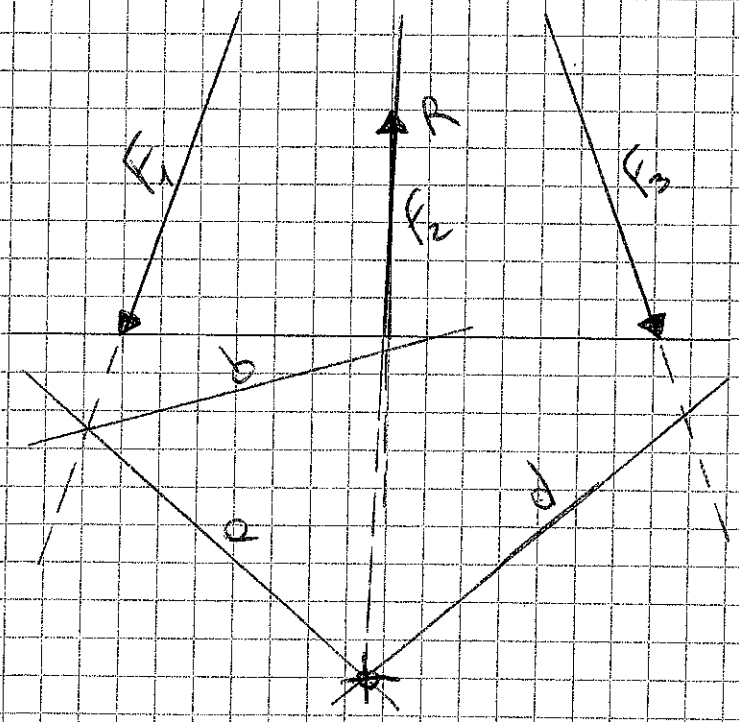
$M_{(P)} = \vec{F}_1 \cdot d$

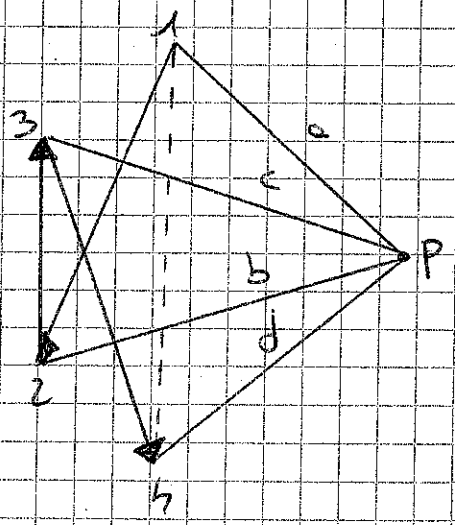
$M_{(P)} = F \cdot b$

$M_{(P)} = 5 \cdot 3 = 15$ (N m)

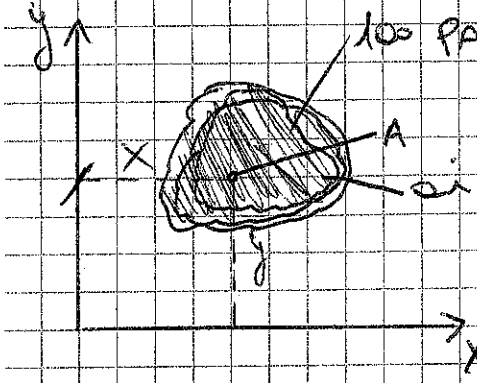
$M_{(O)} = F \cdot b$

ESERCIZIO: POLIGONO FUNICOLARE





IL BARICENTRO È IL PUNTO AL QUALE È APPLICATA LA FORZA RISULTANTE DI TUTTE LE FORZE PESO PARALLELE. PUÒ COINCIDERE CON IL CENTRO DI MASSA DI UN CORPO, E ANCHE CON IL SUO CENTRO DI GRAVITÀ, IL CHE PORTA SPESSO A RITENERE QUESTI TRE TERMINI INTERCAMBIABILI.



$S_x = A \cdot y$ $\quad S_x = \sum_1^n a_i \cdot y$

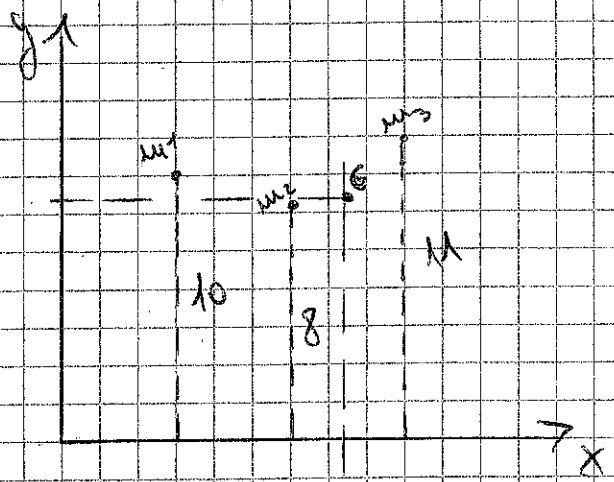
$S_y = A \cdot x$ $\quad S_y = \sum_1^n a_i \cdot x$

$m^2 \cdot m = m^3$

$$\boxed{S = A \cdot d} \quad [m^3]$$

$m_1 = 10\ kg$
$m_2 = 8\ kg$
$m_3 = 11\ kg$

$X_G = ?$
$y_G = ?$



$S_y = (m_1 \cdot x_1 + m_2 \cdot y_2 + m_3 \cdot y_3) = (10 \cdot 4) + (15 \cdot 6) + (30 \cdot 10) = 430$

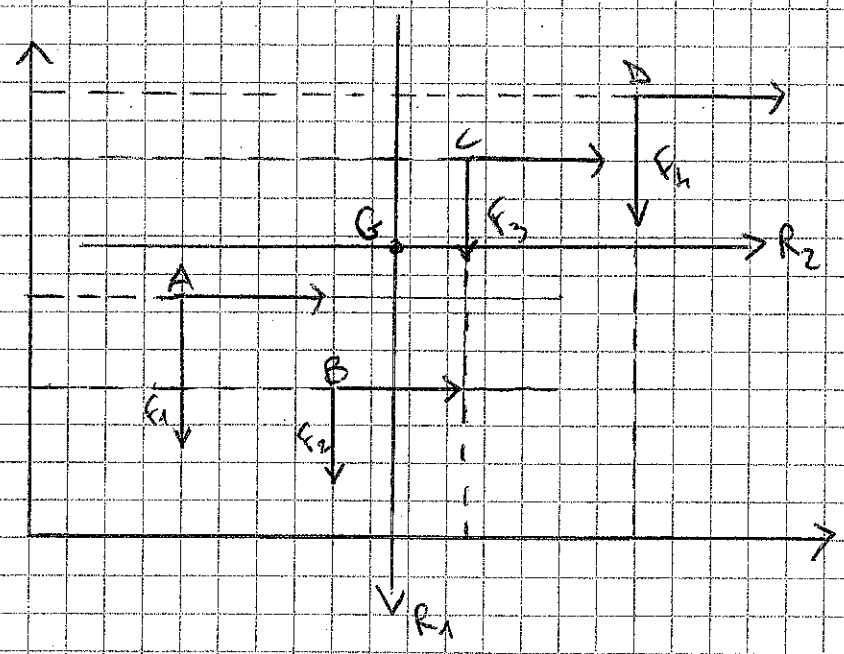
$X_G = \frac{S_y}{\sum A} = \frac{430}{55} = 7{,}81\ m$

$S_x = (m_1 \cdot 10) + (m_2 \cdot 8) + (m_3 \cdot 11) = 550\ kg \cdot m$

$y_G = \frac{S_x}{A} = \frac{550}{55} = 10\ m$

$G(7{,}81 ; 10)$
$\quad x \quad y$

Pg. 54 n: 1

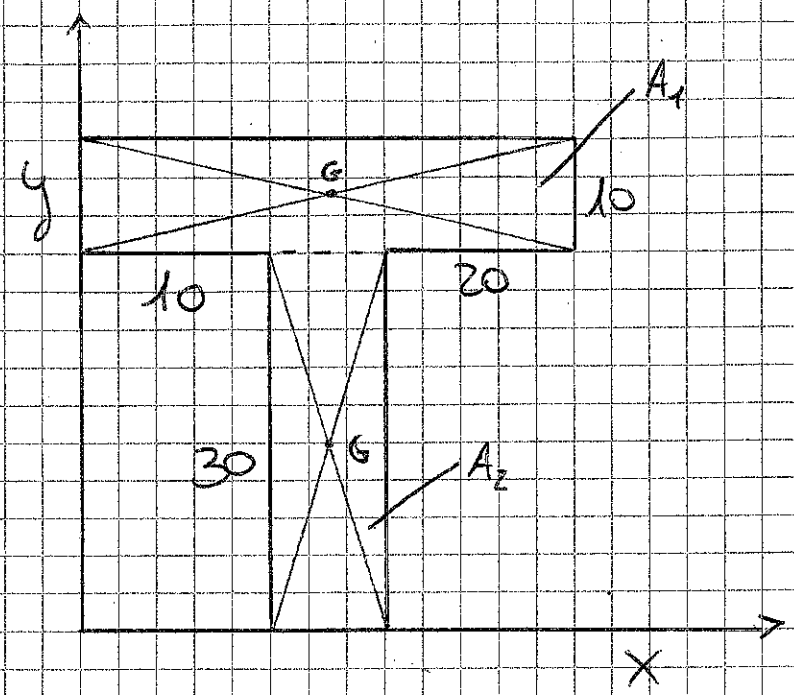


FORZE	INTENSITÀ	MOMENTO RISPETTO ALL'ASSE Y		MOMENTO RISPETTO ALL'ASSE X	
		X cm	$M_y = F \cdot X$ (N cm)	Y cm	$M_x = F \cdot y$ (N cm)
F_1	- 6	+ 3	- 18	+ 9	- 54
F_2	- 4	+ 8	- 32	+ 6	- 24
F_3	- 4	+ 12	- 48	+ 14	- 98
F_4	- 3	+ 20	- 60	+ 16	
	$\Sigma F = -20$	$\Sigma (F \cdot X) = -194$		$\Sigma (F \cdot y) = -224$	

$$X_G = \frac{\Sigma (F \cdot X)}{\Sigma F} = \frac{-194}{-20} = 9{,}7 \text{ cm}$$

$$y_G = \frac{\Sigma (F \cdot y)}{\Sigma F} = \frac{-224}{-20} = 11{,}20 \text{ cm}$$

ESERCIZI IN CLASSE



$X_1 = 20$ cm $\quad y_1 = 35$ cm

$X_2 = 15$ cm $\quad y_2 = 15$ cm

$A_1 = (40 \cdot 10) = 400 \text{ cm}^2$

$A_2 = (30 \cdot 10) = 300 \text{ cm}^2$

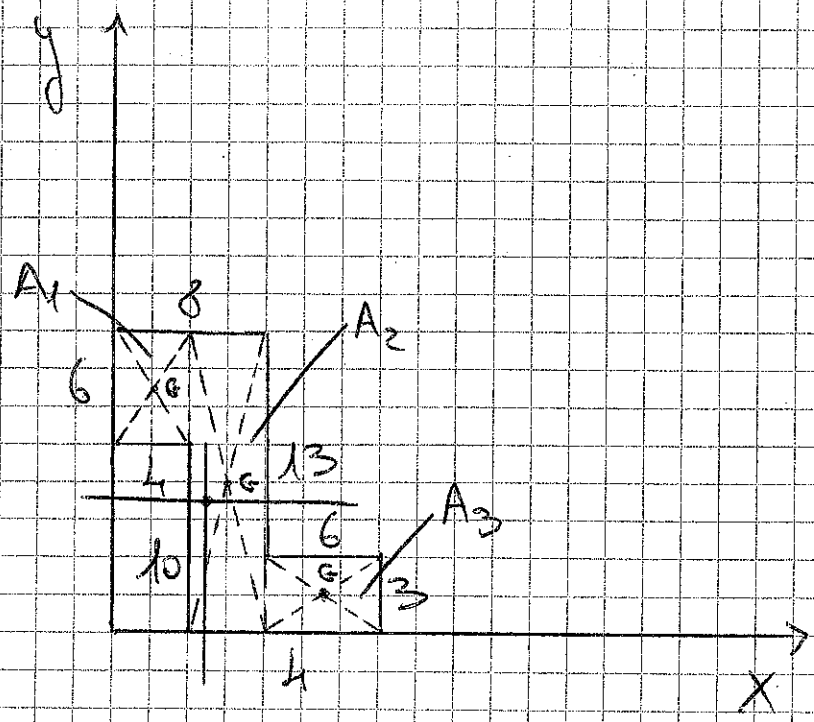
$A = 400 + 300 = 700 \text{ cm}^2$

$S_x = (400 \cdot 35) + (300 \cdot 15) = 14000 + 4500 = 18\,500 \text{ cm}^3$

$y_G = \frac{S_x}{A} = \frac{18\,500}{700} = 26{,}43 \text{ cm}$

$S_y = (400 \cdot 20) + (300 \cdot 15) = 8000 + 4500 = 12\,500 \text{ cm}^3$

$X_G = \frac{S_y}{A} = \frac{12\,500}{700} = 17{,}86 \text{ cm}$



$X_1 = 2$ cm $\quad y_1 = 13$ cm

$X_2 = 6$ cm $\quad y_2 = 8$ cm

$X_3 = 11$ cm $\quad y_3 = 1,5$ cm

$A_1 = 6 \cdot 4 = 24$ cm²

$A_2 = 16 \cdot 4 = 64$ cm²

$A_3 = 6 \cdot 3 = 18$ cm²

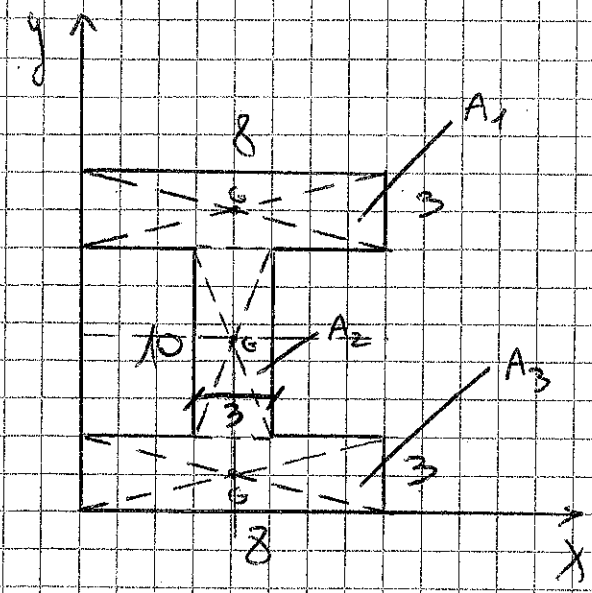
$A = 24 + 64 + 18 = 106$ cm²

$$S_y = (A_1 \cdot X_1) + (A_2 \cdot X_2) + (A_3 \cdot X_3) = (24 \cdot 2) + (64 \cdot 6) + (18 \cdot 11) = 630 \text{ cm}$$

$$X_G = \frac{S_y}{A} = \frac{630}{106} = 5,94$$

$$S_x = (A_1 \cdot y_1) + (A_2 \cdot y_2) + (A_3 \cdot y_3) = (24 \cdot 13) + (64 \cdot 8) + (18 \cdot 1,5) = 851$$

$$y_G = \frac{S_x}{A} = \frac{851}{106} = 8,03$$



$X_1 = 4$ cm $y_1 = 14,5$ cm

$X_2 = 4$ cm $y_2 = 8$ cm

$X_3 = 4$ cm $y_3 = 1,5$ cm

$A_1 = 24$ cm^3 $A_3 = 24$ cm^3

$A_2 = 30$ cm^3

$S_{x(A_1)} = A_1 \cdot y_1 = 24 \cdot 14,5 = 348$ cm^3

$S_{x(A_2)} = A_2 \cdot y_2 = 30 \cdot 8 = 240$ cm^3

$S_{x(A_3)} = A_3 \cdot y_3 = 24 \cdot 1,5 = 38$ cm^2

$$y_G = \frac{\Sigma (S_x)}{\Sigma A} = \frac{348 + 240 + 38}{78} = \frac{626}{78} = 8 \text{ cm}^3$$

$S_{y(A_1)} = A_1 \cdot X_1 = 24 \cdot 4 = 96$ cm^3

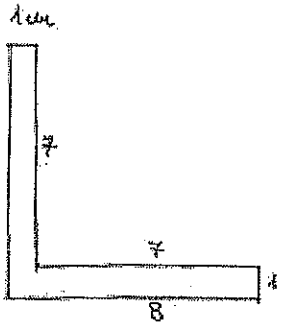
$S_{y(A_2)} = A_2 \cdot X_2 = 30 \cdot 4 = 120$ cm^3

$S_{y(A_3)} = A_3 \cdot X_3 = 24 \cdot 4 = 96$ cm^3

$$X_G = \frac{\Sigma \cdot S(y)}{\Sigma A} = \frac{96 + 120 + 96}{24 + 30 + 24} = \frac{312}{78} = 4 \text{ cm}^3$$

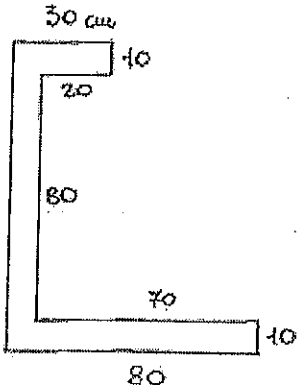
ESERCIZI DI RIEPILOGO

1)

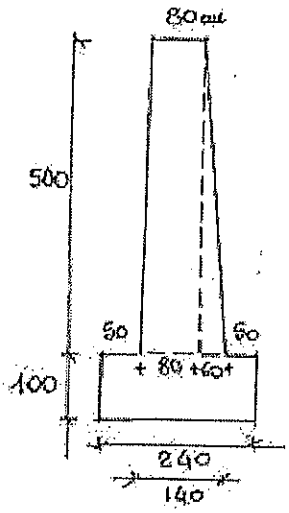


DETERMINARE LE COORDINATE DEI BARICENTRI DELLE SEGUENTI SEZIONI_

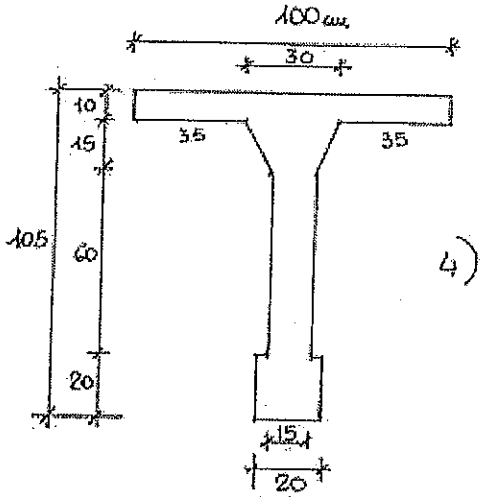
2)



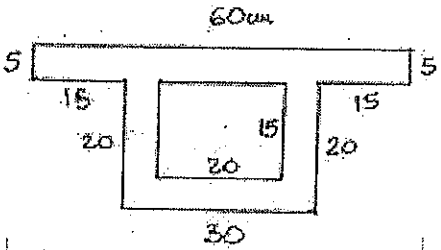
3)



4)

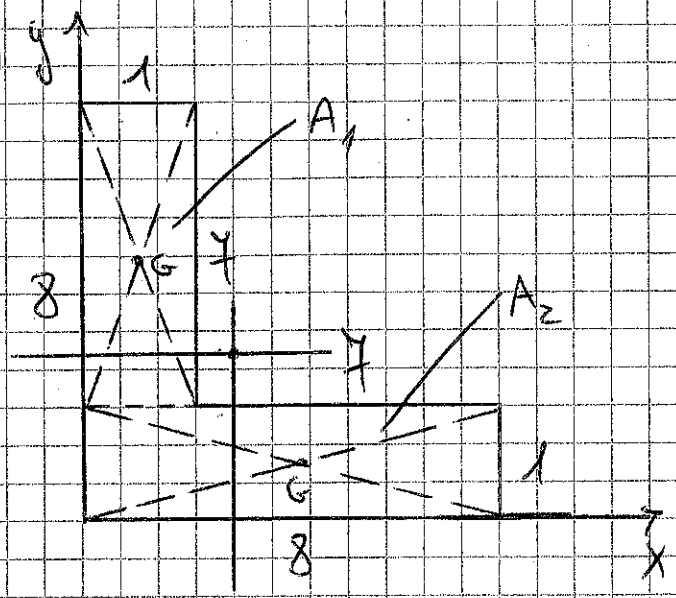


5)



ESERCIZI DI RIEPILOGO

1)



$X_1 = 0,5$ cm $\quad y_1 = 4$ cm

$X_2 = 4$ cm $\quad y_2 = 0,5$ cm

$A_1 = 4$ cm^3 $\quad A_2 = 4$ cm^3

$S_{x(A_1)} = A_1 \cdot y_1 = 4 \cdot 4 = 28$ cm

$S_{x(A_2)} = A_2 \cdot y_2 = 4 \cdot 0,5 = 3,5$ cm

$S_{y(A_1)} = A_1 \cdot X_1 = 4 \cdot 0,5 = 3,5$ cm

$S_{y(A_2)} = A_2 \cdot X_2 = 4 \cdot 4 = 28$ cm

$y_G = \frac{\sum S_{(x)}}{\sum A} = \frac{3,5 + 28}{14} = \frac{31,5}{14} = 2,25$

$X_G = \frac{\sum S_{(y)}}{\sum A} = \frac{28 + 3,5}{14} = \frac{31,5}{14} = 2,25$

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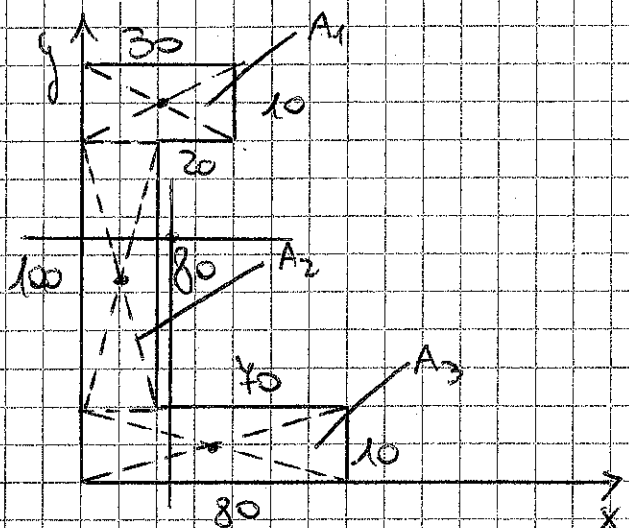
2)



$X_1 = 15$ cm $\quad y_1 = 95$ cm

$X_2 = 5$ cm $\quad y_2 = 50$ cm

$X_3 = 40$ cm $\quad y_3 = 5$ cm

$A_1 = 300$ cm^3 $\quad A_2 = 800$ cm^3 $\quad A_3 = 800$ cm^3

$S_{x(A_1)} = A_1 \cdot y_1 = 300 \cdot 95 = 28\,500$ cm^3

$S_{x(A_2)} = A_2 \cdot y_2 = 800 \cdot 50 = 40\,000$ cm^3

$S_{x(A_3)} = A_3 \cdot y_3 = 800 \cdot 5 = 4\,000$ cm^3

$S_{y(A_1)} = A_1 \cdot X_1 = 300 \cdot 15 = 4\,500$ cm^3

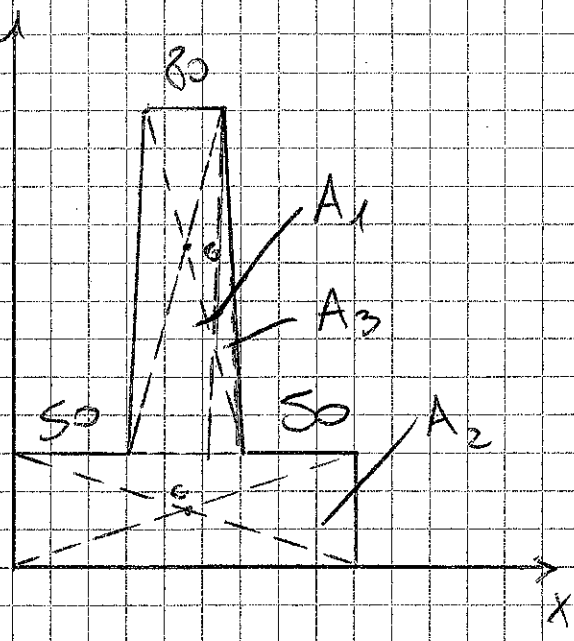
$S_{y(A_2)} = A_2 \cdot X_2 = 800 \cdot 5 = 4\,000$ cm^3

$S_{y(A_3)} = A_3 \cdot X_3 = 800 \cdot 40 = 32\,000$

$y_G = \frac{\sum S_x}{\sum A} = \frac{28\,500 + 40\,000 + 4\,000}{1\,300} = 55,77$ cm^3

$X_G = \frac{\sum S_y}{\sum A} = \frac{4\,500 + 4\,000 + 32\,000}{1\,300} = 31,15$ cm^3

3)



$X_1 = 90$ cm $y_1 = 350$ cm

$X_2 = 150$ cm $y_2 = 266,66$ cm

$X_3 = 120$ cm $y_3 = 50$ cm

$A_1 = 40\,000$ cm^3 $A_3 = 24\,000$

$A_2 = 15\,000$ cm^3

$S_x(A_1) = A_1 \cdot y_1 = 40\,000 \cdot 350 = 14\,000\,000$ cm^3

$S_x(A_2) = A_2 \cdot y_2 = 15\,000 \cdot 266,66 = 3\,999\,900$ cm^3

$S_x(A_3) = A_3 \cdot y_3 = 24\,000 \cdot 50 = 1\,200\,000$ cm^3

$S_y(A_1) = A_1 \cdot y_1 = 40\,000 \cdot 90 = 3\,600\,000$

$S_y(A_2) = A_2 \cdot y_2 = 15\,000 \cdot 150 = 2\,250\,000$

$S_y(A_3) = A_3 \cdot y_3 = 24\,000 \cdot 120 = 2\,880\,000$

$X_G = \frac{\sum S_x}{\sum A} = \frac{19\,199\,900}{79\,000} = 243,04$ cm^3

$X_G = \frac{\sum S_y}{\sum A} = \frac{8\,730\,000}{79\,000} = 110,50$

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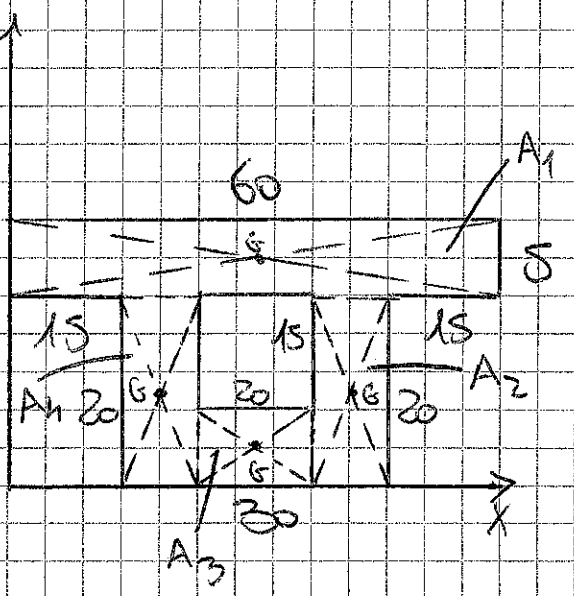
5)



$X_1 = 30$ cm $y_1 = 22,5$ cm

$X_2 = 47,5$ cm $y_2 = 10$ cm

$X_3 = 30$ cm $y_3 = 2,5$ cm

$X_4 = 17,5$ $y_4 = 10$ cm

$A_1 = 300$ cm^3 $A_3 = 100$ cm

$A_2 = 100$ cm^3 $A_4 = 100$ cm

$S_x(A_1) = A_1 \cdot y_1 = 300 \cdot 22,5 = 6\,750$ cm^3

$S_x(A_2) = A_2 \cdot y_2 = 100 \cdot 10 = 1000$ cm^3

$S_x(A_3) = A_3 \cdot y_3 = 100 \cdot 2,5 = 250$ cm^3

$S_x(A_4) = A_4 \cdot y_4 = 1000 \cdot 10 = 1\,000$ cm^3

$S_y(A_1) = A_1 \cdot X_1 = 300 \cdot 30 = 9000$ cm

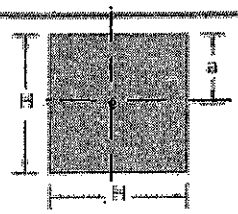
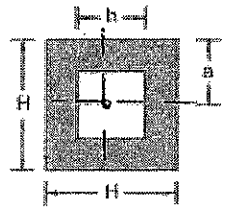
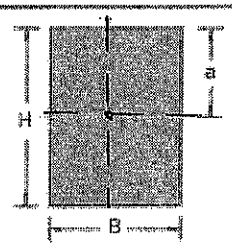
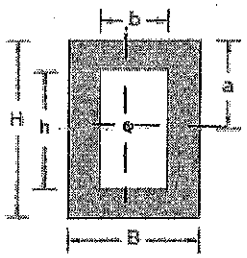
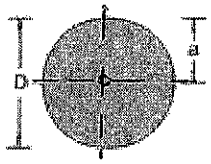
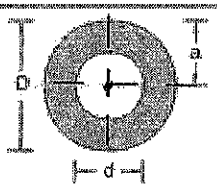
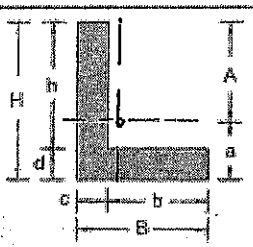
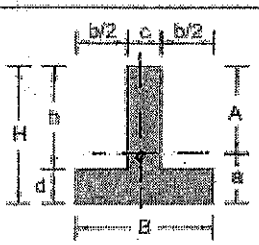
$S_y(A_2) = A_2 \cdot X_2 = 100 \cdot 47,5 = 4750$ cm

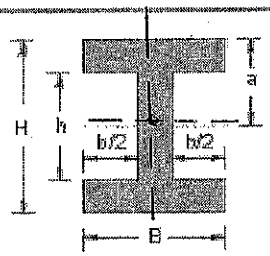
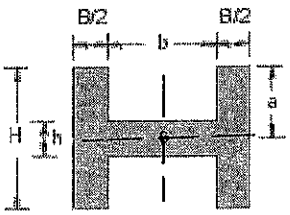
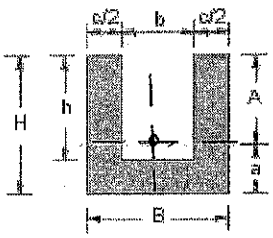
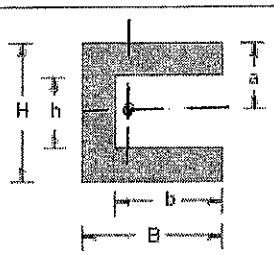
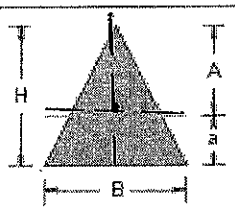
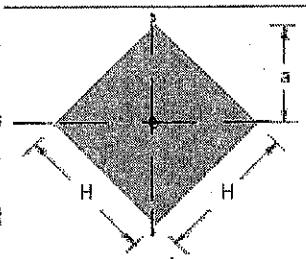
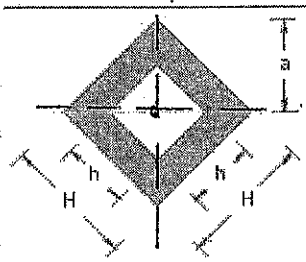
$S_y(A_3) = A_3 \cdot X_3 = 100 \cdot 30 = 3000$ cm

$S_y(A_4) = A_4 \cdot X_4 = 100 \cdot 17,5 = 1750$ cm

$y_G = \frac{\sum S_x}{\sum A} = \frac{9\,000}{600} = 15$ cm^3

$X_G = \frac{\sum S_y}{\sum A} = \frac{18\,500}{600} = 30,83$ cm

Sezione	Area della sezione A cm^2	Distanza dal baricentro a cm	Momento di inerzia J cm^4	Modulo di resistenza W cm^3
	H^2	$\frac{H}{2}$	$\frac{H^4}{12}$	$\frac{H^3}{6}$
	$H^2 - h^2$	$\frac{H}{2}$	$\frac{H^4 - h^4}{12}$	$\frac{H^4 - h^4}{6\,H}$
	$B \cdot H$	$\frac{H}{2}$	$\frac{B \cdot H^3}{12}$	$\frac{B \cdot H^2}{6}$
	$BH - bh$	$\frac{H}{2}$	$\frac{1}{12} \cdot (BH^3 - bh^3)$	$\frac{1}{6H} \cdot (BH^3 - bh^3)$
	$\frac{\pi \cdot D^2}{4}$	$\frac{D}{2}$	$\frac{\pi \cdot D^4}{64}$	$\frac{\pi \cdot D^3}{32}$
	$\frac{\pi \cdot (D^2 - d^2)}{4}$	$\frac{D}{2}$	$\frac{\pi \cdot (D^4 - d^4)}{64}$	$\frac{\pi \cdot (D^4 - d^4)}{32 \cdot D}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{I}{A}$ $W_a = \frac{I}{a}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{J}{A}$ $W_a = \frac{J}{a}$

Sezione	Area della sezione A cm^2	Distanza dal baricentro a cm	Momento di inerzia $= I$ cm^4	Modulo di resistenza W cm^3
	$BH - bh$	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$
	$BH - bh$	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$
	$BH - bh$	$A = H - a$ $a = \frac{1}{2} \cdot \frac{cH^2 + bd^2}{cH + bd}$	$\frac{Ba^3 - b(h - A)^3 + cA^3}{3}$	$W_A = \frac{I}{A}$ $W_a = \frac{I}{a}$
	$BH - bh$	$\frac{H}{2}$	$\frac{BH^3 - bh^3}{12}$	$\frac{BH^3 - bh^3}{6H}$
	$\frac{B \cdot H}{2}$	$A = \frac{2H}{3}$ $a = \frac{H}{3}$	$\frac{B \cdot H^3}{36}$	$W_A = \frac{B \cdot H^2}{24}$ $W_a = \frac{B \cdot H^2}{12}$
	H^2	$\frac{H}{2} \cdot \sqrt{2}$	$\frac{H^4}{12}$	$\frac{H^3}{6\sqrt{2}}$
	$H^2 - h^2$	$\frac{H}{2} \cdot \sqrt{2}$	$\frac{H^4 - h^4}{12}$	$\frac{H^4 - h^4}{6H\sqrt{2}}$

I = raggio di inerzia = $\sqrt{\frac{\text{Momento di inerzia}}{\text{Area della sezione}}} = \sqrt{\frac{I}{A}}$

APPUNTI E CONCETTI BASE PER LA MATERIA DI COSTRUZIONI PER GLI ALUNNI 3°B ITG "NERVI" - ALTAMURA

Si può dire che, in generale, lo scopo della **scienza** e della **tecnica** delle costruzioni è quello di stabilire le condizioni di **sicurezza** e di **funzionalità** delle strutture.

Nel corso di costruzioni degli istituti per geometri si raggiungono le competenze necessarie a dimensionare alcuni semplici elementi di una struttura.

Questo risultato si può ottenere esercitandosi nello studio delle **azioni** agenti sull'intera struttura e sui singoli elementi strutturali, e nell'analisi delle **sollecitazioni** e delle **tensioni** interne presenti in ognuna delle sue sezioni.

Le azioni sulle costruzioni sono costituite essenzialmente da **forze** e **momenti**, il cui studio è previsto nei corsi di fisica del biennio (cinematica, statica).

Innanzitutto: cos'è un elemento strutturale? È una parte (elemento) della struttura che può essere studiato singolarmente. Sono elementi strutturali le fondazioni, le murature, i pilastri, le travi e tutte quelle parti dell'edificio senza le quali non si terrebbe in piedi. Non sono elementi strutturali gli infissi (porte e finestre), i muri divisori, i pavimenti ed i rivestimenti, et cetera.

Gli elementi strutturali sono rappresentati in modo schematico con un segmento proporzionale alla loro lunghezza. Con questa rappresentazione **bidimensionale** i possibili movimenti si riducono a due **traslazioni** (orizzontali e verticale), dovute a delle forze, ed ad una **rotazione**, dovuta ad una coppia di forze (momento).

I collegamenti degli elementi strutturali tra loro e con il mondo esterno sono simboleggiati dai vincoli.

I più comuni sono:

il **carrello** impedisce la traslazione orizzontale

la **cerniera** impedisce sia la traslazione orizzontale che quella verticale

l'**incastro** impedisce le due traslazioni nel piano e la rotazione

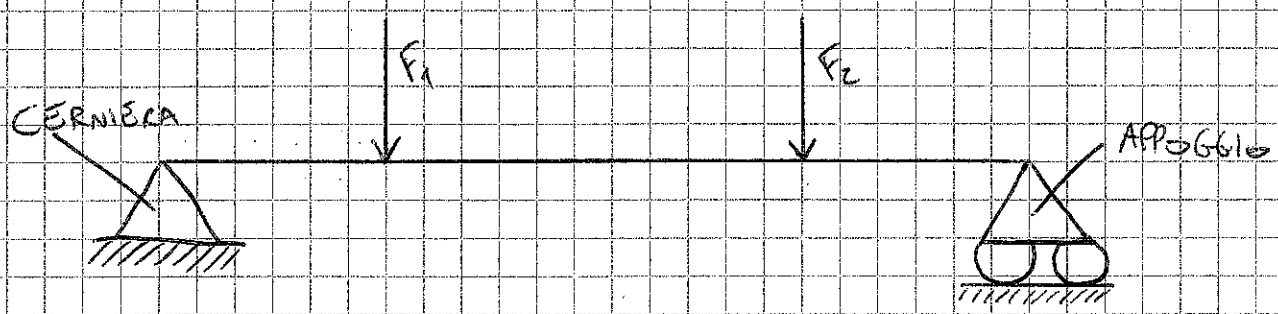
Naturalmente una trave viene progettata in modo che non si muova affatto. I vincoli quindi devono essere disposti in modo da impedire tutti i possibili movimenti. Se ciò accade la struttura è detta **isostatica** (o **iperstatica**), altrimenti la struttura è detta **ipostatica** o **labile**.

Studieremo le principali strutture isostatiche: la trave a mensola, la trave appoggiata (con e senza sbalzi), la trave Gerber, l'arco a tre cerniere.

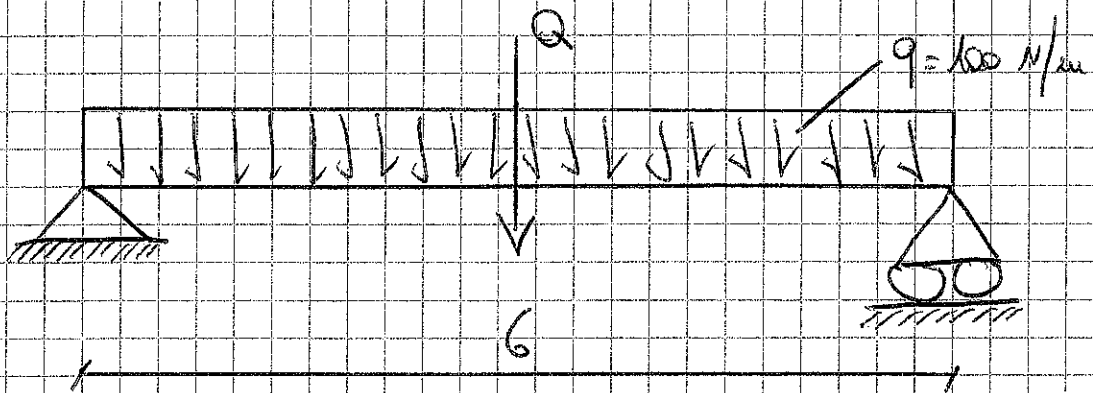
Inizialmente ci limiteremo ad individuare le sole forze esterne (azioni e reazioni vincolari). Passeremo quindi ad analizzare le azioni interne (sollecitazioni e tensioni). Infine proveremo a dimensionare e verificare alcuni semplici elementi strutturali.

TIPOLOGIE DI CARICO

1) PUNTUALI O CONCENTRATO

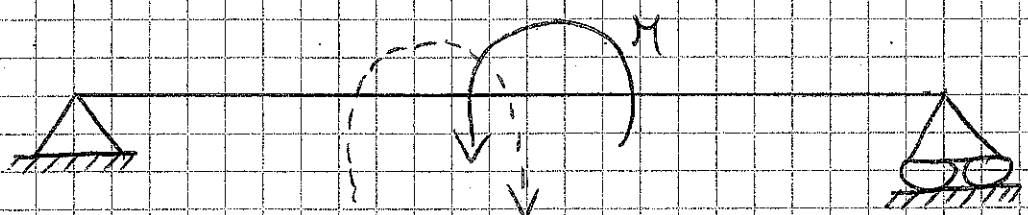


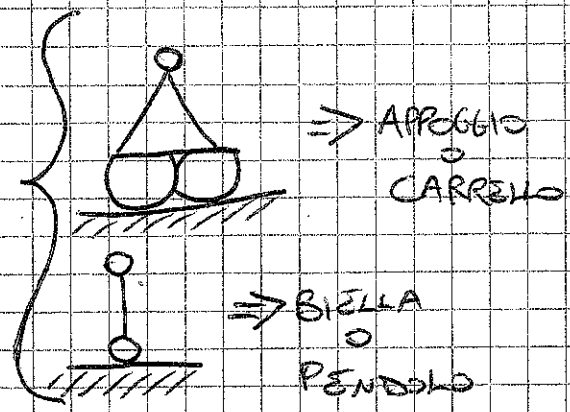
2) UNIFORMEMENTE DISTRIBUITO



$Q = l \cdot q =$
$= 100 \cdot 6 = 600\ N$

3) MOMENTO APPLICATO

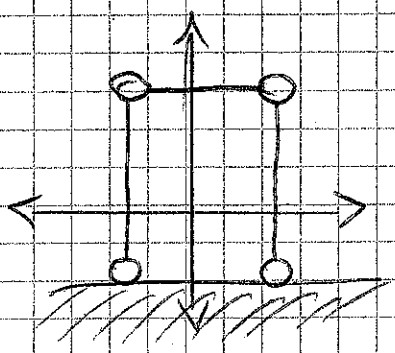




⇒ APPOGGIO O CARRELLO

⇒ BIELLA O PENDOLO

IL CARRELLO IMPEDISCE LA CADUTA DELLA TRAVE.

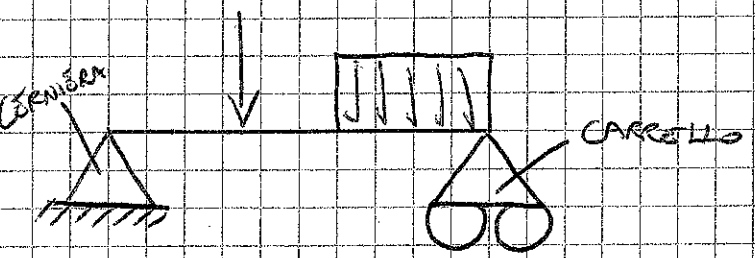


PENDOLO DOPPIO

VERIFICAZIONE ISOSTATICA:

$1 \cdot a + 2 \cdot c + 3 \cdot i = n \cdot 3$

(a: APPOGGIO, c: CERNIERA, i: INCASTRO, n: NUMERO TRAVI)



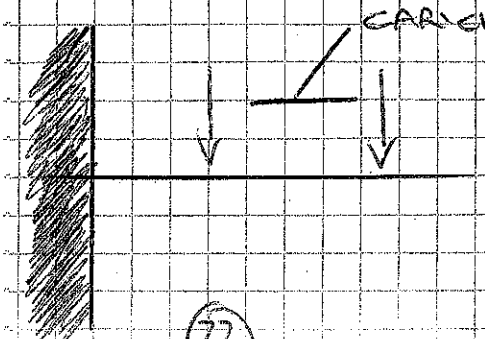
$1 \cdot a + 2 \cdot c + 3 \cdot i = n \cdot 3$

$1 \cdot 1 + 2 \cdot 1 + 3 \cdot 0 = 1 \cdot 3$

$1 + 2 = 3$

$3 = 3$ ISOSTATICA

SCHEMA TRAVE INCASTRATA:



$1 \cdot a + 2 \cdot c + 3 \cdot i = n \cdot 3$

$1 \cdot 0 + 2 \cdot 0 + 3 \cdot 1 = 1 \cdot 3$

$3 = 3$

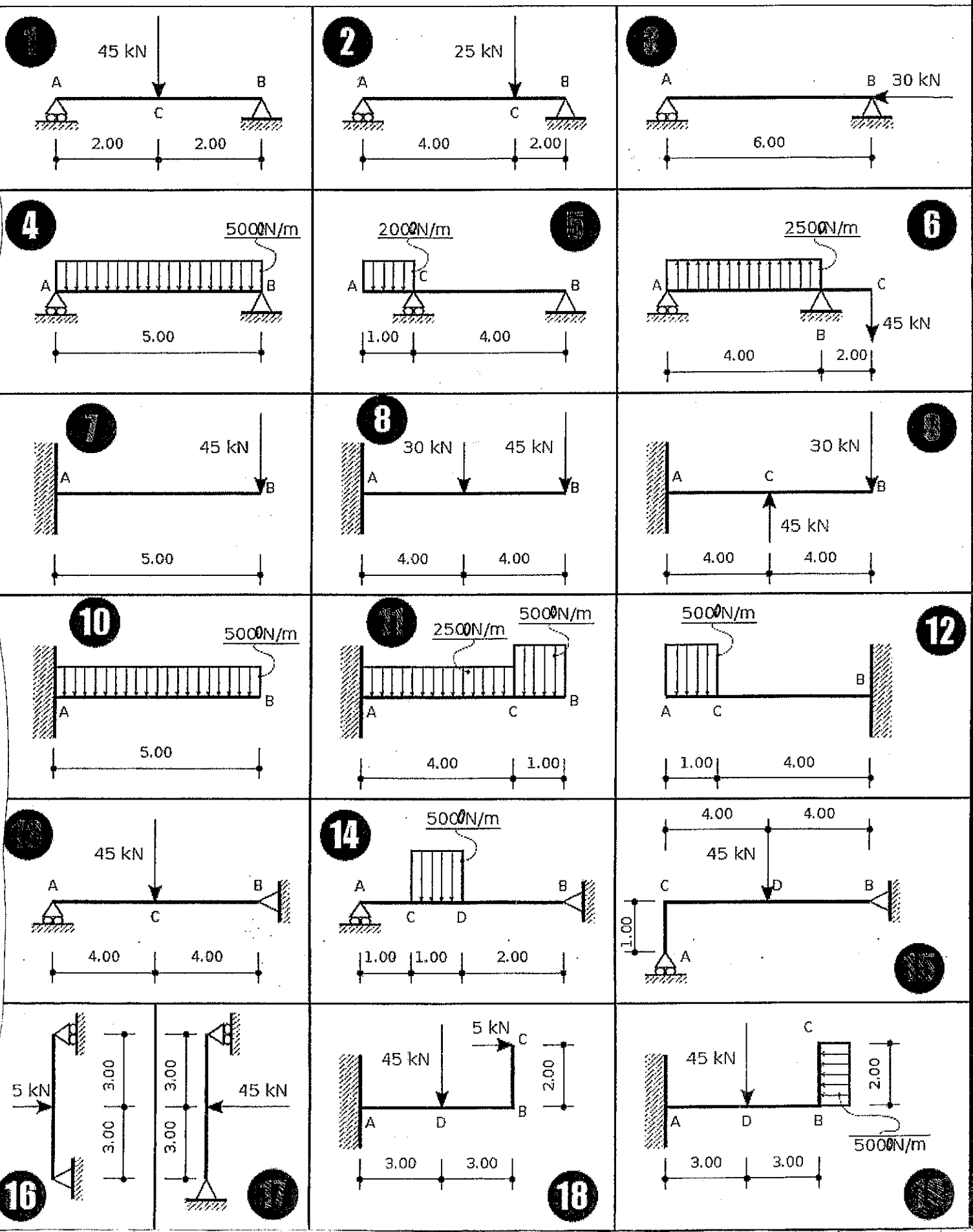
ISOSTATICA

Corso di COSTRUZIONI

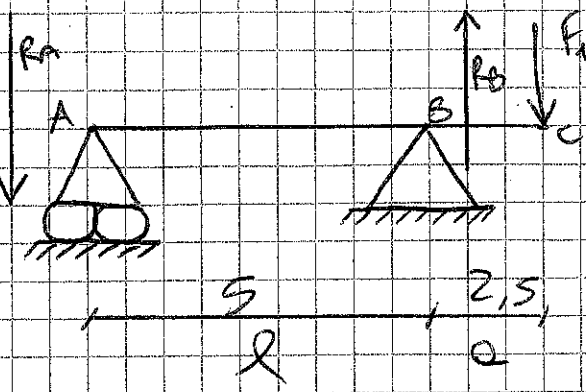
Esercizi su travi isostatiche

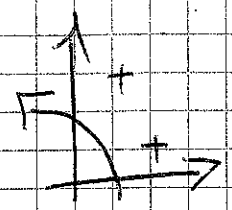
LIVRIERI GIUSEPPE III B

Calcolare le reazioni vincolari e le caratteristiche di sollecitazione delle seguenti strutture isostatiche:



ESERCIZI IN CLASSE:





$F = 100\ kN$

$l = 5{,}00\ cm$

$a = 2{,}5\ cm$

$1 \cdot a + 2 \cdot c + 3 \cdot i = u \cdot 3$

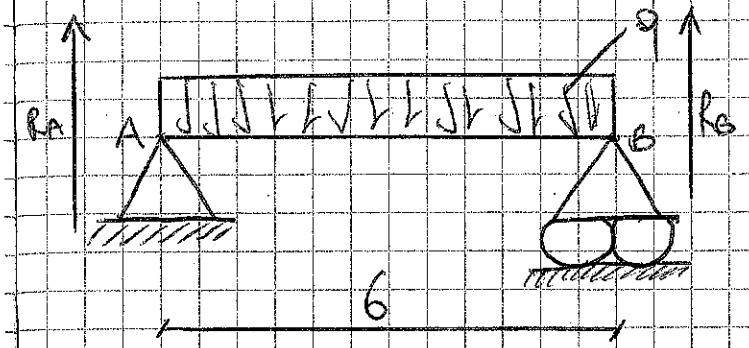
$1 \cdot 1 + 2 \cdot 1 + 3 \cdot 0 = 1 \cdot 3$

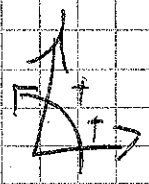
$1 + 2 = 3$

$3 = 3$ ISOSTATICA

$$\begin{cases} \cancel{\Sigma F_x = 0} \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases} \quad \begin{cases} \Sigma F_y = R_A + R_B - F_1 \\ \Sigma M_{(A)} = \cancel{R_A \cdot 0} + R_B \cdot l - F_1 \cdot (l+a) \end{cases} \quad \begin{cases} \Sigma F_y = R_A + R_B - F_1 \\ R_B = \dfrac{F_1 \cdot (l+a)}{l} \end{cases}$$

$$\begin{cases} \Sigma F_y = R_A + R_B - F_1 \\ R_B = \dfrac{100 \cdot (5 + 2{,}5)}{5} = \dfrac{100 \cdot 7{,}5}{5} = \dfrac{750}{5} = 150\ N \end{cases} \quad \begin{cases} R_A = -R_B + F_1 = -150 + 100 = -50\ N \\ R_B = 150\ N \end{cases}$$





$q = 3000\ N/m$

$l = 6\ m$

$R_A = ?$

$R_B = ?$

$Q = q \cdot l = 3000 \cdot 6 = 18000\ N/m$

$1 \cdot a + 2 \cdot c + 3 \cdot \lambda = n \cdot 3$

$1 \cdot 1 + 2 \cdot 1 + 3 \cdot 0 = 1 \cdot 3$

$1 + 2 = 3$

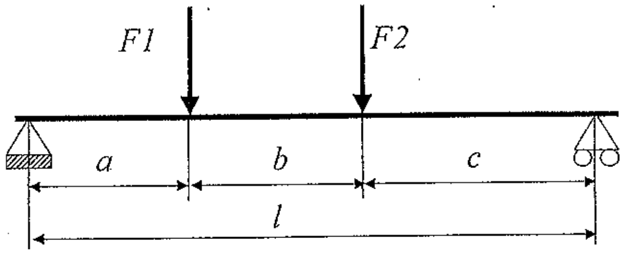
$3 = 3$ ISOSTATICA

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \qquad \begin{cases} \sum F_y = R_A - Q + R_B = 0 \\ \sum M_{(A)} = \cancel{R_A \cdot 0} - Q \cdot \frac{l}{2} + R_B \cdot l \end{cases} \qquad \begin{cases} \sum F_y = R_A - Q + R_B = 0 \\ \sum M_{(A)} = R_B = \frac{Q \frac{l}{2}}{l} \end{cases}$$

$$\begin{cases} \sum F_y = R_A - Q + R_B = 0 \\ R_B = \frac{18000 \cdot \frac{6}{2}}{6} = \frac{18000 \cdot 3}{6} = 9000\ N \end{cases} \qquad \begin{cases} R_A = Q - R_B = 18000 - 9000 = 9000\ N \\ R_B = 9000\ N \end{cases}$$

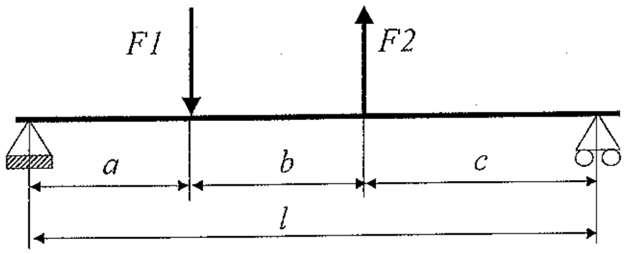
Esercizio n° 1

Data la struttura schematizzata in figura calcolare le reazioni vincolari.
Dimensioni: a= 2 m, b= 2 m, c= 3 m; Forze F1= 10 KN, F2 = 15 KN
Risolvere prima in modo letterale e poi passare ai calcoli.
STAI ATTENTO ! Equazioni della statica; convenzioni verso, VERIFICA CHE LA TRAVE SIA ISOSTATICA , ALTRIMENTI TI METTO 2 !! CAPITO



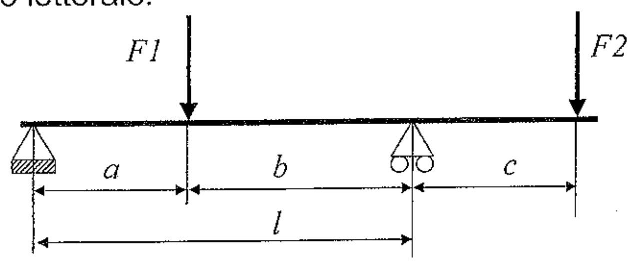
Esercizio n 2

Data la struttura schematizzata in figura calcolare le reazioni vincolari.
Dimensioni: a= 2 m, b= 2,50 m, c= 3 m; Forze F1= 100 KN, F2 = 150 KN
Risolvere prima in modo letterale.



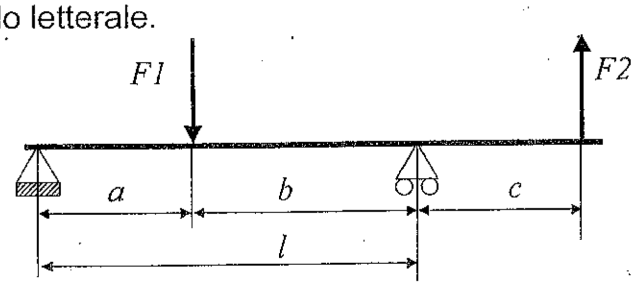
Esercizio n° 3

Data la struttura schematizzata in figura calcolare le reazioni vincolari.
Dimensioni: a= 2 m, b= 2,50 m, c= 1,50 m; Forze F1= 1000 N, F2 = 1500 N
Risolvere prima in modo letterale.

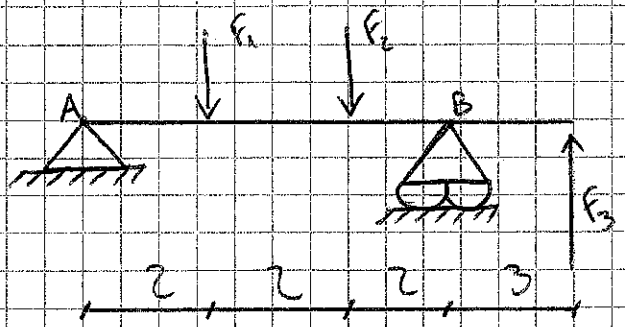


Esercizio n° 4

Data la struttura schematizzata in figura calcolare le reazioni vincolari.
Dimensioni: a= 2,50 m, b= 3,00 m, c= 1,50 m; Forze F1= 3500 N, F2 = 1500 N
Risolvere prima in modo letterale.



ESERCIZI DEL COMPITO



$F_1 = 10\ kN$

$F_2 = 15\ kN$

$F_3 = 20\ kN$

$l = 6\ m$

$1 \cdot a + 2 \cdot c + 3 \cdot x = 3 \cdot m$

$1 \cdot 1 + 2 \cdot 1 + 3 \cdot 0 = 3 \cdot 1$

$1 + 2 = 3$

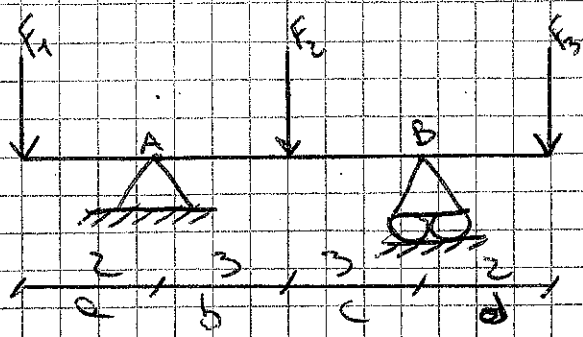
$3 = 3$ ISOSTATICA

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases}$$

$$\begin{cases} \sum F_y = R_A - F_1 - F_2 + R_B + F_3 = 0 \\ \sum M_{(A)} = \cancel{R_B \cdot 0} - F_1 \cdot a - F_2 \cdot 2 \cdot 2 + R_B \cdot l + F_3 \cdot (l+3) = 0 \end{cases}$$

$$\begin{cases} \sum F_y = R_A - F_1 - F_2 + R_B + F_3 \\ \sum M_{(A)} = R_B = \dfrac{F_1 \cdot a + F_2 \cdot 2 \cdot 2 - F_3 \cdot (6+3)}{6} = \dfrac{10 \cdot 2 + 15 \cdot 4 - 20 \cdot 9}{6} = \dfrac{-100}{6} = -16,7\ kN \end{cases}$$

$$\begin{cases} \sum F_y = R_A = F_1 + F_2 + R_B - F_3 = 21,7\ kN \\ R_B = -16,7\ kN \end{cases} \qquad \begin{cases} R_A = 21,7\ kN \\ R_B = -16,7\ kN \end{cases}$$



$F_1 = 35\ kN$

$F_2 = 40\ kN$

$F_3 = 45\ kN$

$l = 6\ m$

$1 \cdot a + 2 \cdot c + 3 \cdot i = 3 \cdot m$

$1 \cdot 1 + 2 \cdot 1 + 3 \cdot 0 = 3 \cdot 1$

$1 + 2 = 3$

$3 = 3$ ISOSTATICA

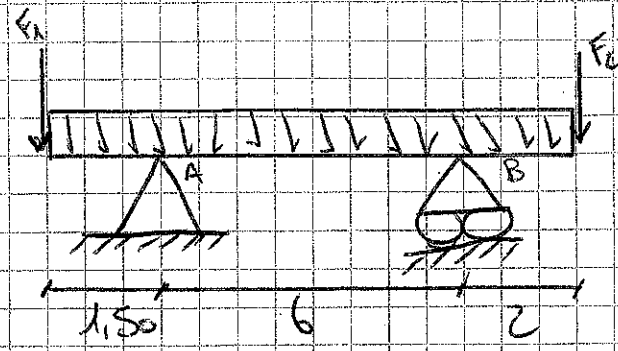
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases}$$

$$\begin{cases} \sum F_y = -F_1 + R_A - F_2 + R_B - F_3 = 0 \\ \sum M_{(A)} = F_1 \cdot 2 + \cancel{R_A \cdot 0} - F_2 \cdot b + R_B \cdot (3+3) - F_3 \cdot (3+3+2) = 0 \end{cases}$$

$$\begin{cases} \sum F_y = -F_1 + R_A - F_2 + R_B - F_3 \\ R_B = \dfrac{+F_1 \cdot a + F_2 \cdot b + F_3 \cdot (b+c+d)}{b+c} = \dfrac{-35 \cdot 2 + 40 \cdot 3 + 45 \cdot 8}{3+3} = \dfrac{410}{6} = -68,3\ kN \end{cases}$$

$$\begin{cases} R_A = F_1 + F_2 - R_B + F_3 = 35 + 40 - 68,3 + 45 = 51,7\ kN \\ R_B = 68,3\ kN \end{cases}$$

$$\begin{cases} R_A = 51,7\ kN \\ R_B = 68,3\ kN \end{cases}$$



$F_1 = 1\ kN$

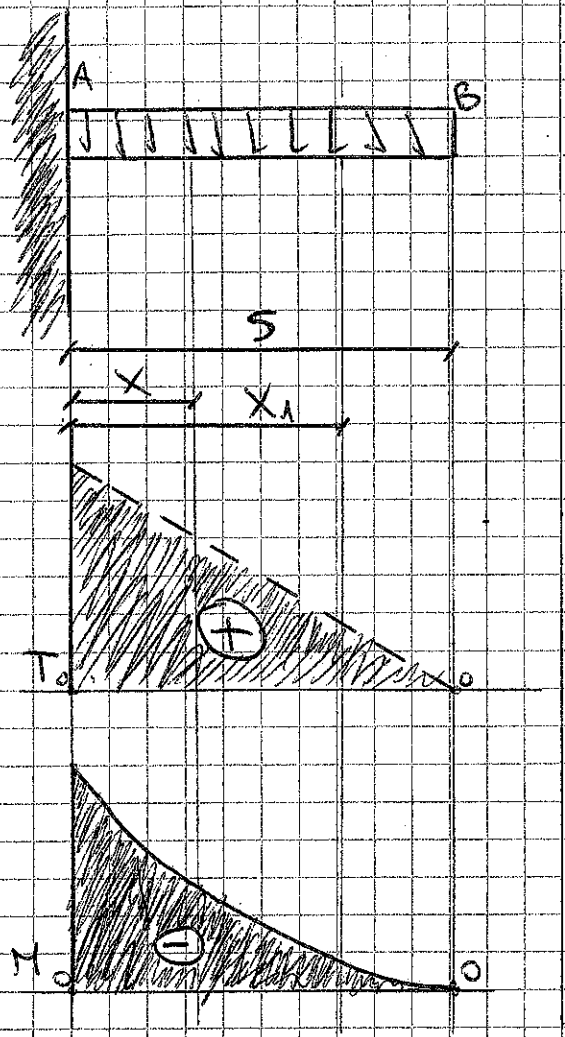
$F_2 = 1\ kN$

$q = 50\ kN/m$

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases} \qquad \begin{cases} \Sigma F_y = -F_1 + R_A - q\cdot(1{,}50+6+2) + R_B - F_2 = 0 \\ \Sigma M_{(A)} = F_1\cdot 1{,}50 + \cancel{0} + (q\cdot 1{,}5)\cdot\frac{1{,}5}{2} - q\cdot\frac{6^2}{2} + R_B\cdot 6 - F_2\cdot(6+2) - 4\infty = 0 \end{cases}$$

$$\begin{cases} \Sigma F_y = -F_1 + R_A - q\cdot(a+b+c) + R_B - F_2 = 0 \\ R_B = \dfrac{-1{,}50 - 56{,}25 + 900 + 8 + 4\infty}{6} = \dfrac{1550{,}25}{6} = 258{,}38\ kN \end{cases}$$

$$\begin{cases} R_A = F_1 + q\cdot(1{,}50+6+2) - R_B + F_2 = 1+425-258{,}38+1 = 218{,}62\ kN \\ R_B = 258{,}38\ kN \end{cases} \qquad \begin{cases} R_A = 218{,}62\ kN \\ R_B = 258{,}38\ kN \end{cases}$$



$q = 10\ kN/m$

$l = 5\ m$

$1 \cdot 2 + 2 \cdot 0 + 3 \cdot 1 = 3 \cdot 1$

$1 \cdot 0 + 2 \cdot 0 + 3 \cdot 1 = 3 \cdot 1$

$3 = 3$ 1=0 STATICA

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0 \end{cases} \qquad \begin{cases} \Sigma F_y = R_A - (q \cdot l) = 0 \\ \Sigma M_{(A)} = R_A \cdot 5 - q \cdot \frac{l}{2} \cdot l = 0 \end{cases}$$

$$\begin{cases} R_A = 50\ kN \\ \Sigma M_{(A)} = -10 \cdot 5 \cdot \frac{5}{2} = 125\ kN/m \end{cases}$$

$R_A - q \cdot x = 0$

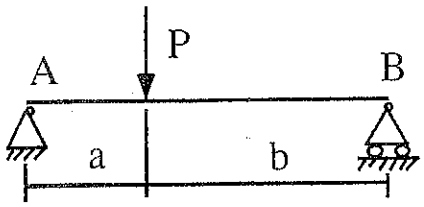
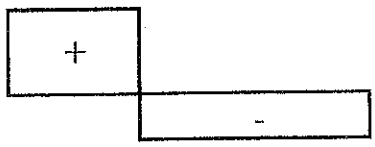
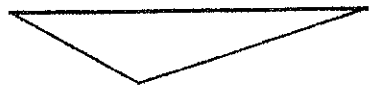
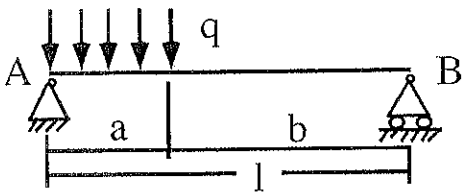
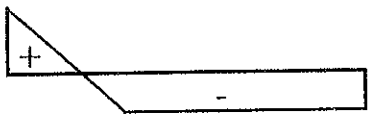
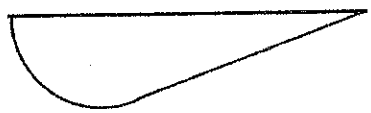
$+q \cdot x = +R_A$

$x = \frac{R_A}{q}$

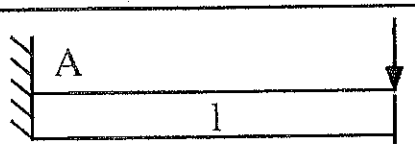
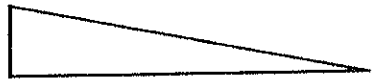
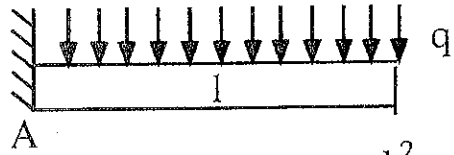
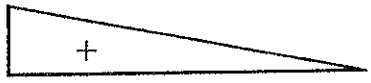
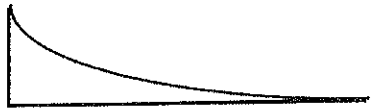
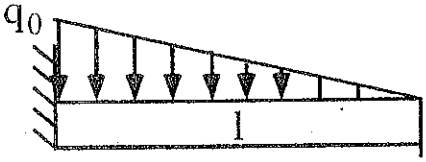
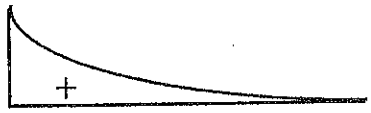
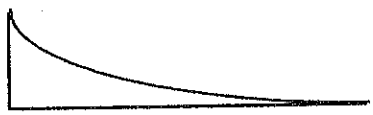
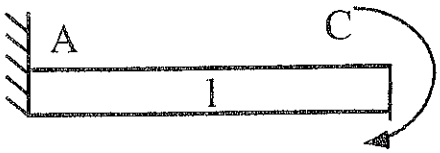
$T_{MAX} = q \cdot l = 50\ kN$

$M_{MAX} = -\frac{1}{2} \cdot q \cdot l^2 = -\frac{1}{2} \cdot 10 \cdot 25 = 125\ kN$

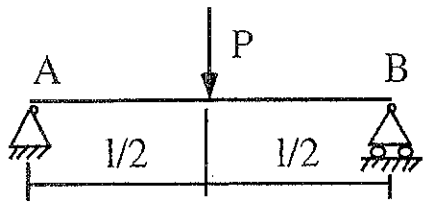
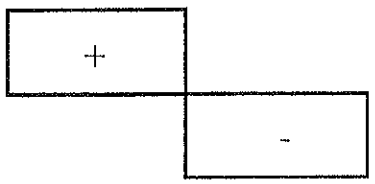
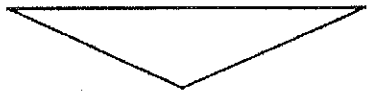
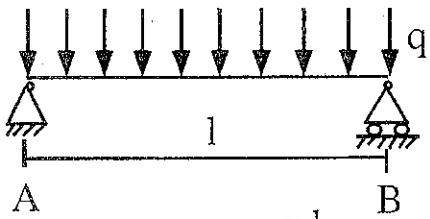
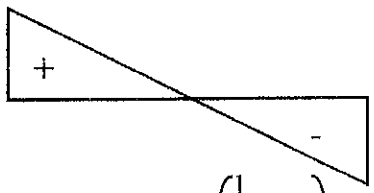
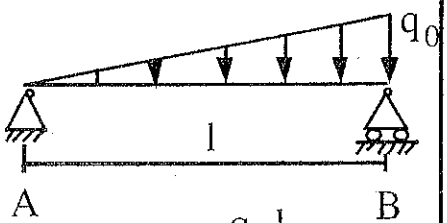
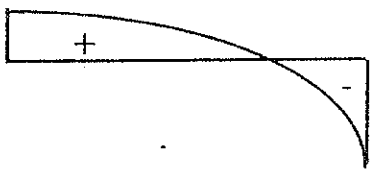
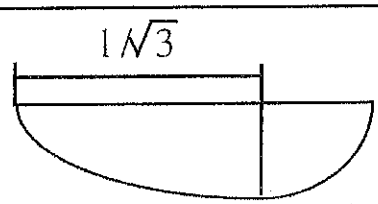
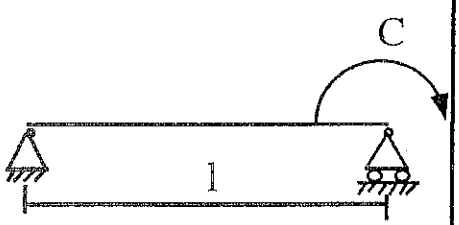
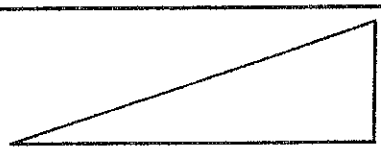
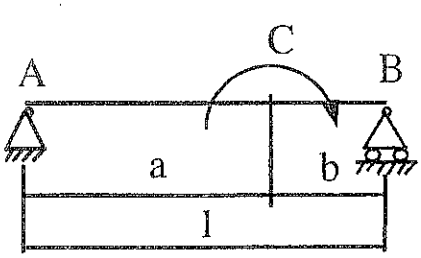
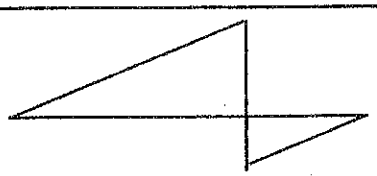
TRAVI A DUE APPOGGI

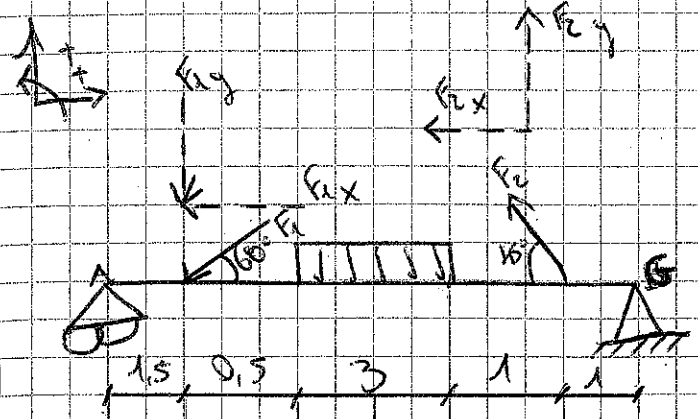
Schema di Carico	Sforzo di Taglio	Momento Flettente
 $V_A = P\,\frac{b}{l}$; $V_B = P\,\frac{a}{l}$	 $T(x) = +P\,\frac{b}{l}$ $T(\bar{x}) = -P\,\frac{a}{l}$	 $M(x) = P\,\frac{b}{l}\,x$ $M(\bar{x}) = P\,\frac{a}{l}\,\bar{x}$ $M_{max} = P\,\frac{a\,b}{l}$
 $V_A = \frac{q\,a}{2\,l}\,(2b + a)$ $V_B = q\,\frac{a^2}{2\,l}$	 $T(x) = q\left(a\,\frac{2b + a}{2\,l} - x\right)$ $T(\bar{x}) = -q\,\frac{a^2}{2\,l}$	 $M(x) = q\,\frac{x}{2}\left(a\,\frac{2b + a}{2\,l} - x\right)$ $M(\bar{x}) = q\,\frac{a^2}{2\,l}\,\bar{x}$

TRAVI A MENSOLA

 $V_A = P$; $M_A = P\,l$	 $T(x) = +P$	 $M(x) = P\,x$; $M_{max} = P\,l$
 $V_A = q\,l$; $M_A = q\,\frac{l^2}{2}$	 $T(x) = +q\,x$; $T_{max} = q\,l$	 $M(x) = q\,\frac{x^2}{2}$ $M_{max} = q\,\frac{l^2}{2}$
 $V_A = \frac{q_0\,l}{2}$; $M_A = \frac{q_0\,l^2}{6}$	 $T(x) = q_0\,\frac{x^2}{2\,l}$ $T_{max} = q_0\,\frac{l}{2}$	 $M(x) = q_0\,\frac{x^3}{6\,l}$ $M_{max} = q_0\,\frac{l^2}{6}$
 $V_A = 0$; $M_A = C$	$T = 0$	 $M(x) = C$

TRAVI A DUE APPOGGI

Schema di Carico	Sforzo di Taglio	Momento Flettente
 $V_A = V_B = \frac{P}{2}$	 $T(x) = +\frac{P}{2} \quad (x < \frac{l}{2})$ $T(x) = -\frac{P}{2} \quad (x > \frac{l}{2})$	 $M(x) = \frac{P}{2} x \quad (x < \frac{l}{2})$ $M_{max} = \frac{P\,l}{4}$
 $V_A = V_B = \frac{q\,l}{2}$	 $T(x) = q\left(\frac{l}{2} - x\right)$ $T_{max} = \frac{q\,l}{2}$	 $M(x) = \frac{q\,x}{2}(l - x)$ $M_{max} = q\frac{l^2}{8}$
 $V_A = \frac{q_0\,l}{6}$ $V_B = \frac{q_0\,l}{3}$	 $T(x) = \frac{q_0}{2\,l}\left(\frac{l^2}{3} - x^2\right)$	 $M(x) = \frac{q_0\,x}{6\,l}(l^2 - x^2)$ $M_{max} = q_0\frac{l^2}{15{,}6}$
 $V_A = -\frac{C}{l}$ $V_B = +\frac{C}{l}$	 $T(x) = -\frac{C}{l}$	 $M(x) = \frac{C}{l} x$
 $V_A = -\frac{C}{l}$ $V_B = +\frac{C}{l}$	 $T(x) = -\frac{C}{l}$	 $M(x) = \frac{C}{l} x$ $M(\bar{x}) = \frac{C}{l}\bar{x}$



$F_1 = 1500$ N

$F_2 = 1000$ N

$q = 500$ N/m

$F_{1y} = F_1 \cdot \text{sen } 60° = 1500 \cdot \text{sen } 60° = 1299,04$ N

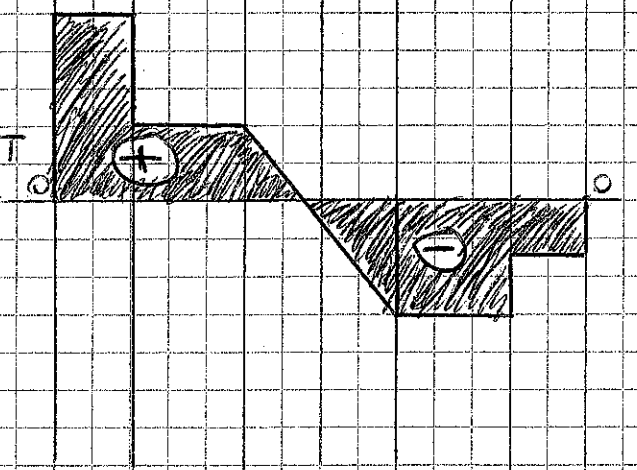
$F_{1x} = 1500 \cdot \cos 60° = 750$ N

$F_{2y} = 1000 \cdot \text{sen } 45° = 707,11$ N

$F_{2x} = 1000 \cdot \cos 45° = 707,11$ N

$Q = q \cdot 3 = 500 \cdot 3 = 1500$ N/m

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \quad \begin{cases} \sum F_x = -F_{1x} - F_{2x} + R_{Bx} \\ \sum F_y = R_A - F_{1y} - Q + F_{2y} + R_{By} \\ \sum M_{(A)} = R_A \cdot 0 - F_{1y} \cdot 1,5 - F_{1x} \cdot 0 - Q \cdot 3,5 - F_{2x} \cdot 0 + F_{2y} \cdot 6 + R_{By} \cdot 7 = 0 \end{cases}$$



$$\begin{cases} R_{Bx} = F_{1x} + F_{2x} = 750 + 707,11 = 1457,11 \text{ N} \\ \sum F_y = R_A - F_{1y} - Q + F_{2y} + R_{By} \\ R_{By} = \dfrac{1299,04 \cdot 1,5 + 1500 \cdot 3,5 - 707,11 \cdot 6}{7} = 422,27 \text{ N} \end{cases}$$

$$\begin{cases} R_{Bx} = 1457,11 \text{ N} \\ R_A = 1299,04 + 1500 - 707,11 - 422,27 = 1669,66 \text{ N} \\ R_{By} = 422,27 \text{ N} \end{cases}$$

$T_A = 1669,66$ N

$T_B = 1669,66 - 1299 = 370$ N

$T_E = 370 - 1500 = -1130$

$T_D = -1130$

$T_{AC} = 370$

$T_{C_1} = R_A - F_{1y} - (q \cdot 1) = 1669 - 1299 - 500 = -130$ N

$T_{C_2} = R_A - F_{1y} - (q \cdot 2) = 1669 - 1299 - 1000 = -630$ N

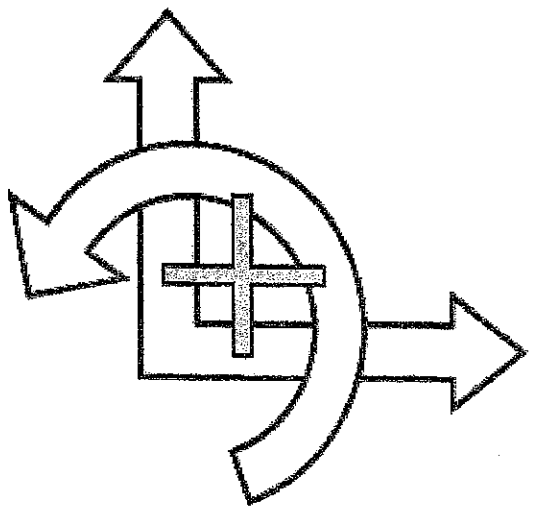
$T_{C_3} = R_A - F_{1y} - (q \cdot 3) = 1669 - 1299 - 1500 = -1130$ N

$T_F = R_A - F_{1y} - (q \cdot 3) + F_{2y} = 1669 - 1299 - 1500 + 707 = -423$ N

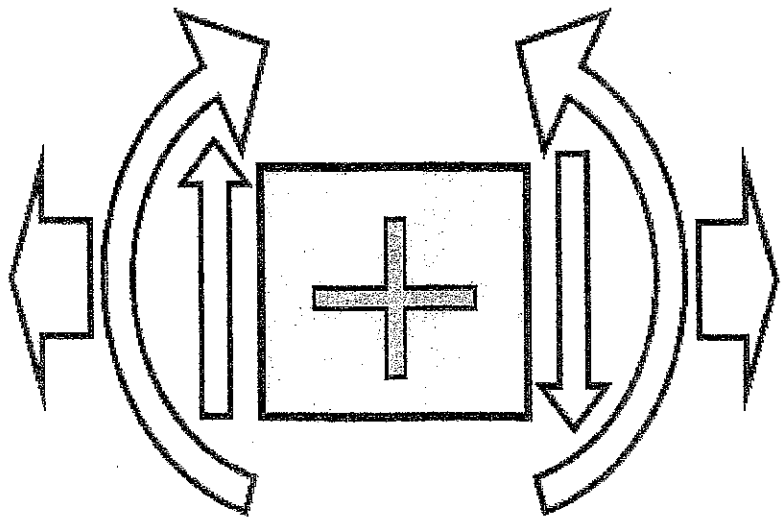
Studio delle Travature Isostatiche

4-1
4-2
4-3

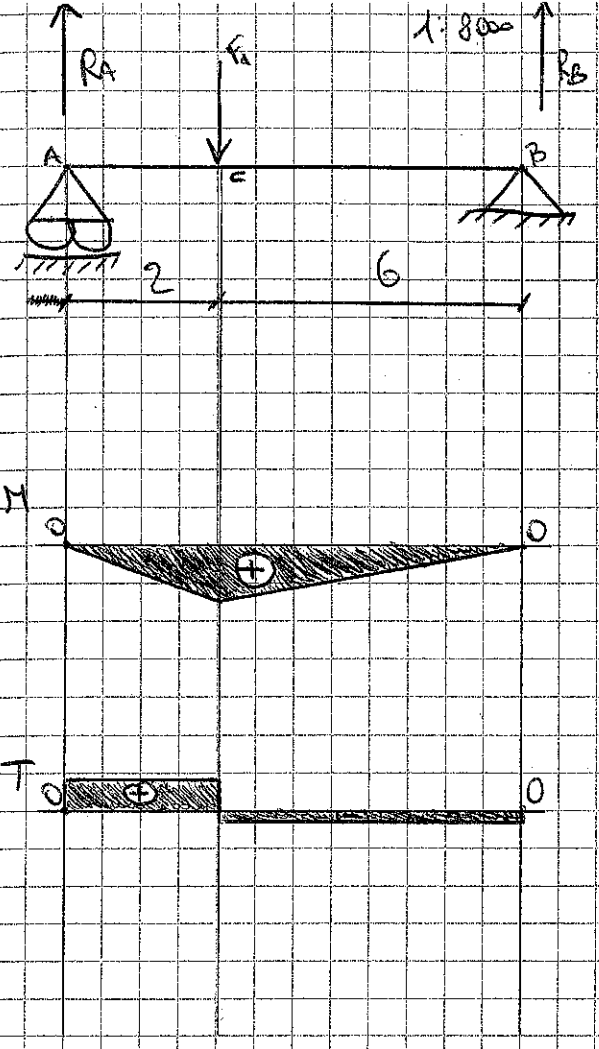
Convenzioni sui segni:



Forze e Momenti



Trazione, Taglio e Momento flettente



$F_1 = 400\ N$

$l = 8\ m$

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \quad \begin{cases} \sum F_y = R_A - F_1 + R_B = 0 \\ \sum M_{(A)} = R_A \cdot 0 - F_1 \cdot 2 + R_B \cdot 8 \end{cases}$$

$$\begin{cases} \sum F_y = R_A - F_1 + R_B = 0 \\ R_B = \dfrac{F \cdot 2}{8} = \dfrac{400 \cdot 2}{8} = \dfrac{800}{8} = 100\ N \end{cases}$$

$$\begin{cases} R_A = F_1 - R_B = 400 - 100 = 300\ N \\ R_B = 100\ N \end{cases}$$

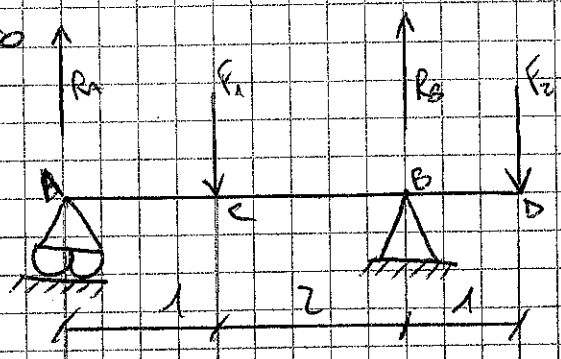
$$\begin{cases} R_A = 300\ N \\ R_B = 100\ N \end{cases}$$

$T_A = 300\ N$

$T_C = 300 - 400 = -100\ N$

$T_B = -100$

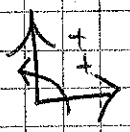
1:3000



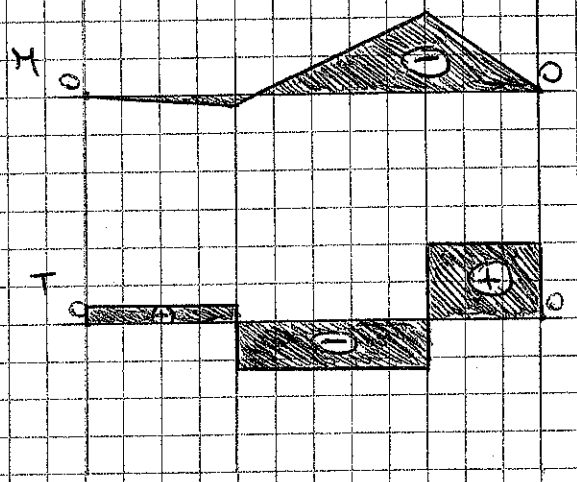
$F_1 = 200$ N

$F_2 = 300$ N

$l = 3$ m



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \quad \begin{cases} \sum F_y = R_A - F_1 + R_B - F_2 = 0 \\ \sum M_{(A)} = R_A \cdot 0 - F_1 \cdot 1 + R_B \cdot 3 - F_2 \end{cases}$$



$$\begin{cases} \sum F_y = R_A - F_1 + R_B - F_2 = 0 \\ R_B = \dfrac{F_1 \cdot 1 + F_2 \cdot 4}{3} = \dfrac{200 + 1200}{3} = \dfrac{1400}{3} = 466 \end{cases}$$

$$\begin{cases} R_A = F_1 - R_B + F_2 = 200 - 466,67 + 300 = 33,33 \\ R_B = 466,67 \text{ N} \end{cases}$$

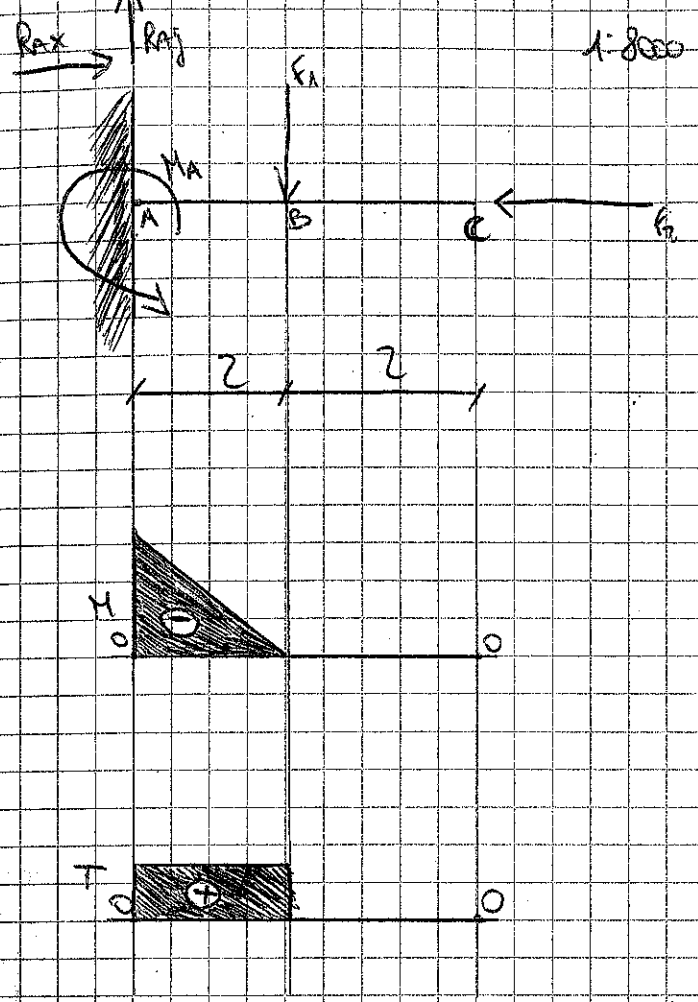
$$\begin{cases} R_A = 33,33 \text{ N} \\ R_B = 466,67 \text{ N} \end{cases}$$

$T_A = 33,33$ N

$T_C = 33,33 - 200 = -166,67$ N

$T_B = -166,67 + 466,67 = 300$ N

$T_D = 300$ N

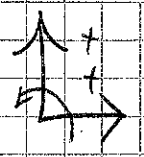


1:800

$F_1 = 600\,N$

$F_2 = 80\,N$

$l = 4\,m$



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \quad \begin{cases} \sum F_x = R_{Ax} - F_2 = 0 \\ \sum F_y = R_{Ay} - F_1 = 0 \\ \sum M_{(A)} = M_A + \cancel{R_{Ax} \cdot 0} + \cancel{R_{Ay} \cdot C} - F_1 \cdot 2 - \cancel{F_2 \cdot 0} = 0 \end{cases}$$

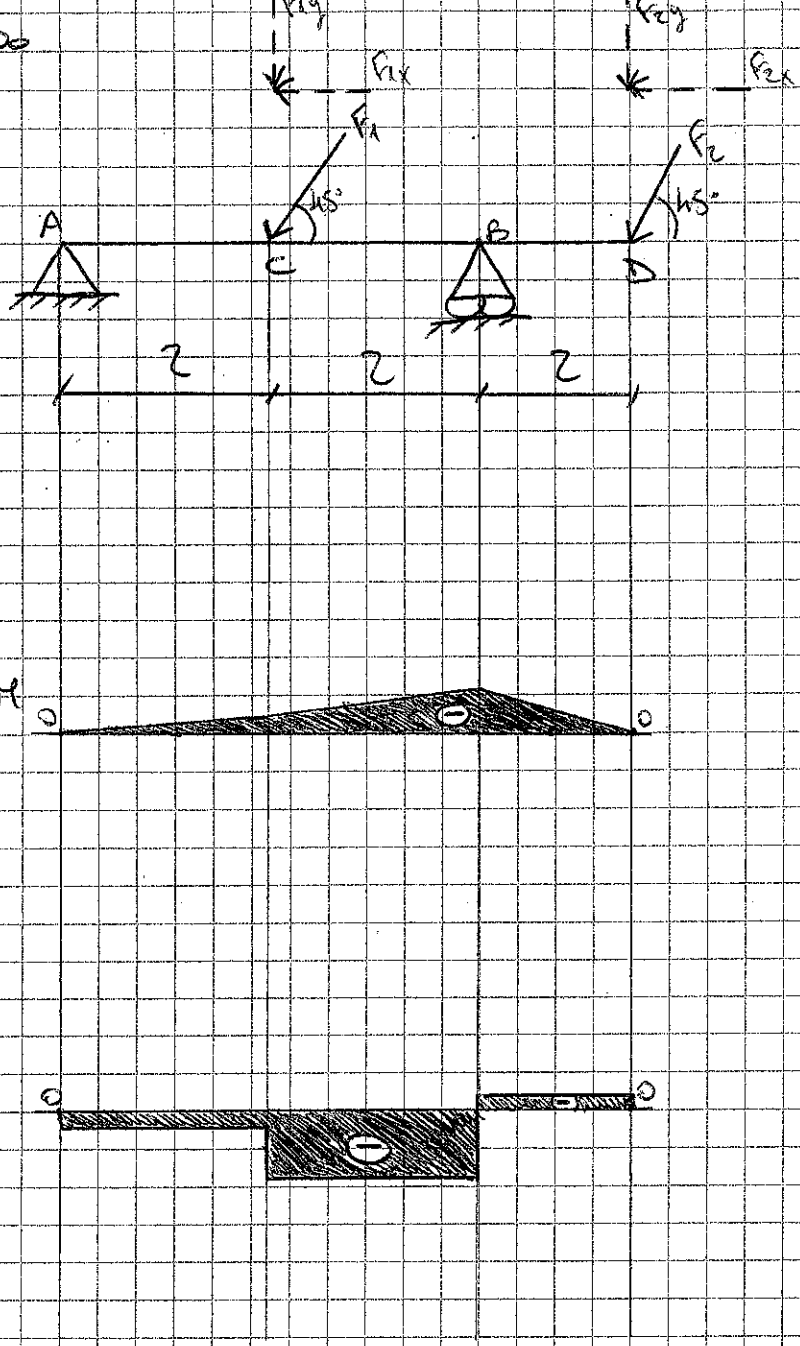
$$\begin{cases} R_{Ax} = F_2 = 80\,N \\ R_{Ay} = 600\,N \\ M_A = 600 \cdot 2 = 1200\,N \end{cases}$$

$T_A = 600$

$T_B = 600 - 600 = 0$

$T_C = 0$

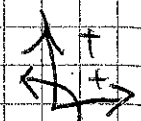
1:3000



$F_1 = 200\ N$

$F_2 = 300\ N$

$l = 4\ m$



$F_{1y} = 200 \cdot \cos 45° = 141,42\ N$

$F_{1x} = 200 \cdot \sin 45° = 141,42\ N$

$F_{2y} = 300 \cdot \cos 45° = 212,13\ N$

$F_{2x} = 300 \cdot \cos 45° = 212,13\ N$

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \quad \begin{cases} \sum F_x = R_{Ax} - F_{1x} - F_{2x} = 0 \\ \sum F_y = R_{Ay} - F_{1y} + R_B - F_{2y} = 0 \\ \sum M_{(A)} = \cancel{R_{Ax} \cdot 0} + \cancel{R_{Ay} \cdot 0} - F_{1y} \cdot 2 + \\ \quad + \cancel{F_{1x} \cdot 0} + R_B \cdot 4 - F_{2y} \cdot 6 \\ \quad + \cancel{F_{2x} \cdot 0} = 0 \end{cases}$$

$$\begin{cases} R_{Ax} = F_{1x} + F_{2x} = 141,42 + 212,13 = 353,55\ N \\ \sum F_y = R_{Ay} - F_{1y} + R_B - F_{2y} = 0 \\ R_B = \dfrac{F_{1y} \cdot 2 + F_{2y} \cdot 6}{4} = \dfrac{141,42 \cdot 2 + 212,13 \cdot 6}{4} = 388 \end{cases}$$

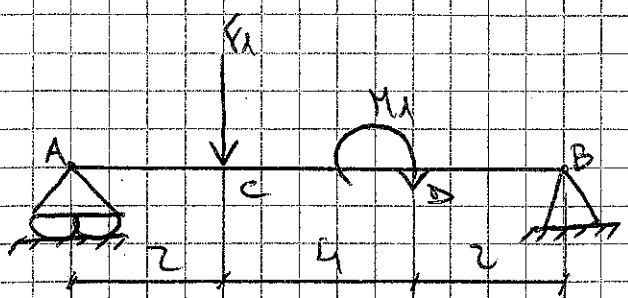
$$\begin{cases} R_{Ax} = 353,55\ N \\ R_{Ay} = F_{1y} - R_B + F_{2y} = 141,42 - 388,91 + 212,13 = -35 \\ R_B = 388,91\ N \end{cases}$$

M

$T_A = -35,36\ N$

$T_C = -35,36 - 141,42 = -176,78\ N$

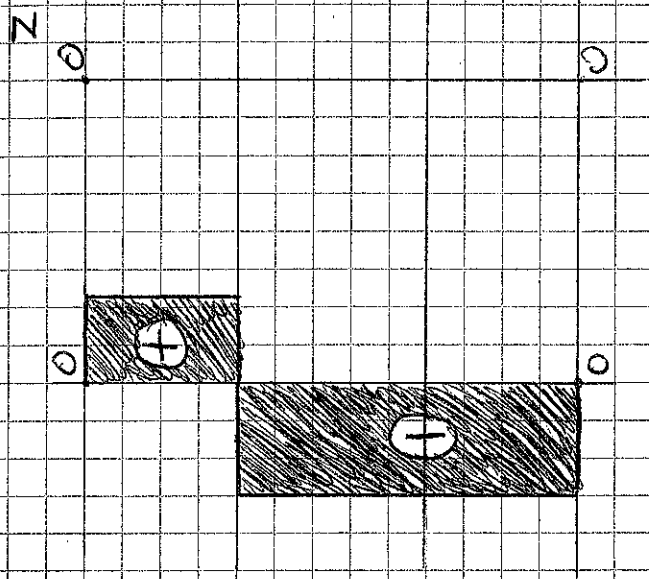
$T_B = -176,78 + 388,91 = 212,19\ N$



$F_1 = 400\ N$

$M_1 = 160\ N \cdot m$

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \quad \begin{cases} \sum F_y = R_A - F_1 + R_B = 0 \\ \sum M_{(B)} = -R_A \cdot 8 + F_1 \cdot 6 - M_1 - R_B \cdot 0 \end{cases}$$



$$\begin{cases} R_B = -400 + 280 = 120\ N \\ R_A = \dfrac{-F_1 \cdot 6 + M_1}{8} = \dfrac{2400 + 160}{8} = 280\ N \end{cases}$$

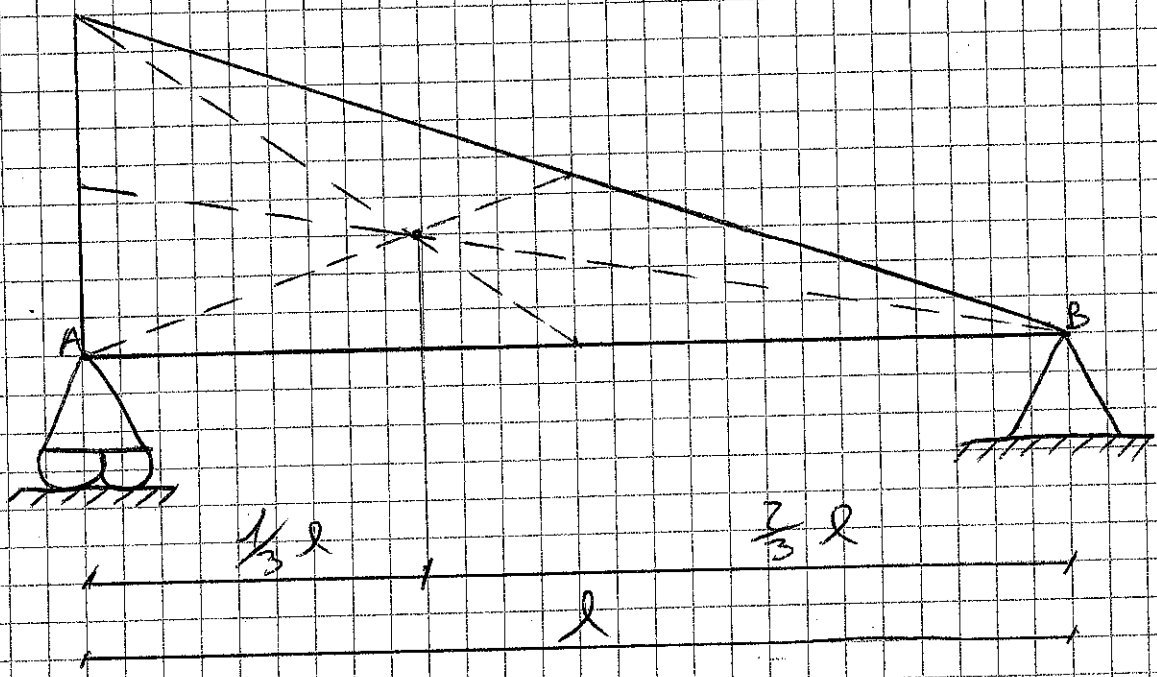
$$\begin{cases} R_B = 120\ N \\ R_A = 280\ N \end{cases}$$

$T_A = 280\ N$

$T_C = 280 + 120 = 400\ N$

$T_B = 400 - 280 = 120\ N$

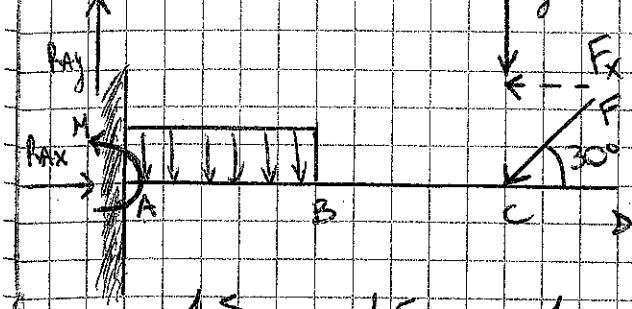
CARICO TRIANGOLARE

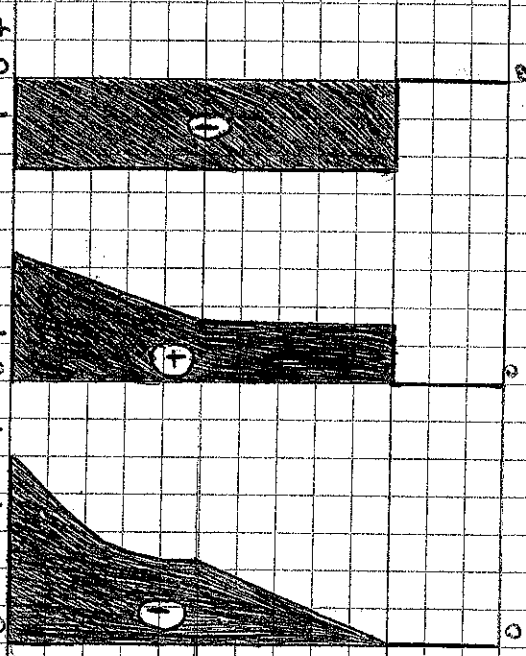


$$Q = \frac{(q_{MAX} + q_{MIN}) \cdot l}{2} = \frac{(1000\ N/m + 0) \cdot 6}{2} = 3000\ N$$

$$R_A = \frac{2}{3} \cdot Q = \frac{2}{3} \cdot 3000 = 2000\ N$$

$$R_B = \frac{1}{3} \cdot Q = \frac{1}{3} \cdot 3000 = 1000\ N$$





$q = 10\ kN/m$

$F = 20\ kN$

$Q = q \cdot 1,5 = 10 \cdot 1,5 = 15\ kN/m$

$F_x = F \cdot \cos 30° = 20 \cdot \cos 30° = 17,32\ kN$

$F_y = F \cdot \operatorname{sen} 30° = 20 \cdot \operatorname{sen} 30° = 10\ kN$

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \quad \begin{cases} \sum F_x = R_{Ax} - F_x = 0 \\ \sum F_y = R_{Ay} - Q - F_y = 0 \\ \sum M_{(A)} = M - Q \cdot 0,75 - F_y \cdot 3 = 0 \end{cases}$$

$$\begin{cases} R_{Ax} = F_x = 17,32\ kN \\ R_{Ay} = Q + F_y = 25\ kN \\ M = Q \cdot 0,75 + F_y \cdot 3 = 15 \cdot 0,75 + 10 \cdot 3 = 41,25\ kN \end{cases}$$

$T_A = R_{Ay} = 25$

$T_B = R_{Ay} - q \cdot 1,5 = 25 - 15 = 10\ kN$

$T_C = R_{Ay} - q \cdot 1,5 - F_y = 0\ kN$

$T_D = 0\ kN$

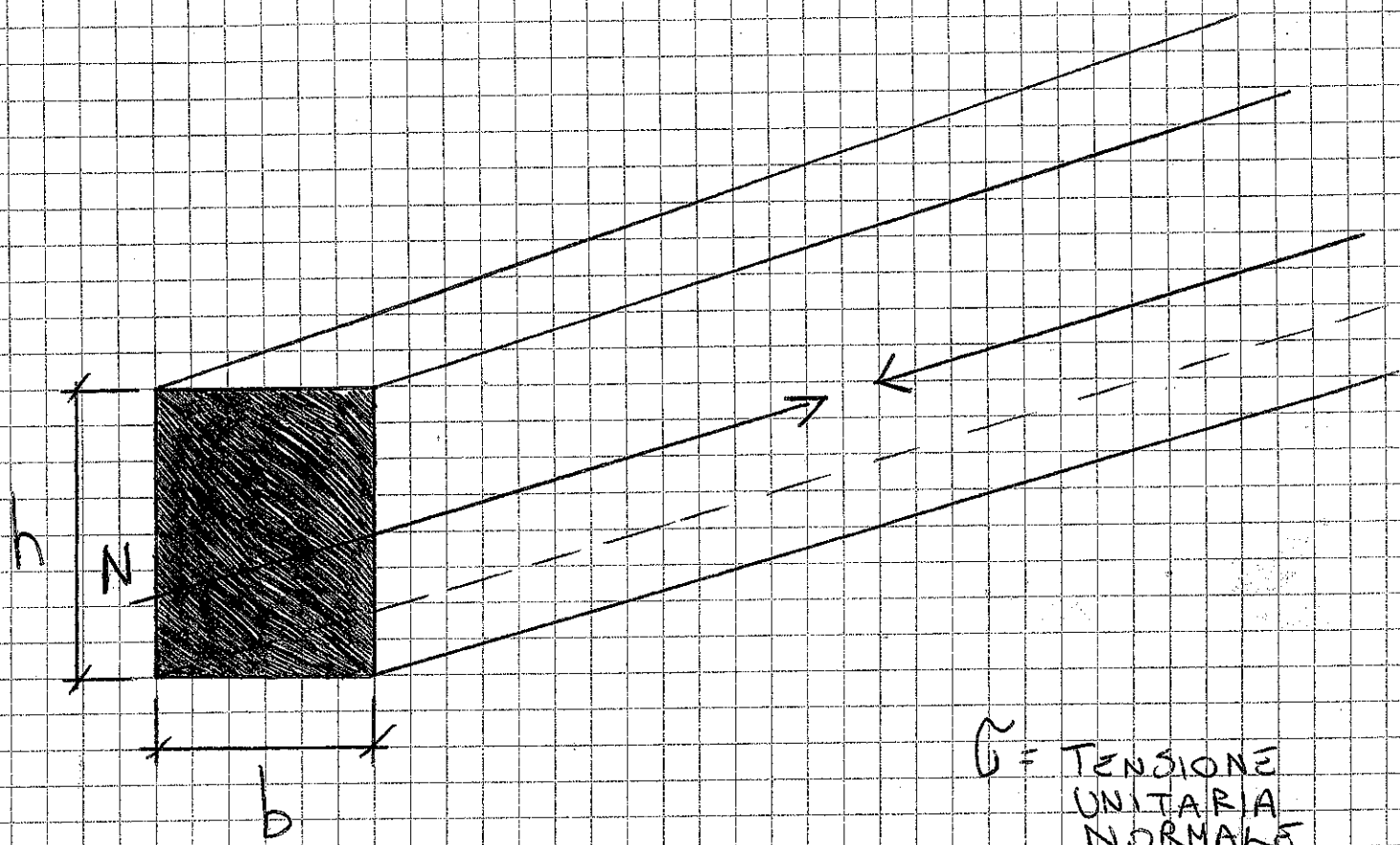
$M_A = -41,25\ kN$

$M_B = -M + R_{Ay} \cdot 1,5 - q \cdot 0,75 \cdot 1,5 = -15\ kN$

$M_C = -M + R_{Ay} \cdot 3 - Q \cdot 2,25 = 0\ kN$

$M_D = 0\ kN$

TRAZIONE E COMPRESSIONE



σ = TENSIONE UNITARIA NORMALE

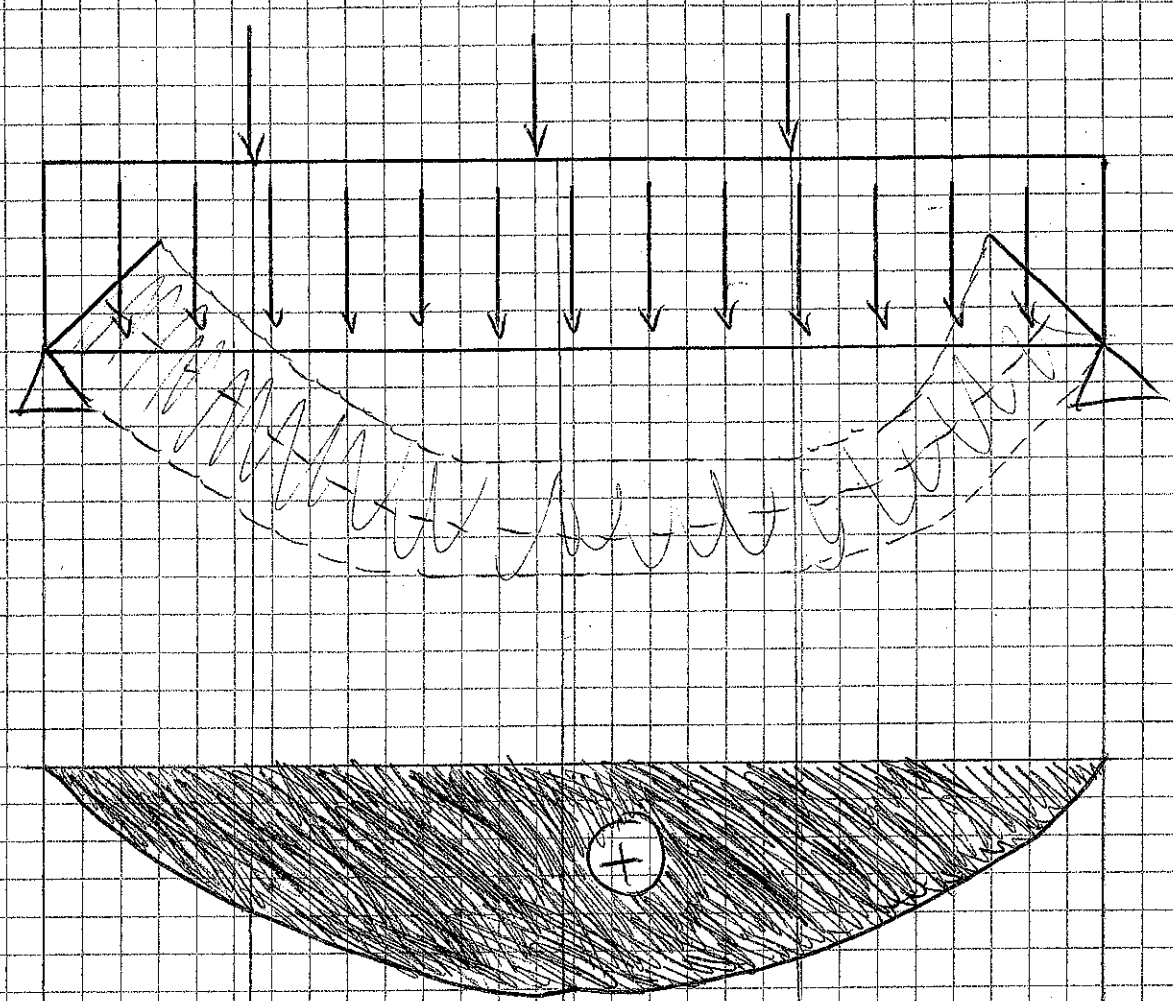
$$\sigma = \frac{N}{A} = \frac{N}{b \cdot h}$$

$$\sigma = +\frac{N}{A} \quad \text{TRAZIONE}$$

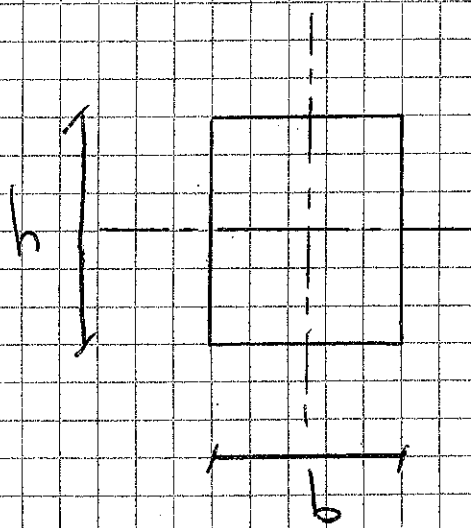
$$\sigma = -\frac{N}{A} \quad \text{COMPRESSION}$$

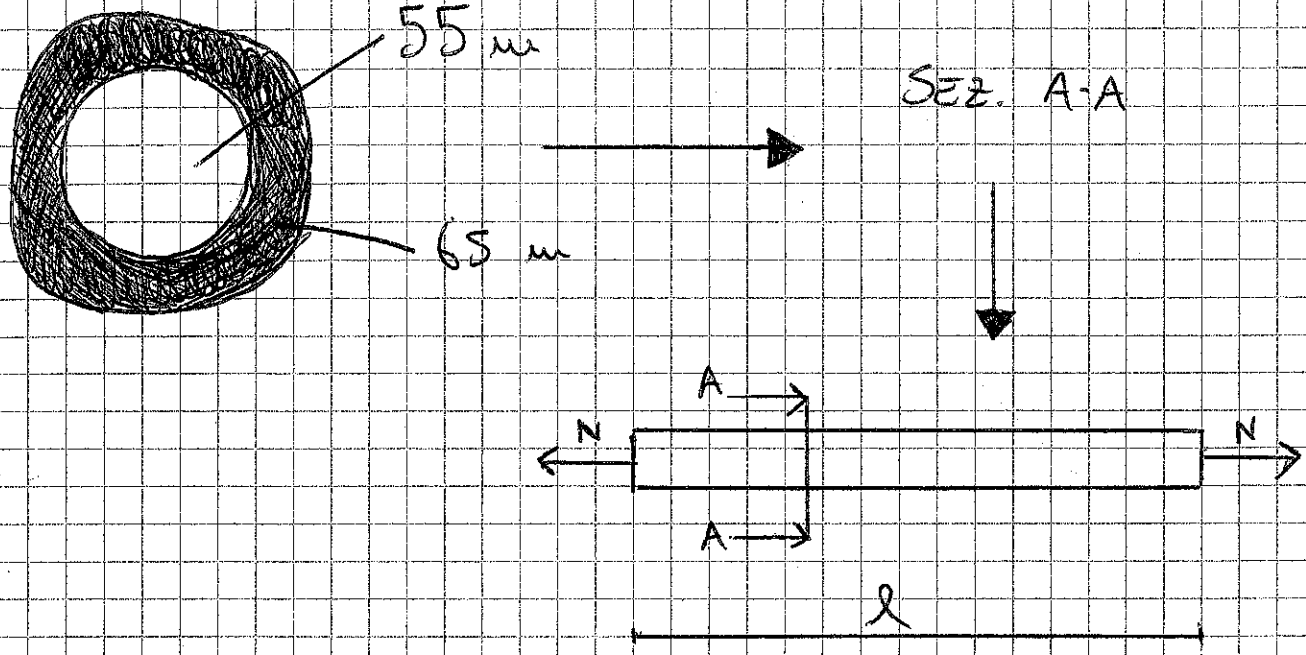
$$\sigma = \frac{N}{A} = \bar{\sigma} \quad \text{AMMISSIBILE}$$

FLESSIONE



$$\sigma = \frac{M}{I} \cdot y\,(MAX)$$





$l = 8$ mm

$N = 50$ kN

$D = 65$ mm

$d = 55$ mm

$$A_{ESTERNA} = \pi \cdot r^2 = \pi \cdot \left(\frac{D}{2}\right)^2 = \frac{\pi \cdot D^2}{4}$$

$$A_{INTERNA} = \frac{\pi \cdot d^2}{4}$$

$$A_{FINALE} = \frac{\pi \cdot 65^2}{4} - \frac{\pi \cdot 55^2}{4} = 3316,62 - 2374,62 = 942 \ mm^2$$

$$\sigma = \frac{N}{A} \left(N/mm^2 \right.$$

LEGGE DI HOOKE

$\sigma = E \cdot \varepsilon$

MKS

m

kg

s

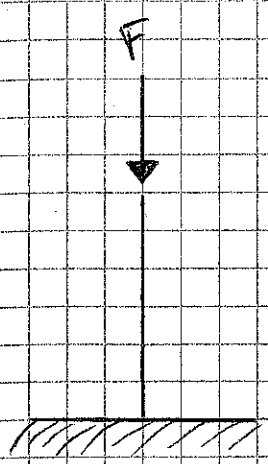
S. I.

mm

N

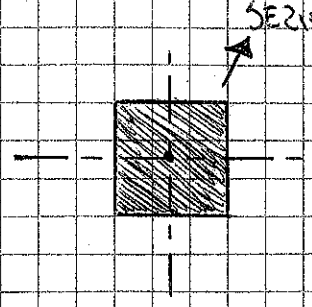
s

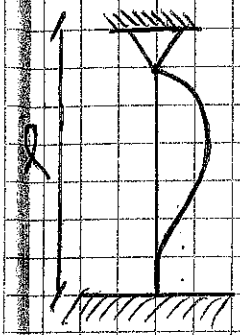
CARICO DI PUNTA



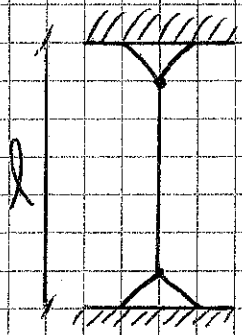
NORMA = 0,7

CORREZIONE = 0,75

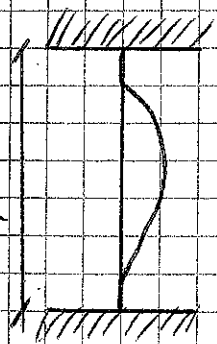




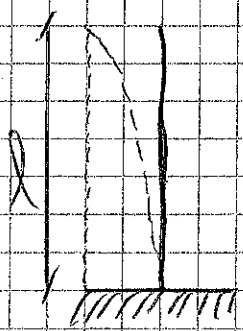
$l_i = 0,7 \cdot l$



$l_i = 1 \cdot l$

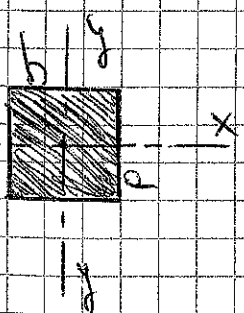


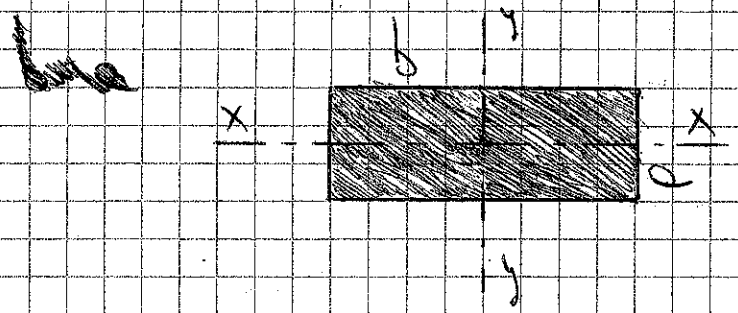
$l_i = 0,5 \cdot l$



$l_i = 2 \cdot l$

$b = a$





$$I_{x-x} = \frac{a^4}{12}$$

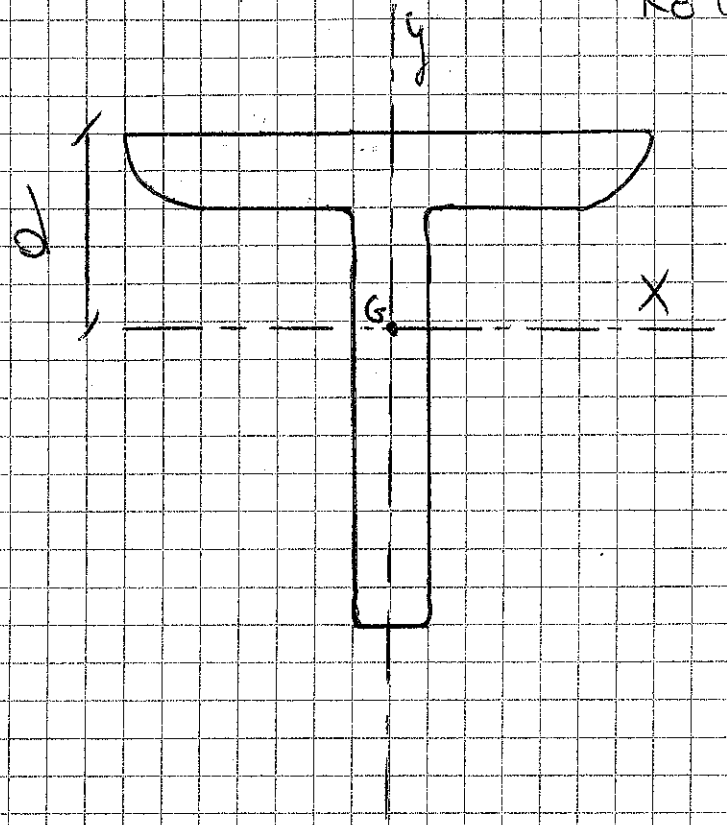
$$I_{x-x} = \frac{b \cdot a^3}{12}$$

$$I_{y-y} = \frac{a \cdot b^3}{12}$$

CALCESTRUZZO $\lambda = 50 \div 100$

LEGNO $\lambda = 50 \div 90$

FLESSIONE SEMPLICE RETTA



$M = -6{,}50 \text{ kN} \cdot \text{m}$

$\sigma = 160 \text{ N/mm}^2$

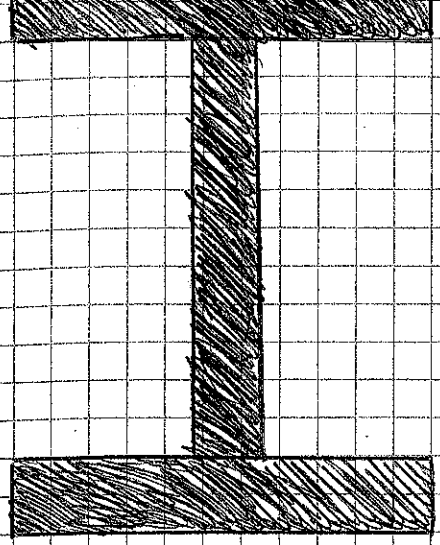
PROFILATO T 120

$$W = \frac{M}{\sigma_s} = \frac{6{,}5 \cdot 10^5}{160 \cdot 10^2} = 40{,}625 \text{ cm}^3$$

PROFILATO T 120 con $W = 42 \text{ cm}^3$; $I = 366 \text{ cm}^4$; $d = 3{,}28 \text{ cm}$

$$\sigma_{M_N} = \frac{M \cdot d}{I} = \frac{6{,}5 \cdot 10^6 \cdot 32{,}8}{366 \cdot 10^4} = 58{,}25 \text{ N/mm}^2$$

$$\sigma_{MAX} = \frac{M \cdot (h - d)}{I} = \frac{6{,}5 \cdot 10^6 \cdot (120 - 32{,}8)}{366 \cdot 10^4} = 154{,}86 \text{ N/mm}^2$$



$l = 4{,}50$ m

$q = 25$ kN/m

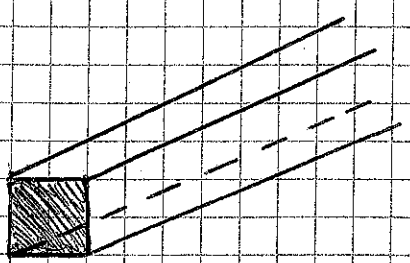
$\sigma = 160$ N/mm²

$M = \frac{1}{8} \cdot 25 \cdot 4{,}50^2 = 63{,}28$ kN/m

$W = \frac{63{,}28 \cdot 10^6}{160} = 395{,}50 \cdot 10^3$ mm³ $= 395{,}50$ cm³

PROFILATO HE 180 B con $W = 426$ cm³ e $J = 3831$ cm⁴

$\sigma = \frac{63{,}28 \cdot 10^6 \cdot \frac{180}{2}}{3831 \cdot 10^4} = 148{,}66$ N/mm²



$M_{MAX} = 9$ kN·m

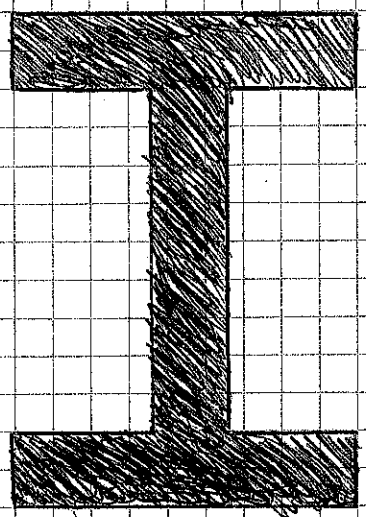
$b = 0{,}7 \cdot h$

$\sigma = 10$ N/mm²

$W = \frac{9 \cdot 10^6}{10} = \frac{1}{6} \cdot 0{,}7 \cdot h^3$

$h = \sqrt[3]{\frac{9 \cdot 10^5 \cdot 6}{0{,}7}} = 194{,}59$ mm $= 200$ mm

$b = 0{,}7 \cdot 200 = 140$ mm



$Hz = 160\ B$

$l = 4{,}00\ m$

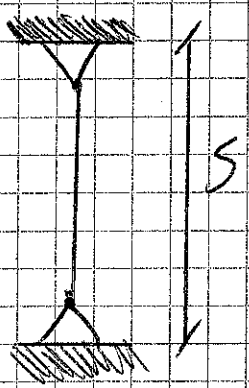
$\sigma = 160\ N/mm^2$

$W = 311\ cm^3$

$P_p = 0{,}426\ kN/m$

$M = \sigma \cdot W = 160 \cdot 311 \cdot 10^3 = 49{,}76 \cdot 10^6\ N \cdot mm = 49{,}76\ kN \cdot m$

$q = \frac{M \cdot 8}{l^2} + P_p = \frac{49{,}76 \cdot 8}{4{,}00^2} - 0{,}426 = 24{,}454\ kN/m$



$A = 16 \times 16\ mm$

LEGNO

$l = 5\ m$

$P = 40\ kN$

$\sigma = 4\ N/mm^2$

$\sigma = W \cdot \frac{P}{A} =$

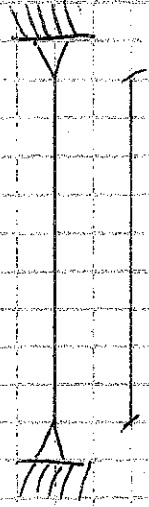
$= 4{,}38 \cdot \frac{40 \cdot 10^3}{256 \cdot 10^2} = 11{,}53\ N/mm^2$

$\lambda = \frac{l_i}{i} = \frac{500}{\sqrt{\frac{I}{A}}} = \frac{500}{4{,}62} = 108{,}22$

$X = \frac{(W_{max.} - W_{min}) \cdot (\lambda_x - \lambda_{min})}{(\lambda_{max.} - \lambda_{min})} = \frac{(7{,}62 - 6{,}30) \cdot (108{,}22 - 100)}{(110 - 100)} = 1{,}08$

$W = 6{,}30 + 1{,}08 = 7{,}38$

(18) $A = \frac{P \cdot W}{\sigma} = \frac{40 \cdot 10^3 \cdot 7{,}38}{4} = \sqrt{1{,}247} = 205{,}35\ mm$



Trave in legno

$A = 16\ 16\ cm$

$l = 5{,}00\ m$

$P = 40\ kN$

$\sigma_{amax} = ?\ N/mm^2$

$$\sigma = \frac{N}{A} \cdot \omega$$

$$i_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{546{,}33\ cm^4}{256\ cm^2}} = 4{,}61\ cm$$

$$\lambda = \frac{l_i}{i_{min}} \Rightarrow \frac{500\ cm}{4{,}61\ cm} = 108{,}45$$

$$A = \frac{N \cdot \omega}{\sigma} = \frac{40 \cdot 10^3\ N \cdot 7{,}41}{7\ N/mm^2} = 42342{,}85\ mm^2 \Rightarrow 205{,}71\ mm$$

$$\lambda = 108{,}45$$

$$X = \frac{(\omega_{max} - \omega_{min}) \cdot (\lambda x - \lambda_{min})}{(\lambda_{max} - \lambda_{min})} =$$

$$= \frac{7{,}62 - 6{,}30) \cdot (108{,}45 - 100)}{(110 - 100)} = 1{,}11$$

$$\lambda_{108{,}45} = 6{,}30 + 1{,}11 = 7{,}41$$

TRAVE IPE 180

$l = 4{,}50$ m

$\sigma = 160$ N/mm²

$P_{max} = ?$

ω = coefficiente moltiplicatore dei carichi

$$\sigma = \frac{P_{max}}{A} \cdot \omega$$

$$\lambda = \frac{l}{i_{min}} \rightarrow \lambda = \frac{450\text{ cm}}{2{,}05\text{ cm}} = 219{,}51$$

$A = 23{,}9$ cm²

$\omega_{219{,}51} = 5{,}98$

$$i_{min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{100{,}85\text{ cm}^4}{23{,}9\text{ cm}^2}} = 2{,}05\text{ cm}$$

$$P_{max} = \frac{\sigma \cdot A}{\omega}$$

$$P_{max} = \frac{160\text{ N/mm}^2 \cdot 2390\text{ mm}^2}{5{,}98} = 63{,}46\text{ KN}$$

INTERPOLAZIONE LINEARE

λ	ω
10	1,02
20	1,05
30	1,15
40	1,20
50	1,25

$\lambda = 36$

$$X = \frac{(\omega_{max} - \omega_{min}) \cdot (\lambda x - \lambda_{min})}{(\lambda_{max} - \lambda_{min})}$$

$$(X) = \frac{(1{,}20 - 1{,}15) \cdot (36 - 30)}{(40 - 30)} = 0{,}03$$

è il valore da aggiungere

$$\omega_{36} = 1{,}15 + 0{,}03 = 1{,}18$$